SCIENCE

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MSS. intended for publication and books, etc., intended for review should be sent to the responsible editor, Professor J. McKeen Cattell, Garrison-on-Hudson, N. Y.

*ON THE FOUNDATIONS OF MATHEMATICS.**

THE American Mathematical Society gives its retiring president the privilege of speaking on whatever he may have at heart. Accordingly, this afternoon I propose to consider with you some matters of importance—indeed, perhaps of fundamental importance—in the development of mathematics in this country; and it will duly appear in what non-technical sense I am speaking 'On the Foundations of Mathematics.'

A VIEW.

Abstract Mathematics.—The notion within a given domain of defining the objects of consideration rather by a body of properties than by particular expressions or intuitions is as old as mathematics itself. And yet the central importance of the notion appeared only during the last century—in a host of researches on special theories and on the foundations of geometry and analysis. Thus has arisen the general point of view of what may be called *abstract mathematics.* One comes in touch with the literature very conveniently by the mediation of Peano's *Revue des Mathématiques.* The Italian school of Peano and the *Formulaire Mathématique,* published in connection with the *Revue,*

* Presidential address delivered before The American Mathematical Society at its ninth annual meeting, December 29, 1902.

misprint. The statement that 'pumice and bombs prove the existence of a molten magma which rises well into the throat of the volcano,' may be questioned, for ancient glassy tuffs and pumice are abundant in the old agglomerates of Martinique, and the bombs are old rocks merely melted on the surface. The estimate of amount of ash sediment discharged, based in part on Russell's expression of the cubical content of a steam cloud, is full of fallacy. The argument is as follows: If a single cloud has a capacity of four billion cubic feet, is charged with one per cent. of solid matter, and is regularly replaced every five minutes by another cloud of the same size, the total discharge of solid matter per day is 11,520,000,000 cubic feet. This is one and a half times the discharge of the Mississippi River per year, and on this basis the discharge of Pelée is greater than that of all the rivers of the world combined, for the same period of time. This argument is concluded with the question, 'what becomes of the void that is being formed in the interior?' The defect in this sort of reasoning lies in the assumption that a primary eruption is continuous for days or even hours. There have been a few moments of violent outburst at certain intervals, which were undoubtedly explosions from great depth, and may be called primary eruption. Secondary explosion continues for weeks in the intervals, and is occasioned by the contact of superficial water and hot deposits. Obviously such explosions are only working over the same material, yet they occasion tremendous puffs that rise many thousand feet, and perfectly simulate deep-seated processes. Professor Heilprin has failed to discriminate primary and secondary eruption when he talks of Mt. Pelée being 'in a condition of forceful activity for upwards of 200 days.' The reviewer questions whether the volcano has been forcefully active from great depths for that many minutes. There have been eight or nine considerable eruptions, and probably none of these lasted more than five or ten minutes. There is no probability of a void in the interior; there is a fissure system, and with the removal of material from the walls, there is probably collapse that is compensated so gradually by subsidence over a wide area, that it makes no appreciable effect even on the height of shore lines.

As a whole the book is a good exposition in popular style of the main facts connected with the Caribbean eruptions of 1902. There are not sufficient maps to make all geographical matters clear, and there is a lack of diagrammatic illustration, much needed to make intelligible certain explanatory or theoretical statements. The scientific results of Professor Heilprin's research would be more easily grasped if they were tabulated; he will doubtless compile tables in more technical forms of publication. His summary of the phenomena, and the description of events in August which came under his immediate observation, will stand as records of permanent value to vulcanology. T. A. J., JR.

SCIENTIFIC JOURNALS AND ARTICLES.

THE opening (January) number of Volume 4 of the *Transactions* of the American Mathematical Society contains the following papers: 'Orthocentric properties of the plane n-line,' by Frank Morley; 'Definitions of a field by independent postulates,' by L. E. Dickson; 'Definitions of a linear associative algebra by independent postulates,' by L. E. Dickson; 'Two definitions of a commutative group by sets of independent postulates,' by E. V. Huntington; 'Definitions of a field (Körper) by sets of independent postulates,' by E. V. Huntington; 'On the invariants of differential forms of degree higher than two,' by C. N. Haskins; 'Über die Reducibilität der Gruppen linearer homogener Substitutionen,' by Alfred Loewy; 'The quartic curve as related to conics,' by A. B. Coble; 'The cogredient and digredient theories of multiple binary forms,' by Edward Kasner; 'On the envelopes of the axes of a system of conics passing through three points,' by R. E. Allardice; 'A Jordan curve of positive area,' by W. F. Osgood.

THE December number of the *Bulletin* of the American Mathematical Society contains: 'Concerning the commutator subgroups of groups whose orders are powers of primes,' by W. B. Fite; 'Note on irregular determinants,' by L. I. Hewes; 'Note on the projec-

tions of the absolute acceleration in relative motion,' by G. O. James; 'Infinitesimal deformation of the skew helicoid,' by L. P. Eisenhart; 'On integrability by quadratures,' by Saul Epsteen; 'The centenary of the birth of Abel,' by E. B. Wilson; 'The English and French translation of Hilbert's Grundlagen der Geometrie,' by E. R. Hedrick; 'Dickson's linear groups,' by G. A. Miller; 'Buckingham's Thermodynamics,' by E. H. Hall; 'Notes'; 'New publications.' The January *Bulletin* contains: 'The October meeting of the American Mathematical Society,' by F. N. Cole; 'Series whose product is absolutely convergent,' by Florian Cajori; 'Three sets of generational relations defining the abstract group of order 504,' by L. E. Dickson; 'Generational relations defining the abstract simple group of order 660,' by L. E. Dickson; 'The Carlsbad meeting of the Deutsche Mathematiker-Vereinigung, September, 1902,' by C. M. Mason; 'Shorter notices'; 'Notes'; 'New publications.' The February *Bulletin* contains: 'On the transformation of the boundary in the case of conformal mapping,' by W. F. Osgood; 'On the quintic scroll having three double conics,' by Virgil Snyder; 'Surfaces referred to their lines of length zero,' by L. P. Eisenhart; 'Supplementary note on the calculus of variations,' by E. R. Hedrick; 'The synthetic treatment of conics at the present time,' by E. B. Wilson; 'Brown's lunar theory,' by F. R. Moulton; 'The doctrine of infinity,' by E. R. Hedrick; 'Some recent German text-books in geometry,' by P. F. Smith; 'Notes'; 'New publications.'

Bird Lore for January–February has an illustrated paper on 'The Mound-building Birds of Australia,' by A. J. Campbell; an article on 'Making Bird Friends,' by Laurence J. Webster, and one on 'The Return of the Nuthatch,' by E. M. Mead; the 'Christmas Bird Census,' taken in various parts of the United States, and a second series of portraits of members of *Bird Lore's* Advisory Council. The article on 'How to Study Birds,' by Frank M. Chapman, treats of 'The Nesting Season,' and Abbott M. Thayer protests against the use of 'Mounted Birds in Illustration,' a subject which has another side to it, shown in the editor's reply.

The American Museum Journal for February contains a few announcements of material received in various departments, and illustrations of the new ptarmigan groups. The important part of the number is the supplement, by William Beutenmiller, devoted to 'The Hawk-moths of the Vicinity of New York.' Besides a key and descriptions there is an illustration of each species, so that the merest tyro should be able, with the aid of this little hand-book, to identify all. This makes the tenth of the valuable 'Guide Leaflets' issued by the American Museum.

SOCIETIES AND ACADEMIES.

BIOLOGICAL SOCIETY OF WASHINGTON.

The 367th meeting was held Saturday, February 21.

D. E. Salmon spoke of 'The Recent Outbreak of the Foot-and-Mouth Disease in New England.' He said that the effects of an outbreak of this kind, if not promptly checked, would be so disastrous financially that the Bureau of Animal Industry was always careful to ascertain that the malady reported was really foot-and-mouth disease; having ascertained the facts in the present case, every means was promptly taken to stamp it out. Dr. Salmon described the symptoms of the disease, saying that it was so extremely contagious, that it was readily carried from barn to barn by men, dogs and even pigeons, and once introduced into a herd, every member was pretty sure to be afflicted. While the distemper did not, in very many cases, cause death, it was extremely painful to the cattle afflicted, destroyed their value as beef for many months, and dried up the milk at once. Foreign governments prohibited the importation of cattle from afflicted districts, and as the United States exported annually 400,000 cattle and 100,000 sheep, the immediate effect of an outbreak in our western cattle regions could readily be seen. Furthermore, if there were no cattle for exportation some steamship lines would be compelled to withdraw their vessels. The only practical way to check the