historian, Moritz Cantor, expressed the opinion that the use of zero was probably due to the Babylonians 1700 B.C. However, it has not been definitely established that zero was in use any earlier than 400 A.D. About this time it was used in India, and several centuries later the Arabs began to employ it. Through the Arabs its use became known to Europeans during the twelfth century. It was not generally adopted in Europe until several centuries later, notwithstanding its great advantages. For a considerable time there were two parties among the European educators—one party, known as the algorists, favored the adoption of the Hindu system of notation (falsely called Arabic) with its position values, while the other, known as the abacists, favored the Roman notation without zero or position value.

The general adoption of the Hindu system was greatly facilitated by the facts that it was explained in most of the calendars for more than a century beginning with 1300, and that the medieval universities frequently offered courses devoted to the use of this notation. With the opening of the medieval universities we approach some of the fundamental discoveries in more modern mathematics. As we considered these on a similar occasion,* we shall merely add a few thoughts on the concept of dimensions which are due to Plücker.

The idea of more than three dimensions can be partially explained in a very simple manner. If the total number of points on a straight line is denoted by ∞ (the symbol for infinity), it is clear that there are ∞^2 points in a plane, since through each point of the given line we may draw a line at right angles to this line. Each of these ∞^2 points of the plane may be taken as the center of an infinite number of circles, and all the circles which have one point as center are distinct from those which have any other point as center. Hence there are ∞^3 circles in a plane, while there are only ∞^2 points in it.

* SCIENCE, Vol. XL. (1900), p. 528.

We arrive at the same result by observing that an infinite number of lines may be drawn through each point of a plane and that each of these lines is tangent to an infinite number of circles going through this point. Hence ∞^2 circles pass through each point of a plane and lie entirely in the plane. As the number of points on a circle is infinite, the number of circles is obtained by multiplying the number of points by ∞. Hence we say that the plane is two-dimensional when the point is considered as the element, but it is three-dimensional if the circle is considered as element. If the ellipse were taken as element it could be readily shown that the plane would be five-dimensional.

Similarly space is three-dimensional if the point is taken as element but it is four-dimensional if the sphere is taken as element. Since there are ∞^6 pairs of points in space and ∞^2 pairs of points on a line there are ∞^4 lines in space, that is there is a 1, 1 correspondence between the lines and spheres of space. This is frequently expressed by saying there are just as many spheres in our space as there are lines, while the number of each of these is infinitely larger than the number of points. From this standpoint there is no limit to the number of dimensions of ordinary space.

G. A. MILLER.

SCIENTIFIC BOOKS.

The Yucceæ. By WILLIAM TRELEASE. From the Thirteenth Annual Report of the Missouri Botanical Garden. Issued July 30, 1902. St. Louis, Mo. Published by the Board of Trustees. 1902. 8vo. Pp. 107.

The Spanish bayonets are shrubby or tree-like plants, principally of the genus *Yucca*, and represented in gardens by short-stemmed species bearing evergreen, erect, sharp-pointed leaves. On the Great Plains a common species is known as the 'Dagger-weed.' In the southwest some of the species attain to the dimensions of trees, as *Yucca australis* and *Y. valida,* and are known as 'bear-grass,' 'palma loca,' 'izote,' etc. Botanically they are closely related to the lilies, and in fact are classed as members of the Lily family, of the tribe *Dracænoideæ,* and subtribe *Yucceæ.* All are natives of North America (including Mexico) and Central America.

In this paper the author describes thirty-four species and forty-five varieties and 'forms.' These are distributed quite unequally among five genera, as follows: *Hesperaloe,* two species, and one variety; *Hesperoyucca,* one species; *Clistoyucca,* one species; *Yucca,* twenty-eight species, and forty-five varieties and 'forms'; *Samuela,* two species. The species of *Hesperaloe* occur in Texas and Mexico, and have narrowed flowers, in contrast with the remaining genera, in which the flowers are broad. The single species of *Hesperoyucca* occurs in California, and may be recognized by its filiform style. In *Clistoyucca,* in which the style is wanting, we find a single branching arborescent species, which attains a height of twenty to twenty-five feet or more, and a stem diameter of nearly two feet. It occurs in the Mohave Desert of California, northwestern Arizona and southwestern Utah, where it is known as the 'Joshua tree.' The numerous species, varieties and 'forms' of *Yucca* are widely distributed, extending from South Dakota southward to central Mexico, and from the Atlantic Ocean to the Pacific. Species occur also in Central America, the Bermuda Islands and the eastern Antilles. The genus is distinguished by the polyphyllous flowers and short style. The plants range from acaulescent, as in *Yucca filamentosa* and *Y. flaccida,* to arborescent, as in *Y. australis* and *Y. valida,* which may attain a height of twenty-five to thirty feet. *Samuela* is a new genus erected by the author to include the species with tubular, gamophyllous flowers. Its two arborescent species are natives of Texas and northern Mexico.

One outcome of the studies on which this monograph is based is the conclusion that most of the Spanish bayonets grown in gardens under the old name of *Yucca filamentosa* are not of that species, but are varieties of the allied species, *Yucca flaccida.* The two species may be distinguished by the more rigid leaves, which bear coarse, curly threads in *Y. filamentosa,* and the more flexible leaves, bearing finer, straighter threads, in *Y. flaccida.*

The yuccas are of some value economically. All possess very fibrous leaves, and it is said that 'local use is made of the fiber almost everywhere that the plants grow.' The trunks of the larger species are locally used in the building of houses, palisades, etc., and the leaves are used for thatching. On account of their saponifying properties the stems and rootstocks of some species are used as a substitute for soap, and the species so used bear the local names 'amole,' 'soapweed,' 'soap plant,' etc. Apparently some use is made of this saponifying constituent in the manufacture of certain proprietary soaps and detergent compounds. The flowers and young leaves of many species are greedily eaten by cattle. In the Nebraska sandhills the present writer has seen many examples of plants which had been broken down and their young leaves eaten by the hungry cattle, and in these regions it is very difficult to find complete flower panicles, on account of the greediness of the cattle in eating the succulent flowers. In Mexico the flower clusters of *Samuela carnerosana* are gathered for feeding to sheep and other domestic animals, and it is the practice of the inhabitants to split open the thick trunks of this species in order that the succulent interior portions may be eaten by stock. The fruits of the baccate species are eaten by the natives, as are the young flower buds of some species when roasted or boiled. The seeds are ground and used as meal or boiled into a mush for human food, in some localities. Lastly, attention may be called to the ornamental value of many species, and for this purpose they are largely employed, especially in gardens and parks of considerable extent. They are not adapted to small grounds, where their sharp-pointed leaves are quite annoying.

It is not necessary to refer at length to the well-known dependence of the yuccas upon certain insects for the deposition of the pollen on the stigmas of the flowers, since that has been so frequently described by many observers. Such dependence seems to be general throughout the group. Even in *Samuela*, with its oddly narrowed, tubular perianth, the common yucca-moth (*Pronuba yuccasella*) is shown to be the agent in pollination.

The monograph is richly illustrated by eighty-eight plates of plants and their fruits and seeds, besides twenty-four maps showing the distribution of the species. American botany is to be congratulated on the publication of this admirable monograph.

CHARLES E. BESSEY.
UNIVERSITY OF NEBRASKA.

A List of North American Lepidoptera, and Key to the Literature of this Order of Insects. By HARRISON G. DYAR. Bulletin 52, U. S. National Museum. 1902 [February, 1903]. Pp. 723.

For many years the guide and companion of the European lepidopterist has been Staudinger's 'Catalogue of the Lepidoptera of the Palæarctic Faunal-region.' The veteran author of that work has now died, leaving us a new edition, prepared in conjunction with Dr. H. Rebel. In America we have had nothing equivalent to Staudinger's catalogue, although Dr. J. B. Smith's useful check-list of 1891 served to indicate the names and classification of the species. At last, however, Dr. Dyar has given us a detailed catalogue, including full references to literature and brief indications of localities. In preparing this work, Dr. Dyar has been assisted by Dr. C. H. Fernald, Rev. Geo. D. Hulst and Mr. August Busck, as is carefully acknowledged on the title-page; he has also utilized previous lists, so far as they proved serviceable. The literature of the subject has been searched with extraordinary care, and full advantage has been taken of the most recent advances in our knowledge of the classification of the Lepidoptera, many of them due to Dr. Dyar himself. While there are of course a few errors in copying or printing, these are extremely few, and the work as a whole is exceedingly well done. If any of us are inclined to regret that a man like Dr. Dyar, one of the most original and gifted investigators in America, should spend his time in preparing a catalogue, we may console ourselves by recollecting the character of some other catalogues, prepared by men of less ability. In truth, the thing was well worth while, and its value to students of American lepidoptera can hardly be overestimated.

The Staudinger and Rebel catalogue for the Palæarctic Region, published in 1901, includes the names of nearly 4,800 species. Dyar's list (including 44 interpolated since it was made up) includes 6,666 species, occurring in America north of the Mexican boundary and the West Indies. This is not precisely equivalent to the Nearctic region, as it excludes the tableland of Mexico, and includes certain Neotropical elements represented in southern Florida. In all probability, our region is much richer in species than the Palæarctic, as it is quite certain that our lists are very incomplete in respect to the smaller moths. In parts of the southwest, indeed, it appears that new species of microlepidoptera are so abundant that the most superficial collector can not fail to find some, while the harvest to be reaped by systematic collecting and breeding is almost unlimited.

It is difficult to determine exactly the degree of resemblance between the lepidopterous faunæ of the Palæarctic and Nearctic regions, but while the two have even a number of species in common, they are in most respects very different. Taking the index of the Palæarctic (Staudinger and Rebel) catalogue, I find 326 valid genera enumerated under the first three letters of the alphabet. Of these, only 97, or less than 30 per cent., are found in Nearctic region. The difference would seem even greater if one took the names just as they stand in the two catalogues, because different views about nomenclature have given us in many cases different names for the same genus. It is very likely that a more exact comparison between the Palæarctic and Ne-