*ON USES OF A DRAWING BOARD AND SCALES IN TRIGONOMETRY AND NAVIGATION.**

It may seem a little strange that any one should think it worth while to call special attention to a drawing board and scales as a means of solving spherical triangles and a few somewhat similar problems. For, accurate results can be obtained through simple computations and rough results by aid of suitable diagrams composed chiefly of curve systems. Such, for instance, are many cartographic projections of the great and small circles of a hemisphere. But where results reliable to about 5′ of arc or angle are required, and where computation is to be altogether dispensed with, it seems to me that the methods about to be described certainly possess merits which have not heretofore been fully recognized.

Suppose the dimensions of the drawing board to be about 22 by 40 inches. Let it be trimmed, as it were, with a metallic border three margins of which are divided into degrees and fractions of degrees so as to form a large rectangular protractor, as sketched in Fig. 1. The border of the fourth side may be graduated uniformly from its center, where is situated a pivot or pin about which the scales may revolve.

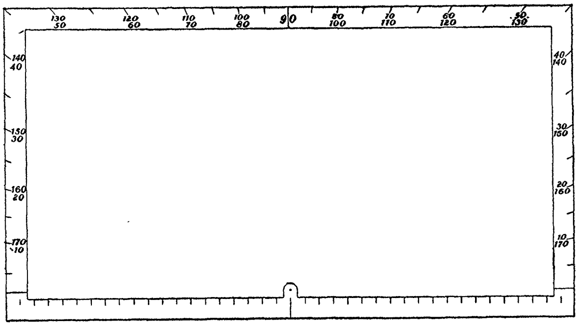

Fig. 1.

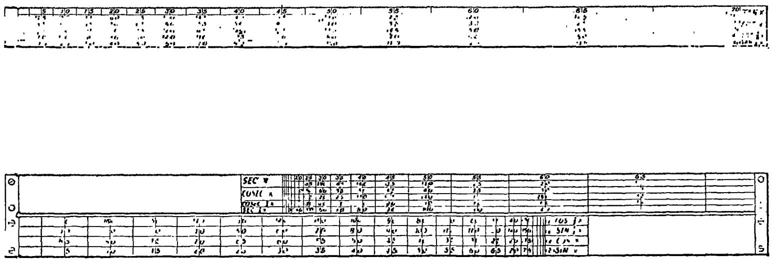

Fig. 2.

The scales to be used in the solution of spherical triangles are scales of sines, cosines, tangents, etc., like those shown in Fig. 2, but having, of course, much finer graduations along the edges. For use upon a 20-by-40-inch board, the extreme length of the scales should be about 30 inches. For increasing the size of the divisions, we shall suppose sines and cosines to have been multiplied by 2 in constructing the scales. In addition to trigonometrical scales it is supposed that we have several

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uniformly graduated scales, which are especially useful in problems involving plane trigonometry, and a set of scales of meridional parts for various latitudes, each scale representing, say, 10 degrees of latitude. In all cases the scales must be straight and beveled on their edges. It is supposed that we have also a T-square with a uniformly graduated blade.

RIGHT SPHERICAL TRIANGLES.

By means of the board and scales we can find such products as cot $b \times \tan c$ by laying off cot b, according to the scale labeled cot x, along the base line of the protractor. Let a straight edge, turning about the

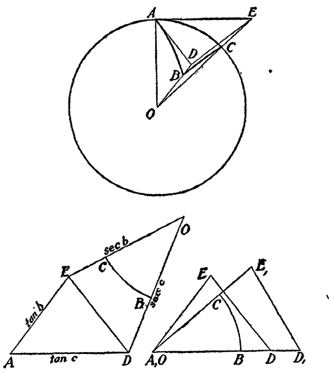

FIG. 3.

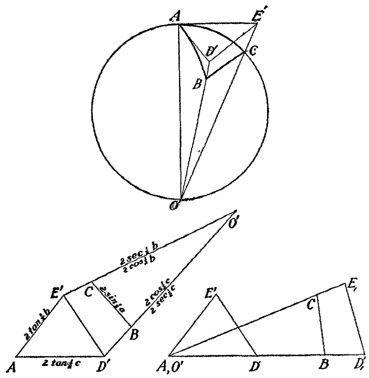

FIG. 4.

pivot, be directed toward value c on the margin. The T-square shows the line extending from the point where cot b was laid off to the straight edge. The distance is the product cot $b \times \tan c$. By reading this distance on a scale cos x, or double this distance on a scale 2 cos x we obtain a certain angle which is the value of the angle A of a triangle right-angled at B. Napier's rules enable one to see at a glance what product is required and how it is to be read. Where tangents and cotangents are involved, the application of this method is, of course, somewhat restricted on account of the length of the scales.

SINE RATIOS IN SPHERICAL TRIANGLES.

If two of the three given parts of a triangle are opposites, the unknown part opposite the third given part becomes known through the equality of the sine ratios; viz.,

$$\sin A : \sin a = \sin B : \sin b = \sin C : \sin c,$$

or

$$\sin A \sin b = \sin B \sin a, \text{ etc.}$$

The process of mechanically solving such problems can be illustrated by taking a particular case or problem; given A, a, B to find b.

Find a on the scale labeled 2 sin x and direct the scale (pivoted to the board) towards B as found on the margin of the board; this locates a certain point. Next direct the scale towards A. Find on the scale as now directed a second point whose altitude is the same as that of the first. This is done by sliding the graduated T-square. The reading of the second point on the scale labeled 2 sin x (still pivoted to the board) is the required side b.

RELATION BETWEEN THE THREE SIDES AND ONE ANGLE.

In treating the problem—given the three sides to find an angle, or given two sides and the included angle to find the remaining side—it is important to consider two methods, or constructions, analogous, but differing somewhat from each other. The first may be referred to as the gnomonic method, and the second as the sterographic.

Let *ABC*, Fig. 3, be the spherical triangle. A plane tangent to the sphere at *A* contains the lines *AD*, *AE*, whose lengths are tan *c*, tan *b*. Imagine the triangle *ODE* revolved about *DE* into the plane *ADE*. In the plane quadrilateral *ADOEA* thus obtained, $OD = \sec c$, $OE = \sec b$, $OB = OC = 1$; also arc $BC = a$, which measures the angle *O*.

Assume now that we have scales of tangents and secants. The quadrilateral constructed by aid of such scales and a protractor gives the angle *A* when *a, b, c* are the known parts, or *a* when *A, b, c* are known. In practice the quadrilateral is not actually constructed; but the work of finding the required unknown part of the triangle is arranged in accordance with the diagram in Fig. 3, which shows *A* and *O* as coinciding. More particulars about this arrangement may be gathered from the example.

Given the two sides *b, c* of a spherical triangle and the included angle *A*, find the side *a* by means of the above-described apparatus.

Lay off with the scale of tangents when pivoted to the drawing board a distance along the initial direction which reads *c;* this fixes on the board the point *D*. Lay off towards *A*, as found on the margin of the board, a distance which reads *b* on the tangent scale; this fixes on the board the point *E*. On beam compasses set the distance *DE*, or mark it off upon a strip of paper. Next remove the scale of tangents and pivot to the board the scale of secants. Lay off on the secant scale, and beyond *D*, a distance which reads *c*; this fixes the point D_1. With the beam compasses centered at D_1, describe an arc. Lay off on the secant scale, still pivoted to the board (and revolving about *O*) a distance which reads *b* and note where it intersects the arc just drawn; this fixes the point E_1. The angle read off on the margin of the board is the side *a*.

The same construction taken in a slightly different order serves for finding *A* where *a, b, c* are the known parts.

We now take up the stereographic method. The problems to be considered are the same as those to which the gnomonic method applies. The advantage of the stereographic method lies in the fact that $\tan \frac{1}{2} x$ does not approach infinity until *x* approaches 180°.

A plane tangent to the sphere at *A*, Fig. 4, contains the lines *AD′*, *AE′* whose lengths are $2 \tan \frac{1}{2} c$, $2 \tan \frac{1}{2} b$. Imagine the triangle *O′D′E′* revolved about *D′E′* into the plane *AD′E′*. In the plane quadrilateral *AD′O′E′A* thus obtained, $O'D' = 2 \sec \frac{1}{2} c$, $O'E' = 2 \sec \frac{1}{2} b$, $O'B = 2 \cos \frac{1}{2} c$, $O'C = 2 \cos \frac{1}{2} b$ and line $BC = 2 \sin \frac{1}{2} a$.

With suitable scales and a protractor the quadrilateral *AD′O′E′A* could be constructed and the required part of the spherical triangle could be thus determined; but the more practical arrangement is that shown in the figure where *O′* is made to fall upon *A*. Moreover, it is convenient to omit the factor 2 before tangents and secants.

Given *b, c, A* to find *a* by means of the above-described apparatus.

Lay off by means of the scale labeled $\tan \frac{1}{2} x$, pivoted to the drawing board, a distance along the initial direction which reads *c*; this fixes a point *D′*. Lay off on this scale now directed towards *A*, as found on the margin of the board, a distance which reads *b*; this fixes on the board a point *E′*. On beam compasses set the distance *D′E′*, or mark it off upon a strip of paper. Next remove the $\tan \frac{1}{2} x$ scale and pivot to the board the double scale shown in Fig. 2. Lay off with the scale labeled sec $\frac{1}{2}$ x, and beyond D', a distance which reads c; this fixes a point D_1'. With the beam compasses centered at D_1', describe an arc. Lay off on the sec $\frac{1}{2}$ x scale, still pivoted to the board (and revolving about O'), a distance which reads b and note where it intersects the arc just drawn; this fixes a point E_1'. Along the directions AD_1' and AE_1' locate the points B, C by laying off distances which read c and b upon the scale labeled 2 cos $\frac{1}{2}$ x. The reading of the distance BC upon the scale labeled 2 sin $\frac{1}{2}$ x is the value of the side a.

The same construction taken in a slightly different order serves for finding A when a, b, c are the known parts.

In passing, it may be well to note that plane trigonometry applied to the triangles ADE and OED gives

$$\overline{DE}^2 = \overline{AD}^2 + \overline{AE}^2 - 2\overline{AD}\,\overline{AE}\cos A$$

$$= \overline{OD}^2 + \overline{OE}^2 - 2\overline{OD}\,\overline{OE}\cos a$$

$$\therefore \tan^2 c + \tan^2 b - 2\tan c \tan b \cos A = \sec^2 c + \sec^2 b - 2\sec c \sec b \cos a.$$

Noting that $\sec^2 = 1 + \tan^2$ and multiplying through by $\cos b \cos c$, we have, after transposing,

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

The same equation follows from the second method by noting that

$$\overline{D'E'}^2 = \overline{AD'}^2 + \overline{AE'}^2 - 2\overline{AD'}\,\overline{AE'}\cos A\,;$$

and

$$\overline{D'E'} : \overline{BC} = \sec \tfrac{1}{2} c : \cos \tfrac{1}{2} b$$

where $\overline{BC} = 2 \sin \frac{1}{2} a$, and later on that

$$2\sin^2 \tfrac{1}{2} a = 1 - \cos a,\ 2\cos^2 \tfrac{1}{2} a = 1 + \cos a.$$

POLAR TRIANGLES.

Cases analogous to any of the above, but having sides and angles interchanged throughout, can be solved by the foregoing methods provided we first subtract all known parts from 180°, interchanging capital and small letters, and then after having solved this polar triangle, subtract the parts from 180°, and finally interchange the capital and small letters.

APPLICATIONS.

The methods already described may be used to advantage in some classes of plane-table work. To obtain the azimuth of the sun by one mechanical solution of the triangle, it is necessary that the telescope of the alidade be supplied with a vertical circle. The direction of the sun could be ascertained by means of a good watch, but the spherical triangle would then have to be solved for two of its parts.

The azimuth and hour angle of the sun or other heavenly body can be obtained from an observed altitude with sufficient accuracy for enabling one to lay down a Sumner line at the assumed or dead-reckoning latitude, and for ascertaining the variation of the magnetic needle at sea.

Tables of sunrise and sunset can be computed with great facility by means of the stereographic method. In fact, the true zenith distance (BC) of the rising or setting body is a constant for all latitudes and dates. The distance between pole and zenith at any particular latitude is constant for all dates. (That is, D', B and D_1' are fixed points for a given latitude, and about B, the zenith, a circle can be described with 2 sin $\frac{1}{2}$ a as radius.) Since the polar distance of sun or moon never exceeds 120°, scales of moderate length will suffice for all possible cases.

Consider now the question of great circle sailing. The distance and initial course between two points specified by their latitudes and longitudes require the solution for two parts of a triangle whose given parts are two sides and the included angle. The longitude and latitude of the vertex involve the solution of a right-angled triangle for two of its parts.

Since it seems to be almost certain that a proposed great circle track will in reality be sailed as a series of rhumb lines each terminating in or near the great circle, the methods of Mercator sailing will still be found useful. By aid of a set of meridional scales, problems in Mercator sailing can be worked with great facility. For, the board, the uniformly divided scales, and the T-square constitute an ordinary traverse table, and the departure is readily converted into difference of longitude through the equation

$$\text{Diff. long.} = \frac{\text{merid. diff. lat.}}{\text{true diff. lat.}} \times \text{departure.}$$

To do this, suppose the meridional difference of latitude to be laid off upon a uniform scale rotating about the pin or pivot. Let the true difference be laid off along, or parallel to, the initial line. Rotate the former scale until the T-square indicates that the point representing meridional difference is directly above that representing true difference. Now slide the T-square along until a point in the initial line is reached which denotes the value of the departure. The reading of the rotated scale directly above this point is the difference of longitude.

In conclusion, it should be said that the aim has been to use the drawing board proper merely as a surface upon which to locate points or lines temporarily, the accuracy of the work depending upon the fact that the scales and border of the board are not subject to any considerable atmospheric or temperature changes. R. A. Harris.

SCIENTIFIC BOOKS.

Zoology: Descriptive and Practical. By Buel P. Colton, A.M. Boston, D. C. Heath & Co. 1903. Part I., Descriptive, pp. x + 375. Part II., Practical, pp. xvii + 204.

Colton's 'Practical Zoology,' which was published seventeen years ago, did excellent pioneer work as a laboratory guide for secondary schools. This useful hand-book, revised and amplified, now appears in connection with an excellent descriptive zoology.

In the latter the author introduces each of the larger groups of animals by a description of a typical example, treating of its morphological and physiological characteristics and paying especial attention to its habitat, movements, senses, capture of prey, taking of food and manner of self defense.

Naturally, Arthropods, and particularly Insects, have a prominent place at the beginning, followed by a brief account of the Annulata, a somewhat longer description of the Mollusca and an extended discussion of the Chordata. Thereupon the Protozoa, Porifera, Cœlenterata, Echinodermata, Platyhelminthes, Trochelminthes and Molluscoidea are taken up in the order given. This is an excellent practical arrangement on the whole, though it might have been still better to have placed the Annulata and Echinodermata last and thus have preserved the ascending order throughout each of the two sections, for the sake of avoiding those misconceptions which are wont to arise in the mind of the beginner, to whom position in a text-book has a profound significance.

The strongest feature of the book is its broad treatment of animal life, in other words, its natural history. The author has a keen sense of what is interesting. His style is simple and direct, and the book is thoroughly readable.

The author did not cease to do pioneer work when he published his 'Practical Zoology' seventeen years ago. In the present book he makes free use of 'tho,' 'thru,' 'thoro' and their various compounds, while 'celom,' 'cecum,' 'hemal' and a few other words have been stripped of superfluous letters. He does not attempt to set right names like Amœba, which are apparently protected by their Latin form, but one is surprised that 'cœlenterates,' 'diaphragm' and a few other terms should not have been pruned. Spelling reform has much in its favor, and it must be introduced