Brown, Arthur Erwin, Phila.
Curtis, Winterton C., Columbia, Mo.
Dahlgren, Ulric, Princeton, N. J.
Davison, Alvin, Lafayette College, Easton, Pa.
Duerden, J. E., Chapel Hill, N. C.
Evermann, Barton W., Washington, D. C.
Glover, M. Allen, Cambridge, Mass.
Guyer, M. F., Cincinnati, O.
Hall, Robert William, South Bethlehem, Pa.
Herrick, Francis Hobart, Cleveland, O.
Hunter, Walter David, Cosmos Club, Washington, D. C.
Jenkins, O. P., Stanford University.
Jones, Lynds, Oberlin, O.
Knower, H. McE., Baltimore, Md.
Linton, Edwin, Washington, Pa.
McGregor, James Howard, New York City.
Mead, A. D., Providence, R. I.
Nachtrieb, H. F., Minneapolis, Minn.
Neal, H. V., Galesburg, Ill.
Osgood, W. H., Washington, D. C.
Rand, Herbert Wilbur, Cambridge, Mass.
Rankin, Walter M., Princeton University, Princeton, N. J.
Raymond, Pearl, Ann Arbor, Mich.
Rice, E. L., Delaware, O.
Torrey, Harry Beal, Berkeley, Calif.
Weysse, Arthur W., Boston, Mass.
Wilder, Harris Hawthorne, Northampton, Mass.
Zeleny, Charles, Chicago, Ill.

Section G:

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Banker, Howard J., Greencastle, Ind.
Berry, Edward W., Passaic, N. J.
Blodgett, Frederick H., College Park, Md.
Burrill, Thomas J., Urbana, Ill.
Cannon, W. A., Tucson, Ariz.
Coker, Wm. C., Chapel Hill, N. C.
Coulter, S. M., St. Louis, Mo.
Duval, Joseph W., Washington, D. C.
Ferguson, A. McG.
Fitzpatrick, Iowa City, Iowa.
Holferty, George M., St. Louis, Mo.
Jeffrey, E. C., Cambridge, Mass.
Kirkwood, Jos. E., Syracuse, N. Y.
Piper, C. V., Washington, D. C.
Pond, Raymond H., 87 Lake St., Chicago, Ill.
Rose, J. N., Washington, D. C.
Shull, G. H., Cold Spring Harbor.
Spillman, Wm. Jasper, Washington, D. C.
Thornber, J. J., Tucson, Ariz.
Wylie, R. B., Sioux City, Iowa.

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Baird, John Wallace, Baltimore, Md.
Churchill, William, New Haven, Conn.
Dellenbaugh, Century Club, New York City.
Fracker, George Cutler, New York City.
Haines, Thomas Harvey, Columbus, O.
Jones, Adam Leroy, New York City.
Kirkpatrick, E. A., Fitchburg, Mass.
Messenger, James Franklin, Winona, Minn.
Spaulding, Edward G., New York City.
Witmer, Lightner, Philadelphia.
Woodbridge, Frederick J. E., New York City.

Section I:

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Clark, Judson F., Montreal, Canada.
Du Bois, William E. B., Atlanta, Ga.
Edmonds, Richard H., Baltimore, Md.
Foote, Allen Ripley, Home Ins. Bldg., Chicago.
MacVannel, John Angus, New York City.
Stoke, Alfred Holt, Greenville, Miss.
Stokes, Anson Phelps, New York City.

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Abbott, Alexander C., University of Pennsylvania.
Burton-Opitz, Russell, New York City.
Dexter, E. G., Urbana, Ill.
Flexner, Simon, Rockefeller Institute, New York City.
Lindley, Ernest H., University of Indiana, Bloomington, Ind.
Loeb, Leo, University of Pennsylvania.
Meyer, Adolf, New York City.
Smith, Allen J., University of Pennsylvania.
Yerkes, Robert Mearns, Cambridge, Mass.

SCIENTIFIC BOOKS.

Elements of the Differential and Integral Calculus. By W. A. Granville. Boston, Ginn and Company. Pp. xiv + 463.

A characteristic feature of mathematics in the last half century is the increasing attention paid to the foundations and rigorous development of this science. In analysis this movement began with Gauss, Cauchy and Abel in the early years of the nineteenth century and found its greatest exponent in Weierstrass. The movement thus begun has been continued by such men as Riemann, Dedekind, Hankel, Cantor, Jordan, Dini, Stolz, Harnack, Peano and a host of younger men.

As a result of these investigations it was found that much of the reasoning hitherto employed and in current use among mathematicians was either worthless or required to be modified, restricted or completed. It thus became necessary to rewrite textbooks on analysis or to prepare new ones more in harmony with the new teachings. In this way arose the new edition of Jordan's 'Cours d'Analyse' and Harnack's edition of Serret's 'Calcul,' as well as the new works of Stolz, 'Allgemeine Arithmetik,' and 'Grundzüge'; Tannery, 'Théorie des fonctions d'une variable'; Dini, 'Fondamenti per la teorica delle funzioni di variabili reali.'

In England and America more progressive teachers have felt for some time the need of a modern text-book on the calculus, which is at once rigorous and elementary. The task of writing such a work is not easy. On the one hand, it is necessary to avoid the worthless and even vicious forms of reasoning which mar so many elementary treatises and which are simply intolerable to one educated according to modern standards of rigor. On the other hand, the author must not introduce subtilties of reasoning and logical refinements beyond the needs and comprehension of those who are to use the book.

The volume under review is an attempt to solve this difficult problem. To our mind the efforts of its author have been abundantly crowned with success. In perusing Dr. Granville's book one feels throughout that the author has in mind the requirements of modern rigor. The demonstrations, it is true, often rest on intuition; but this is necessary in a first course, as all will admit. They are, however, usually correct as far as they go, and free from the defects we have mentioned above. We believe the present volume is eminently a safe book to put in the hands of the beginner. He will get no false notions which afterwards will have to be eradicated, with much difficulty; he will, on the other hand, acquire a considerable acquaintance with the principles of the calculus and a good working knowledge of its methods.

We make now a number of criticisms and suggestions.

The definition of limit given in § 29 is not the one given by Cauchy and Weierstrass and now universally accepted. Looked at carefully, we see it supposes that all variables are functions of an auxiliary variable, the time. This leads to unnecessary complications in the definition of the limit of a function in § 32. We believe the strict Weierstrassian definition should be given and used. As an aid to comprehension, the author's notions in these articles might prove useful. In § 34 the notion of a graph is explained; but not with sufficient care, to our mind. How is the reader to know from their graphs that x and log x are continuous functions? The three properties of the exponential function given in this article result from their arithmetical properties and not from their graph, as the author seems to imply.

The definition of the derivative given in § 41 is not satisfactory; what the author really defines is the differential coefficient at a point. It is their aggregate that forms the derivative.

In § 55 the author has avoided an error which is very prevalent. His passage to the limit is, however, not completely justified. He has yet to show that

$$\lim_{\Delta x=0} \frac{\Delta y}{\Delta v} \cdot \frac{\Delta v}{\Delta x} = \lim_{\Delta v=0} \frac{\Delta y}{\Delta v} \cdot \lim_{\Delta x=0} \frac{\Delta v}{\Delta x}.$$

The demonstration in § 56 should, it seems to us, be replaced by a simpler one. The author obtains the equation

$$1 = \frac{dy}{dx} \cdot \frac{dx}{dy} \qquad (1)$$

and then remarks: if $dx/dy \neq 0$, we have

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

He should see that there can be no need of making the further assumption, $dx/dy \neq 0$: for if it were, the equation (1) could not exist.

In § 133 the author introduces a double limit without any explanation. As such limits are used in connection with double integrals, § 231, *seq.*, they should be explained with care. The footnote on page 194 is unintelligible to us and certainly will give rise to misapprehension.

The theory of total differentiation does not meet our approval at all. The author has treated the subject from the standpoint that the variables $x_1, x_2, \cdots x_n$ are all functions of some one variable. Instead of true total differentials, he gets *total derivatives*. The *du* in § 137 are not *total* differentials, but differentials of functions of one variable. In the differentiation of implicit functions the author assumes merely the existence of the partial derivatives. He should assume also their continuity. The form of demonstration is bad, as it requires him to assume (tacitly) the existence of the very thing he is seeking, viz., dy/dx.

In the treatment of envelopes, § 141, the author does not as usual give sufficient conditions for the validity of his reasoning, but contents himself with the vague statement in a footnote that the process is all right 'in all applications made in this book.' This blemish, which a few lines will remedy, should be removed in another edition. The definition of an infinite series given in § 147 is not felicitous. In avoiding the lax definition usually given the author has gone to the opposite extreme. The simplest way seems to be to consider

$$a_1 + a_2 + a_3 + , \dots \text{in inf.}$$

as a symbol to which a meaning is attached as to other symbols, as $> < =$, etc. The solution of Ex. 3, § 152, is not quite rigorous, as it postulates the covergence of G. In § 160 *undefined* arithmetical operations are performed on series.

We can not agree with the author that the remainder in Taylor's series for several variables is too complicated to be given. The treatment of maxima and minima can be made much more complete without complications or difficulty. The reasoning given at the bottom of page 248 can be made not only 'plausible,' but entirely conclusive, using no more space that that required by the author.

In the reduction of indefinite integrals the author proves the trivial formulæ

$$\int (du + dv - dw) = \int du + \int dv - \int dw,$$

$$\int a dv = a \int dv,$$

but omits entirely the demonstration relative to the transformation of the variable. This is all the more surprising as this transformation is constantly employed, even in establishing important theorems. Two chapters, XXIX. and XXX., are devoted to definite integrals. In the first we arrive at the notion of a definite integral by means of the notion of area; in the second, by means of the limit of a sum. In our opinion the first treatment is not only superfluous, but should be entirely omitted on several counts.

The relatively few blemishes in this work, the reviewer is glad to state, will be removed in the next edition. James Pierpont.

Yale University.

The Study of the Atom, or the Foundations of Chemistry. By F. P. Venable. Easton, Pa., The Chemical Publishing Co. Pp. 290.

The history of an important scientific theory is an interesting study, where it is possible, as it often is, to trace the orderly development of that theory from stage to stage. The evolution of the atomic theory is a subject which has claimed the attention of many writers, and the story has been told so often and so well in works on the history of chemistry, that one wonders whether it is not familiar to most chemists. A careful perusal of this book does not disclose any new point of view, or anything new in the method of treatment, though the matter is generally presented in a satisfactory manner, especially Chapter V., which deals with the periodic system. In the last chapter of the book the author considers the most recent hypotheses regarding the constitution of matter by J. J. Thomson, Rutherford and others. The book is generally clear, conservative in tone and, on the whole, well-proportioned, though 75 pages, or one fourth of the contents, seems rather too much to devote to the conception of the atom before the time of Dalton, especially as this material must be taken entirely from secondary sources. The book may be commended as a good summary for students. E. T. Allen.

SOCIETIES AND ACADEMIES.

NEW YORK ACADEMY OF SCIENCES. SECTION OF GEOLOGY AND MINERALOGY.

The section was called to order at 8:15 p.m., November 21, 1904, with Vice-president Kemp in the chair and forty persons present.