sorbed by the single corpuscle as its full charge. Nernst's 'Theoretische Chemie' (1900, p. 394) gives the most reliable estimate of the number of molecules of oxygen, or any other gas, in a cubic millimeter at standard temperature and pressure. This is 55 thousand millions of millions (which may be written 55TMM). Calculated from this, the oxygen taken in by the single blood corpuscle as a full charge is found to be about 28 hundred millions of molecules. But as the combination is known to be regularly one molecule of the gas to one molecule of hæmoglobin, this result, or in round numbers three thousand millions, is approximately the number of hæmoglobin molecules in the blood-corpuscle (3 TM).

Dividing this last number into the volume of the hæmoglobin in a corpuscle, we obtain the volume of the cubic 'room' assigned by chemists to each molecule, and the cube root of this will give the length of the imaginary walls of said room, also nearly the diameter of the molecule regarded as a sphere in a solid state. The volume is approximately $1/10^{17}$ cubic millimeters, and the linear dimension of the side of a molecule 'room' is about 1/500,000 of a millimeter. The 'rooms' of the oxygen molecules in the gaseous condition are much larger than these, because the gases rejoice in spacious apartments; in fact, the volume of gas which is insorbed by the blood is nearly twice as great as that of the devouring hæmoglobin.

Nernst states that by multiplying the absolute atomic weight of hydrogen upon the molecular formula of any proteid, we may obtain the absolute weight of the proteid. This involves, we think, the assumption that no condensation has occurred in building up proteid molecules. In order to test the rule by hæmoglobin, we find that this rule gives as the absolute molecular weight $1.35 \times (10)^{-17}$ of a milligram. By the method of the quantitative absorption given above of oxygen the value comes out as $1.30 \times (10)^{-17}$ of a milligram. The two results differ by less than 4 per cent. This close harmony does not prove that the estimated weight of the atom of hydrogen is right, for it enters into both methods; but it does prove non-condensation, and also confirms the quantitative results of Hüffner and others as to the absorption of oxygen. It may be added that the oxygen absorbed is, when estimated in its fluid form, about 1/470 the volume of the absorbing hæmoglobin.

But probably if the oxygen were examined in the liquefied or solidified condition, its molecular sphere of action would be found not to be so very widely divergent from its rightful proportion of 32 to 16,669.

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SCIENTIFIC BOOKS.

Leçons sur les séries divergentes. Par ÉMILE BOREL, maître de conférences a l' Ecole Normale Supérieure. Paris, Gauthier-Villars. 1901. Pp. vi + 182.

La Série de Taylor et son prolongement analytique. Par JACQUES HADAMARD. Scientia, série physico-mathématique. Chartes, imprimerie Durand. 1901. Pp. viii + 102.

These two works can appropriately be classed together, on account of both their authorship and their contents. Among the younger French mathematicians who have taken their doctors' degrees within the past dozen years none are to-day more conspicuous than Hadamard and Borel. Their theses were published in 1892 and 1894 respectively. A few years later both writers were recipients of prizes from the French Academy of Sciences. In 1896 Hadamard received the 'Prix Bordin' for his work on geodesics, while Borel won the 'Grand Prix des sciences mathématiques' in 1898 for his investigations upon divergent series. Recently also they have been bracketed in a list of nominees to fill a vacancy in the Academy of Sciences.

We have here to consider two representative works. Each has the special interest that it is devoted to that branch of the theory of functions in which its author first attained distinction. Hadamard aims to give a concise, almost an encyclopedic, résumé of the present state of our knowledge concerning the analytic continuation of Taylor's series. Borel, on the other hand, gives a more detailed exposition of a single chapter of this subject, the divergent series. On this account his book will have the greater interest for the mathematical public and will be reviewed at somewhat greater length.

Two other works of equal size and somewhat similar character have been previously published by Borel, his 'Leçons sur la théorie des fonctions' (treating the 'Eléments de la theorie des ensembles et applications') and his 'Leçons sur les fonctions entières.' Together with the present work they form a unique series, embodying the results of much recent investigation in the theory of functions. It is indeed a piece of rare good fortune in any province of mathematics to have the important recent work thus promptly picked out and thrown into accessible form by such a mathematician as Borel. For this reason the publication of these lectures cannot be too warmly welcomed.

It is safe to say that no previous book upon divergent series has ever been written. Borel opens up a field of research which is still very new and promises rich reward to the investigator. In the process of evolution the divergent series has passed through several curious stages of development. At first a divergent series was accepted on faith and used with great *naïveté*. Thus Leibnitz, for example, when considering the expansion of $1/(1+x)$ into the series $1-x+x^2-x^3+\ldots$ remarks that if $x=1$, the sum of n terms takes alternately the values 1 and 0, and the sum of the series must therefore be equal to the mean value $\frac{1}{2}$. After the introduction of exact analysis by Cauchy and Weierstrass such a loose mode of treatment could no longer be tolerated. The mass of inconsistencies to which it would lead was clearly perceived, and a divergent series was therefore considered by the mathematician to be meaningless, good for nothing but to be thrown away. However, a few of the great mathematicians were visibly perturbed over the situation. Thus we find Cauchy complaining in 1821:

"J'ai été forcé d'amettre diverses propositions qui paraîtront peut-être *un peu dures;* par exemple, qu'une série divergente n'a pas de somme."

We know also that Abel was only prevented by his premature death from attacking the problem. But the view that a divergent series had no place in mathematical analysis soon became orthodox, and search after a legitimate basis for its use was abandoned. Nevertheless the astronomers, in utter disregard of this opinion, still continued to employ divergent series and to obtain from them a sufficient degree of approximation for practical purposes.

The impetus to a new mathematical treatment of the subject may be said to have come simultaneously from Stieltjes and Poincaré, although prior to this, in 1880, the legitimacy of the conclusion of Leibnitz had been established by Frobenius in a memoir which was suggestive of the beautiful theory developed later by Borel. According to the new view a divergent power-series is considered as having value in two distinct ways, either as enabling one to find an approximate value of some corresponding function (Poincaré and Stieltjes) or as a source of another algorithm which is convergent and therefore defines a proper function (Stieltjes, 1894).

The treatise of Borel begins with an interesting historical introduction. The body of the book can be divided roughly into four parts, which take up successively the four chief theories of divergent series; the asymptotic theory of Poincaré, the continued fraction theory of Stieltjes, the theory of Borel—characterized by the use of definite integrals containing a parameter z—, and, finally, the theory of Mittag-Leffler. The crowning achievement is without doubt Borel's own work, and his presentation of it is the most interesting feature of the book. No adequate idea, however, of the treatment of Stieltjes can be obtained without direct reference to the famous memoir of 1894, as Borel frankly states. This inadequacy of presentation is offset by the addition of an important supplement which Borel himself contributes to the theory of Stieltjes. We regret the omission of the method of Lindelöf. Its dismissal with a half dozen lines and without even a reference to his article in the *Acta Societatis Fennicae* is possibly due to a certain haste in preparation which we have fancied we have detected in several places. While the method of conformal representation (or transformation of the variable) which Lindelöf employs has been applied only to a restricted class of divergent series, it seems probable that it could be developed so as to give a more general theory.

On account of its somewhat abstract character Borel's treatise will probably be of greater interest to the pure mathematician than to the astronomer or student of applied mathematics. Few applications of the various theories have been given, probably because but few applications have yet been made, except in the case of the asymptotic theory of Poincaré. The author leaves us in some uncertainty as to how far his own theory has been carried and applied to differential equations. We hope that in a subsequent edition the important applications will be more fully developed.

We turn now to the little book of Hadamard. This is one of a series of short monographs published under the general title 'Scientia' and devoted to the 'Exposé et développement des questions scientifiques à l'ordre du jour.' The special topic taken up by Hadamard, as has already been stated, is the analytic continuation of a power-series, $a_0+a_1z+a_2z^2+\ldots$. In the consideration of this question two problems of the greatest importance and difficulty present themselves. These are: (1) The determination of the nature and position of the singular points of the analytic function defined by the series, and (2) the calculation of the value of the function at points exterior to the circle of convergence.

Hadamard has had the extremely difficult task of compressing into a few pages what has been done on these problems. In this he has succeeded admirably. It is extraordinary what an amount of information is packed away in the space of one hundred pages. Yet the work is no dry compilation of facts. Nowhere is the skill of the author more fully shown than in the manner in which he has woven his materials together. The theorems are analyzed, their significance is pointed out, and their demonstrations are outlined sufficiently to show the manner in which the subject is treated. Attention should also be called to the excellent bibliography with which the book opens and to which reference is constantly made. In correlating the one hundred and fifty memoirs here included Hadamard has performed a very important service. His admirable report is not suited to the reader who has little acquaintance with the general subject, but to the specialist and investigator it will be invaluable.

E. B. VAN VLECK.

The Elementary Principles of Chemistry. By A. V. E. YOUNG, Professor of Chemistry in Northwestern University. New York, D. Appleton & Company. 1901.

This book differs so radically from those in general use that if reviewed at all, it must be at some length. The author has used this method successfully for thirteen years; his object being to instruct the student during the first year by this method, which he calls the quantitative method. He says that its inception is due to Professor Josiah P. Cooke, of Harvard; he believes it 'both scientifically and pedagogically an improvement on prevailing methods.' The presentation of a topic in the text is to be studied by the student after performing the laboratory experiment illustrating the same.

The first 97 pages of the book are devoted to the physical and chemical properties of substances and to simple theoretic chemistry, including the fundamental quantitative laws of chemical action, the gas laws, atomic and molecular theory, kinetic theory of gases, structure and stereoisomerism. The author lays particular stress on the quantitative laws, and also on the laws of Gay Lussac, Dulong and Petit, Mitscherlich and Raoult, as illus-