NOTE ON THE MULTIPLE IMAGES FORMED BY TWO PLANE INCLINED MIRRORS.

Having noticed errors in the statement of the number of images in a couple of text-books, I have recently made a canvass of forty-one books on optics or general physics, and was surprised to find a general lack of information on this subject. Fifteen of the books did not consider it at all, many of the others took up only special cases where the angle is 60° or 90°, and eight contained misstatements as to the number of images. It does not seem to be generally recognized that the number of possible images depends upon the position of the object, and that the number of these which are visible depends upon the position of the observer's eye. As in only five of the forty-one books examined was the dependence of the number of images upon the position of the object correctly stated, I have thought it well to write out the following analysis.

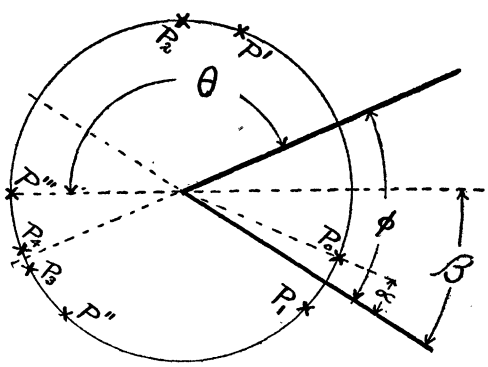

Fig. 1.

Let φ be the angle between the mirrors, a the angular distance of the object P_0 from one of them, P_1, P_2, etc., the images formed by first reflection in the latter, P', P'', etc., the images formed by first reflection in the other mirror. Then, if n be a whole number, the angular distance from P_0 to any image may be stated as follows:

P_{2n}	$2n\phi$
P_{2n+1}	$2n\phi + 2a$
P^{2n}	$2n\phi$
P^{2n+1}	$2n\phi + 2(\phi - a)$

If the p-th image fall behind both mirrors in set P_{2n}

$$\pi + \phi - a > 2n\phi > \pi - a$$

in set P_{2n+1}

$$\pi + a > 2n\phi + 2a > \pi - \phi + a$$

or

$$\pi + \phi - a > (2n+1)\phi > \pi - a$$

These conditions are the same and may be written

$$\pi + \phi - a > p\phi > \pi - a$$

or

$$\frac{\pi - a}{\phi} + 1 > p \geqq \frac{\pi - a}{\phi}$$

where the equality sign is brought in so that p may represent the number of images in this set as well when the last one falls on the line of one of the mirrors.

Similarly, if the r-th image fall behind both mirrors in set P^{2n}

$$\pi + a > 2n\phi > \pi - \phi + a$$

in set P^{2n+1}

$$\pi + \phi - a > 2n\phi + 2(\phi - a) > \pi - a$$

or

$$\pi + a > (2n+1)\phi > \pi - \phi + a$$

Hence

$$\frac{\pi + a}{\phi} > r \geqq \frac{\pi + a}{\phi} - 1$$

and the total number of images $= k = p + r$. If φ divide evenly into π, $p = r = \pi/\varphi$ and the last images of the two sets coincide. Then

$$k = p + r - 1 = \frac{2\pi}{\phi} - 1,$$

and k is independent of a.

This much is correctly worked out in Violle, Heath and several smaller works on optics, but in no text on general physics in English that I have seen.

If the eye be behind the plane of one mirror, only the images finally formed by the other are visible. If the eye be between the mirrors (angularly) all the images formed in front of either mirror are visible. An image behind both mirrors will be visible only to an eye between the mirrors whose position makes an angle less than $\pi - \theta$ with the surface in which the last reflection takes place, where θ is the angle between this surface and the position of the image considered.

A few examples may make the subject more clear.

If $\phi = 72°$, $\alpha = 36°$; then $p = 2$, $r = 2$, $k = 4$,

and all the images are visible to an eye anywhere between the two mirrors.

If $\phi = 72°$, $\alpha = 10°$; $p = 3$, $r = 2$, $k = 5$,

but only three of the five images will be visible to an eye placed within 25° of the mirror furthest from P_0.

If $\phi = 55°$, $\alpha = 25°$; $p = 3$, $r = 3$, $k = 6$.

If $\phi = 55°$, $\alpha = 10°$; $p = 4$, $r = 3$, $k = 7$.

If the eye be placed within the angle β only five of the seven images are visible. This is the case shown in the figure.

It is interesting to observe the results experimentally and see one of the images disappear or merge into another at critical angles. For $\varphi = 55°$, a critical case is given by $\alpha = 15°$.

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THE PLANET EROS.

THE planet Eros, as we noted last week, was rediscovered on the morning of August 2 by Dr. Chas. J. Ling with the twenty inch refractor of the Chamberlin Observatory at University Park, Colo. It was found pretty close to the place derived from an ephemeris computed by Miss Mary Clark Traylor. But it was estimated to be considerably brighter than had been expected. In view of the fact that the law of its variability is unknown the results of photometric measures will be awaited with interest. As Eros is now low in the east when the morning twilight begins, and is advancing in right ascension nearly as fast as the sun, it will not be an easy object for some months to come. The position on August 11 at $15^h\ 25^m\ 19^s$ Univ. Park M. T. was

AR $5^h\ 36^m\ 35.03^s$.
Decl. $+31°\ 56'\ 17.7''$.

THE MARINE BIOLOGICAL LABORATORY AND THE CARNEGIE INSTITUTION.

AT a meeting of the corporation of the Marine Biological Laboratory held at Woods Hole on August 12, it was voted to transfer the laboratory and its equipment to the Carnegie Institution. This action was taken after it had been stated to the members of the corporation that the executive committee of the Carnegie Institution would recommend to the trustees that the laboratory should be accepted, that its debts should be paid, that new buildings should be erected, that $20,000 a year should be allowed for maintenance and that the scientific management should rest as heretofore with the naturalists of the country. A motion that a three fourths vote of the members present be required for the transfer of the property of the corporation was defeated by a vote of 32 to 19. It was voted without dissent that an account of the action of the corporation be made public.

SCIENTIFIC NOTES AND NEWS.

DR. N. L. BRITTON sailed for England on August 16, where he will complete some investigations upon American *Carices* and *Crassulaceæ* which will be issued as the first parts of the 'Systematic Botany of North America' to be published by the New York Botanical Garden. Dr. MacDougal will be acting director of the garden during his absence.

PROFESSOR J. E. WOLFF, of the U. S. Geological Survey, is continuing the investigation of the areal and structural geology in the crystalline areas of New Jersey, in cooperation with the state geologist.

DR. ROBERT BELL, of the Geological Survey of Canada, has been engaged in the geological survey of Baffin Land, which he finds to have an area of about 300,000 square miles, making it the largest island with the exception of Australia and Greenland. The results of Dr. Bell's explorations are to be presented to the Royal Geographical Society.

MR. BORCHGREVINK, the Norwegian explorer, has taken out naturalization papers, so as to Americanize the Antarctic expedition he is planning under the auspices of the National Geographical Society.

MEDICAL exchanges state that the committee of the Liverpool School of Tropical Medicine is making arrangements to enable Major Ron-