SECTION A, MATHEMATICS AND ASTRONOMY.

THE meeting of the section at Pittsburgh compared favorably with former meetings both in the number and character of the papers presented and in the attendance. The number of papers presented was twenty-four, mathematical papers predominating somewhat over astronomical. The attendance was better in the morning than in the afternoon sessions, on account of the large and attractive list of excursions planned for the afternoons by the Local Committee. The list of papers follows:

On the Adaptability of the Glycerine Clock to the Diurnal Motion of Astronomical Instruments, particularly those used in Photographing Solar Eclipses: Professor DAVID P. TODD, Amherst College Observatory.

In outline the glycerine clock is an accurately constructed cylinder, about four inches in diameter, in which travels a piston, the flow of the glycerine being controlled at any required speed or rate by means of a series of needle valves. By attaching a mirror or objective to a frame, equatorially mounted, the glycerine clock can be set under one arm of it, at any convenient distance from the axis, and the requisite rate for counteracting the diurnal motion of the sun can be given by means of the needle valves. This permits very heavy weights to be thrown on the piston, and therefore the vibration of the instruments by wind can be precluded. When the run of the piston is finished, the glycerine is pumped out of the top of the cylinder and forced back into the bottom, and the run is commenced over again.

On a Convenient Type of Finder for very large Equatorials: Professor DAVID P. TODD, Amherst College Observatory.

The object of a 'finder' is convenience. But in equatorials above twenty inches in aperture, the ordinary finder is necessarily mounted so far away from the axis of the great telescope that its use occasions much inconvenience, simply because of the distance of its eyepiece from that of the great tube. To obviate this difficulty, Professor Todd proposes to construct the finder with a pair of reflectors, either planes or prisms, set at 45°, and to mount the main part of its tube in rings or bearings. By turning the tube in these, the finder's eyepiece can be brought as near the eyepiece of the great tube as is desired, or pushed away from it to admit attachment or adjustment of subsidiary apparatus.

Series whose Product is Absolutely Convergent: Professor FLORIAN CAJORI, Colorado College.

This paper is a continuation of the subject as developed by the author in his previous papers (*Trans. of the Am. Math. Soc.,* Vol. II., pp. 25–36, 1901; SCIENCE, Vol. XIV., p. 395, 1901; *Bulletin of the Am. Math. Soc.,* Vol. VIII., pp. 231–236, 1902) and in the article of Alfred Pringsheim (*Trans. of the Am. Math. Soc.,* Vol. II., pp. 404–412, 1901). Some of the results previously obtained, relating to absolutely convergent products of two or more series, are generalized and the method of treatment is simplified. The construction of pairs of divergent series with real or complex terms is given, such that the product of the two series is not only absolutely convergent, but equal to any desired value, including zero.

A New Treatment of Volume: Professor G. B. HALSTED, University of Texas.

After the establishment of a sect-calculus, the area of a triangle is defined as the product of its base by half its altitude, this product being proved independent of the choice of base. Then the volume of a tetrahedron is defined as the product of the area of its base by one third its altitude, this product being proved independent of the choice of base. The volume of a polyhedron is defined as the sum of the volumes of a set of tetrahedra into which it is cut, this sum being proved independent of the mode of partition into tetrahedra. Then a prismatoid is defined, and its volume proved

$$V = \frac{a}{4}(B + 3S).$$

Then all the ordinary solids are forms of prismatoids.

A New Solar Attachment: HERBERT A. HOWE, Director of the Chamberlin Observatory, University Park, Colorado.

This was a description of a small solar of simple construction devised by Mr. Orville F. Shattuck, a former pupil of Professor Howe. It was shown how the device, when attached to a universal instrument or engineer's transit, may be used for some simple astronomical observations, and for illustrating the principles of the equatorial coudé, the prism transit, the sextant and the almucantar.

On the Periodic Solutions of the Problem of Three Bodies: Professor E. O. LOVETT, Princeton University.

Lagrange found five exact solutions of the problem of those bodies in each of which the bodies preserve an unvarying configuration which revolves with a uniform velocity. When the third body is of infinitesimal mass compared with the other two, it can describe small periodic orbits in the vicinity of the points where exact solutions exist. The latter points were called centers of libration by Gyldén, and Darwin calls the infinitely small body an oscillating satellite. Hill pointed out the fertility of the notion and made a splendid application of it in his lunar theory. Poincaré elaborated the mathematical theory in his celebrated researches and we owe to Darwin an extended collection of examples of periodic orbits.

One of the most recent investigations of such orbits is a suggestive paper by Charlier, in No. 18 of the *Meddelanden från Lunds Astronomiska Observatorium.* In the *Monthly Notices of the Royal Astronomical Society* for November, 1901, Plummer has discussed some of Charlier's results in a more general manner.

It is the object of Professor Lovett's paper to determine the imaginary centers of libration and their corresponding orbits, and thus complete the analytical solution proposed by Charlier. The results cannot be expected to fit the sky, but they may be of some interest to mathematical astronomers. It appears that there are real periodic orbits corresponding to imaginary centers of libration.

The Rate of the Riefler Sidereal Clock, No. 56: Professor CHARLES S. HOWE, Case School of Applied Sciences.

In this paper Professor Howe gave the details and results of some careful series of experiments of a Riefler clock enclosed in a glass case from which the air had been partially exhausted. The mean daily rate for a trifle over three months was .116 of a second. The average daily variation from this mean was .015, and the maximum variation .022 of a second. The paper will be published in the *Astronomische Nachrichten.*

A Representation of the Coordinates of the Moon in Power Series which are Proved to Converge for a Finite Interval of Time: DR. F. R. MOULTON, University of Chicago.

It is proved in this paper that the differential equations which the motion of the moon must fulfill can be integrated as power series in certain parameters, and that the series converge for at least a certain finite, determinable interval of time. The equations to be integrated are of the type

$$(1)\quad \frac{dx_i}{dt} = X_i(x_1, \cdots, x_n;\, a_1, \cdots, a_j;\, \beta_1, \cdots, \beta_k;\, t) \qquad (i = 1, \cdots, n),$$

where the x_i are any variables defining the position and motion of the moon, and the α's and β's are parameters occurring in the differential equations.

Solutions as power series in the α's are sought of the form

$$(2)\quad x_i = \sum_{\mu_1, \cdots, \mu_j = 0}^{\infty} x^{(i)}_{\mu_1, \cdots, \mu_j}\, a_1^{\mu_1} a_2^{\mu_2} a_j^{\mu_j}, \qquad (i = 1, \cdots, n),$$

where the

$$x^{(i)}_{\mu_1, \cdots, \mu_j}$$

are functions of the time to be determined. Substituting (2) in (1) and equating to zero the coefficients of the various powers of the α's, it is found that after the

$$x^{(i)}_{0, \cdots, 0}$$

have been found the other coefficients are determined by linear non-homogeneous differential equations which can always be solved. The proof of the convergence of these series is made by employing suitable comparison differential equations.

There is nothing to prevent any of the β's being numerically equal to any of the α's. In fact, on the start all the parameters are α's, but before the integrations made those which occur in a special manner, as in the trigonometrical functions, may be called β's. When this is done in an appropriate manner all the

$$x^{(i)}_{\mu_1, \cdots, \mu_j}$$

are purely periodic functions of the time except the angular variables, each of which has one term which is proportional to the time. After a finite number of terms have been found they may be rearranged as Fourier series whose coefficients are power series in the α's, giving expressions of the same form as usually found by lunar theorists.

The advantages of this method are: (*a*) The series are known to converge, (*b*) every step is defined in advance and contains nothing arbitrary, and (*c*) the work is divided up in a convenient manner.

The Mass of the Rings of Saturn: Professor A. HALL, South Norfolk, Conn.

The mass of these rings was first determined by Bessel in 1831 from the motion of the apsides of the orbit of Titan. This motion is about half a degree in a year. But the action of the figure of the planet, and the attractions of the other satellites were neglected; and, as Bessel pointed out, the resulting mass of the rings was too great. This mass is 1/118, the mass of Saturn being taken as the unit.

In this paper an equation was formed containing two indeterminate quantities depending on the figure of the planet, the mass of the rings, and the masses of the three brighter satellites, Rhea, Dione and Tethys. The small resulting action of the other satellites was estimated. The coefficiency of these six indeterminate quantities can be computed with sufficient accuracy. The uncertainty in finding the mass of the rings arises chiefly from the lack of good values of the masses of the satellites. These masses must be found from the mutual perturbation of the satellites. Substituting the values of the masses of the satellites determined by Professor H. Struve, the principal coefficient depending on the figure of the planet was assumed to be 0.0222. The mass of the rings is 1/7092. It is probable that Struve's masses of the satellites are too small, and the above mass of the rings too great.

Saturn will soon return to our northern skies, and it is hoped that further observations and their dimensions will give good values of the constants of this interesting system.

On a Class of Real Functions to which Taylor's Theorem does not apply; and

On a Class of Transcendental Functions with Line-Singularities: Professor JOHN A. EISTAND, Thiel College.

In the first paper a class of real functions to which Taylor's theorem does not apply was discussed. Examples of such functions were given and the non-identity of the expansion with the function expanded was shown.

In the second paper a new type of transcendental functions fulfilling certain conditions within and on the unit-circle was discussed. These conditions are: *The function together with all its derivatives is finite and continuous within as well as on the unit circle;* which is a singular line for the function. The form of the functions is as follows:

$$f(z) = \Pi \frac{z-(1+a_\nu)e^{2\pi i\nu a}}{z-(1+b_\nu)e^{2\pi i\nu a}} e^{\Phi_\nu(z)},$$

where

$$\lim a_\nu = 0, \quad \lim b_\nu = 0,$$

a is an incommensurable number, and

$$\Phi_\nu(z) = \frac{(a_\nu - b_\nu)^{2\pi i\nu a}}{z-(1+b_\nu)e^{2\pi i\nu a}} + \frac{1}{2}\left[\frac{(a_\nu - b_\nu)e^{2\pi i\nu a}}{z-(1+b_\nu)e^{2\pi i\nu a}}\right]^2 + \cdots + \cdots + \frac{1}{\nu-1}\left[\frac{(a_\nu - b_\nu)e^{2\pi i\nu a}}{z-(1+b_\nu)e^{2\pi i\nu a}}\right]^{\nu-1}.$$

On a General Method of Subdividing the Surface of a Sphere into Congruent Parts: Mr. HAROLD C. GODDARD, Amherst College.

The problem was incidental to the practical problem of constructing a steel sphere one hundred feet in diameter, in connection with a new method of mounting a telescope, as outlined in an article in the *American Journal of Science* for June, 1902, by Professor David P. Todd, of Amherst College.

If a regular dodecaedron be inscribed in a sphere, planes determined by the center and each edge of the dodecaedron cut out on the sphere twelve equal regular spherical pentagons. If the vertices of each pentagon be connected with its center by arcs of great circles the surface of the sphere is divided into sixty congruent isosceles spherical triangles, whose angles are determined as 60°, 60° and 72°.

A Possible New Law in the Theory of Elasticity: Professor J. BURKITT WEBB, Stevens Institute of Technology.

Owing to the absence of Professor Webb at the time this paper was called for, it was presented only in abstract. The law referred to in the title is: "If the forcible change of the distance between two points in an elastic sytem changes the distance of two other points by a certain amount, then the same force applied to alter the distance of the two other points will change the distance of the first two points by the same amount."

On Extracting Roots of Numbers by Subtraction: Dr. ARTEMUS MARTIN, Washington, D. C.

A paper on 'Evolution by Subtraction' was published in the *Philosophical Magazine* for September, 1880, communicated by the Rev. F. H. Hummell, who ascribed the method to his friend and neighbor, the Rev. W. B. Cole. The rule given in Mr. Hummell's paper for finding the square root of any number is:

From any square number subtract the even numbers in succession, beginning with 2, until the remainder is less than the next even number to be subtracted. This remainder will be the square root sought.

The statement of the rule may be simplified as follows:

For the nth subtrahend add 2 to the preceding subtrahend. The last remainder will be the square root sought.

Or: For the nth subtrahend, multiply n by 2; the last remainder will be the square root sought.

To find the cube root of a number the rule is:

For the nth subtrahend, multiply n by 6 and add the preceding subtrahend; the last remainder will be the cube root sought.

Thus the first subtrahend is 6; the next $6\times2+6=18$; the third, $6\times3+18=36$; and so on.

For the fourth root, the rule is:

To find the nth subtrahend, multiply n^2 by 12 and add 2 plus the preceding subtrahend; the last remainder will be the root sought.

In the paper rules are given for finding the fifth, sixth, seventh, eighth, ninth and tenth roots, with examples.

General formulas for the nth subtrahend of any root (the mth) are:

$$s_n=(n+1)^m-(n^m+1),$$

or

$$s_n=s_{n-1}+(n+1)^m+(n-1)^m-2n^m.$$

It is shown in the paper that in all cases for all roots the number of subtractions to be performed is one less than the number of units in the root sought, and consequently the root equals number of subtractions plus 1.

A table of subtrahends containing the first ten subtrahends for the first eleven roots is appended to the paper. This table can be extended to any desired extent by the rules and formulas given.

The paper will be published in the *Mathematical Magazine.*

On the Determination of the Places of the Circumpolar Stars: Professor MILTON UPDEGRAFF, U. S. Naval Observatory.

The contents of this paper are: A sketch of previous work done on circumpolar stars, (2) a statement of the kind of work needed, and (3) some suggestions as to the best methods to be used in redeterminations of the coordinates of the circumpolar stars. The paper will be published in one of the astronomical journals.

Report on Quaternions: Professor ALEXANDER MACFARLANE, Lehigh University.

This paper will be printed in full in the *Proceedings* of the Association.

The Definite Determination of the Causes of Variation in Level and Azimuth of Large Meridian Instruments: Professor G. W. HOUGH, Dearborn Observatory, Evanston, Ill.

This was an elaborate discussion of the various styles of mounting for meridian instruments, and of the effects of changes of temperature in causing variation. The results of several long series of observations upon this effect were exhibited. Professor Hough's conclusion was that stone piers give the best results. The paper gave rise to some spirited discussion.

A New Founding of Spherics: Professor G. B. HALSTED, University of Texas.

Professor Halsted presented under this title some abstracts from a book which he is about to publish. The author made a simple set of assumptions: (1) of association, (2) of betweenness, (3) of congruence, and he then showed how, without the assumption that the straight line is the shortest distance between two points, or that the shortest path between two points on a sphere is on the great circle through them, or even that two sides of a triangle are together greater than the third, all the projective and metric properties of spherics are established.

Report on the Theory of Collineations: Professor H. B. NEWSON, University of Kansas.

Owing to Professor Newson's absence from the meeting, this paper could be presented only by title. It will be printed in full in the *Proceedings.*

Second Report on Recent Progress in the Theory of Groups of Finite Order: Professor G. A. MILLER, Stanford University.

In the absence of Professor Miller this report was presented in abstract by Dr. W. B. Fite, of Cornell University. It will be published in the *Bulletin of the American Mathematical Society.*

Displacements Polygons: Professor J. BURKITT WEBB, Stevens Institute of Technology.

Owing to the absence of Professor Webb at the time this paper was called for it was read by title.

Some Theorems on Ordinary Continued Fractions: Professor THOMAS E. MCKINNEY, Marietta College.

Let D be any positive integer not a perfect square, and let its square root be represented by an ordinary continued fraction. This paper determines the form of D so that the continued fraction representing its square root may have a period with one, two, three or four elements, and applies the results to the determination of the number of reduced forms in the class to which the indefinite quadratic form (1, 0, D) belongs.

On the Forms of Sextic Scrolls of Genus One: Dr. VIRGIL SNYDER, Cornell University.

In his classification of sextic scrolls of genus 1, Dr. Snyder employed the method of point correspondence between two plane sections and made use of the following theorems which were proved in one of his former papers: (1) The nodal curve (simple or composite) is of order 9; (2) every generator cuts four others, and (3) any non-reducible plane curve lying on the surface is of genus 1.

Thirty-three types are found, ten of which have a multiple conic. It will be published in the *American Journal of Mathematics.*

Transformation of the Hypergeometric Series: Professor EDGAR FRISBY, U. S. Naval Observatory.

If in the differential equation of the second order connecting the elements of the hypergeometric series

$$P = 1 + \frac{\alpha\beta}{\gamma}x + \frac{\alpha\beta(\alpha+1)(\beta+1)}{1\cdot2\cdot\gamma\cdot(\gamma+1)}x^2 + \text{etc.}$$

$x^\mu P'$ be substituted for P, new relations are obtained in which P' takes the place of P, and the new elements are functions of the original elements. μ is determined from the condition that the new series must be of the same general form as the old. If, in addition, x be replaced by $1/x$ another series is obtained. From these two new series, by proper substitution of the new derived elements, are obtained almost by inspection, the twenty different series ordinarily given in works on differential equations.

EDWIN S. CRAWLEY,
Secretary.

SECTION G, BOTANY.

SECTION G of the American Association met in the Botanical Hall of Phipps Conservatory on the mornings of June 30 and July 3, 1902. In the absence of Professor D. H. Campbell, Stanford University, Professor C. E. Bessey, of the University of Nebraska, was elected acting Vice-President.

The abstracts of papers presented are as follows:

The Prevalence of Alternaria in Nebraska and Colorado During the Drought of 1901: GEORGE GRANT HEDGCOCK, Lincoln, Nebr.

This paper gives a brief synopsis of observations made in various sections of Nebraska and Colorado during the severe period of drought in July and August of