As the chief authority on the iron-bearing formations of the Upper Peninsula, Brooks was asked in 1869 to take charge of the Economic State Geological Survey of that district, and he accepted on condition that he should be allowed to secure all the private aid possible. The necessity for this provision is apparent when it is understood that during four years the State paid toward the work but $9,000, while he spent $2,000 of his own means and received no pay himself. The results of his work are embodied in Volumes II. and III. of the Michigan State Geological Survey. His reports are direct in style, simple in treatment and extremely practical in substance. They are models of excellence as economic geologic reports. The most original chapters, and those which still possess most practical value, relate to the principles of mine management and of magnetic observations in prospecting for iron ores. For many years the chapters on geology were standards of reference, and they have been replaced only by most elaborate studies, based largely on Brooks' work and carried out with the most refined methods of modern geology. Says Van Hise, Brooks' successor: "Notwithstanding the immense advantage which it has been to have Brooks' work as a foundation, it has taken many years of labor fairly to complete the structural story to which Brooks contributed important chapters. Only those who have labored in the Lake Superior region and who understand its peculiar difficulties can give Brooks credit for the remarkable work he did. His geological work is my ideal of what should be done in a new region of complex geology."

In 1873 Major Brooks' health gave out under the stress of overwork to which he drove himself. He sought relief abroad, and resided in London and Dresden while completing his State reports. He became a Fellow of the Geological Society of London and Corresponding member of the Geological Society of Edinburgh. Returned to this country in 1876, he resided at Monroe and at Newburg, N. Y., and after 1883, during the winters, at Bainbridge, Ga., living the life of a country gentleman and farmer. His interest in science and engineering practice never abated, and he was always ready with wise counsel, even though strength failed him for action.

Major Brooks was characterized by intense energy, which exhausted his physique before he reached middle age; by originality, which combined with common sense made him a most efficient man of affairs; by keen powers of observation and deduction, which he applied untiringly to scientific research; by geniality and affection, generosity, truthfulness and loyalty to principle, which made him beloved and stamped him as a man whose memory will be honored and revered.

Bailey Willis.
U. S. Geological Survey.

SCIENTIFIC BOOKS.

THE APPRECIATION OF NON-EUCLIDEAN GEOMETRY.

Histoire des Mathématiques. Par Jacques Boyer. Paris, Carré et Naud. 1900. 8vo. Pp. xi + 260. Price, 5 francs.

Geometry: Ancient and Modern. By Professor Edwin S. Crawley. *Popular Science Monthly* (January). 1901. Pp. 257–266.

Non-Euclidean Geometry. By Professor Henry Parker Manning. Boston, Ginn and Company. 1901. 8vo. Pp. vi + 95.

The last section of Boyer's attractive book is headed 'Géométrie Euclidienne et Géométries non-Euclidiennes.' He says, p. 240, "The last quarter of the nineteenth century witnessed the building up of interesting theories." The next page continues: "But beyond contradiction the most original researches of this period pertain to the non-Euclidean geometries, and it is by them that we will terminate this incomplete exposition of contemporary science." The brief account which follows (less than five

pages) is certainly stimulating, and is adorned by a full-page portrait of Lobachevski, taken from that given in the Kazan edition of his collected works.

But there is extant another picture of Lobachevski far more impressive, showing him in the plenitude of his powers, which I first saw at Kazan, a daguerreotype from the life, a copy from which you may see as the frontispiece of Engel's monumental 'Nikolaj Iwanowitsch Lobatschefskij.'

As for Boyer's account of what he rates so high, it begins as follows: "From long ago it has been sought to demonstrate the famous axiom laid down twenty centuries ago by Euclid, to wit: through a point only one parallel to a given straight can be drawn."

This of course is not Euclid's postulatum, but rather a paraphrase of what is called (even by Cajori) Playfair's axiom, though Playfair explicitly credits it to Ludlam, namely: "That two straight lines which cut one another can not be both parallel to the same straight line." Boyer continues: "These attempts remained unfruitful. However, at the end of the eighteenth century, an Italian Jesuit, Saccheri, wished to found a geometry resting on a principle different from the celebrated postulate." It was certainly not *the end* of the eighteenth century, for Saccheri died in 1733. Nor did he wish to set up any geometry different from Euclid, since the very title of his book was 'Euclid vindicated from every fleck.'

"Finally," continues Boyer, "at the opening of the nineteenth century, a Russian, Lobachevski, and a Hungarian, John Bolyai, perceived at about the same time the impossibility of this demonstration." In the index, citing to this page, Boyer gives as the dates of the birth and death of John Bolyai 1775–1856, the dates for his father Wolfgang Bolyai (Bolyai Farkas). Lombroso makes this same confusion and identification of father and son, and from it draws testimony for his thesis that great wits to madness are allied.

John Bolyai (Bolyai János) was born December 15, 1802, and died January 27, 1860. At the celebration of his centenary next year in Hungary I hope to be present. Boyer continues: "Their works published independently of each other had without doubt been inspired by the doctrines of the philosopher Kant who, in a passage of his *Critique of the pure reason*, indicated a new consideration of space. For this latter, space existed *a priori*, precedent to all experience, as completely subjective form of our intuition."

In regard to this bold attribution of influence, I may be allowed to say that not a particle of evidence has appeared to show that John Bolyai ever heard of even the existence of Kant. I examined Bolyai's papers, his correspondence, his 'Nachlass', at Maros-Vásárhely, and never found even the name of Kant. As for Lobachevski, he might have had his attention called to Kant by Bronner the professor of physics at Kasan, at one time an admirer of the 'Kritik der reinen Vernunft,' but that Kant influenced him is merest conjecture, and unnecessary, since we sufficiently know the path of his mental on-going. Of Lobachevski's doctrine, Boyer says, p. 242, "He declares at the beginning the following axiom: through a point can be drawn *many parallels* to a given straight."

What Lobachevski does assume is that through a given point can be drawn innumerable distinct straight lines in a plane which will never meet a given straight in that same plane; but of these, only those two are parallel to the given straight which approach it asymptotically.

Continuing, Boyer says of Lobachevski, p. 245, "When he died in 1856 he occupied the position of Rector of the University which he had entered as simple student."

Unfortunately Lobachevski had been deprived of his position of Rector for ten years before he died.

Passing on to Riemann's geometry, Boyer says: "To construct this Geometry, its inventor rejects the postulatum and the first axiom of Euclid: two points determine a straight."

But the postulatum is: "And if a straight cutting two straights makes with them interior angles lying on the same side, which together are less than two right angles, then the two straights intersect if continually produced on the side upon which these angles lie."

In the 'spherical' or Riemannian geometry

here referred to, so far from this being rejected, it actually remains true. In the 'elliptic' or Clifford-Klein geometry its last clause becomes unmeaning, because the straight line does not divide the elliptic plane into two separated regions. Here we cannot distinguish two sides of a straight. Without crossing a given straight we can pass from any one point of the plane to any other point. In this elliptic geometry, the other assumption mentioned by Boyer, that two straight lines cannot meet at more than one point, is retained. Riemann's epoch-making contribution was that the universe while unbounded still may very well be finite.

This gives us the assumption that a straight line has no point at infinity, that is, that every straight line is actually cut by every other straight line coplanar with it.

Now dropping Euclid's implicit, not explicit, assumption that the straight line is infinite, but retaining all his postulates and axioms, especially Postulate 1 (Simon's Euclid, 1901, p. 30), "Let it be granted that one and only one sect can be drawn from any one point to any other point," we have the elliptic geometry.

On p. 245, line 23, is a misprint, 'joints' for 'points.'

Peirce (C. S.), p. 247, is identified with his father in the index, his name being given as Benjamin and his dates as 1809–1880.

Mr. C. S. Peirce is still alive, having an article in the January, 1901, number of the *Popular Science Monthly*, which contains the charming résumé by Professor Crawley entitled, 'Geometry: Ancient and Modern.' Perhaps it could be wished that this article had more definitely emphasized Euclid.

The advertisement of Boyer's 'Histoire' calls mathematics the science of Euclid and Newton.

In writing of 'The Wonderful Cenury,' Alfred Russel Wallace says of all time before the seventeenth century: "Then, going backward, we can find nothing of the first rank except Euclid's wonderful system of geometry, perhaps the most remarkable mental product of the earliest civilizations."

The new departure, the non-Euclidean geometry, is absolutely epoch-making, but fortunately it has intensified admiration for that imperishable model, already in dim antiquity a classic, the immortal *Elements* of Euclid.

Professor Crawley's exposition of the non-Euclidean geometry is exceedingly interesting. But as soon as it gets beyond two dimensions it becomes obscure.

He says, p. 265, "If we proceed beyond the domain of two dimensional geometry we merge the ideas of non-Euclidean and hyper-space."

If we do, we are very apt to blunder. Thus Professor Crawley says, p. 266, "Professor Newcomb has deduced the actual dimensions of the visible universe in terms of the measurement of curvature in the fourth dimension."

This mistake of supposing that a non-Euclidean space requires or needs a space of higher dimensionality has often been publicly corrected.

On page 293 of his 'Nicht-Euklidische Geometrie, I.' Felix Klein puts in pillory the unfortunate title of Newcomb's contribution as follows: "Elementary theorems relating to the geometry of a space of three dimensions and of uniform positive curvature in the fourth dimension. (Die letzten Worte des Titels sind sehr merkwürdig *und deuten auf ein Missverständniss.*)"

After 'we merge the ideas,' Professor Crawley's very next sentence is: "The ordinary triply-extended space of our experience is purely Euclidean." This naïve assertion not only Professor Crawley does not know and cannot prove, but, strangely enough, no one can ever know, no one can ever prove. For Euclidean space the angle-sum of a rectilineal triangle must be exactly two right angles. Such absolutely exact metric results experience can never give.

In connecting a geometry with experience there is involved a process which we find in the theoretical handling of any empirical data, and which therefore should be familiarly intelligible to any scientist. The results of any observations hold good, are valid, always only within definite limits of exactitude and under particular conditions. When we set up the axioms, we put in place of these results statements of absolute precision and generality. In this idealization of the empirical data our addition is at first only restricted in its arbitrariness in

so much as it must seem to approximate, must apparently fit, the supposed facts of experience, and, on the other hand, must introduce no logical contradiction. Thus to-day the ordinary triply-extended space of our experience may be purely Bolyaian, or purely Euclidean, or purely Cliffordian, or purely Riemannian.

In Manning's 'Non-Euclidean Geometry' America has taken a step in advance of all the world in thus putting forth an intended, available class-book for elementary instruction in this fascinating subject.

The book is very gratifying to me, in that the method of treatment that has been taken as the basis of the first chapter is Saccheri's, drawn from my translation, the first ever made, which appeared in the *American Mathematical Monthly*, beginning in June, 1894. My copy of Saccheri is still, so far as I am aware, the only copy on the Western Continent.

It is also matter for congratulation that so many of the further proofs have been taken unchanged from Lobachevski and Bolyai. We rejoice that the world will be rich now in pupils of those who in life had never a disciple.

It perhaps should be noted that though the book says (p. 93), "The Elliptic Geometry was left to be discovered by Riemann," it gives only the simple elliptic, or single elliptic, or Clifford-Klein geometry. It never even mentions the double elliptic or spherical or Riemannian geometry, which Killing maintains was the only form which ever came before Riemann's mind.

GEORGE BRUCE HALSTED.

AUSTIN, TEXAS.

Commercial Organic Analysis. By ALFRED H. ALLEN, F.I.C., F.C.S. Volume II., Part II.; Hydrocarbons, Petroleum, and Coal Tar Products, Asphalt, Phenols, and Creosotes. Third Edition, with revisions and additions by the Author and HENRY LEFFMANN, M.A., M.D. Philadelphia, P. Blakiston's Son and Co. 1900. Pp. viii+322. Price, $3.50.

In the revision of this volume, most of the notes of the second edition have been incorporated in the text, the text condensed to a certain extent, by minor changes, and by omissions, and many valuable additions made. Much matter has been added in regard to the testing of lubricating oils and phenols, and the technology of acetylene. The section on asphalt has been largely increased. The claim of the preface that the nomenclature of the Geneva convention has been applied does not seem justified. Some of the analytical operations are not described in sufficient detail, notably the method for the assay of calcium carbide (page 32). The determination of sulphur in petroleum does not receive the attention it deserves, the most important method—combustion in a current of oxygen or air, and collection of the sulphur dioxide in standard alkali—receiving only passing mention in the section on asphalt. The method for the detection of β-napthol 'suggested' by the American Association of Official Agricultural Chemists was indeed described by the referee in his report, but has not yet been adopted officially by that body. It should be credited rather to Dr. W. D. Bigelow, the referee. It was considered necessary to call attention to these points, but in considering them, the general excellence of the revision should not be lost sight of.

G. S. FRAPS.

L. M. B. C. Memoirs on Typical British Marine-Plants and Animals. Edited by W. A. HERDMAN, D.Sc., F.R.S. V. *Alcyonium* by SYDNEY J. HICKSON, M.A., D.Sc., F.R.S. London, Williams & Norgate. 1901.

A student or amateur zoologist visiting the sea-shore is apt to find the ordinary text-books of zoology somewhat too general to be of much service to him in unraveling the structural details of many of the forms which attract his attention, and the number of forms described in the more special laboratory manuals being necessarily limited, he may find no mention in these of the special organism which interests him. To meet this difficulty the Liverpool Marine Biology Committee has undertaken the publication of a series of memoirs giving detailed descriptions of a number of common animals and plants occurring in the district under investigation by the Committee. The fifth of these memoirs, on *Alcyonium* by Professor Sydney J. Hickson, has just appeared, its predecessor being memoirs on *Ascidia* by Professor Herdman, on *Cardium* by Mr. J. Johnstone, on *Echinus* by Mr. H. C. Chadwick and on *Codium*

by Mr. R. J. H. Gibson and Miss Helen Auld. The editor announces twenty-five other memoirs in course of preparation.

The present little volume, which may be taken as a sample of the series, opens with a brief introduction containing a definition of the order Alcyonaria, and then follow sections on the general appearance of a colony of *Alcyonium digitatum*, its reproduction, the anatomy of the colony, the anatomy of the polyps, the development of the colony and finally its physiology. Three plates containing twenty-four figures complete the little volume, which consists of but twenty-two octavo pages and is sold for the modest sum of eighteenpence.

It is almost needless to say that a description of *Alcyonium* by Professor Hickson is well done, and if the remaining volumes prove as satisfactory as the present one, the editor is to be congratulated upon the initiation of so admirable and useful a series. It would seem that the zoologists of this country might profitably undertake a similar series of memoirs and the idea may be commended to the attention of the officers of the Marine Biological Laboratory. J. P. McM.

SCIENTIFIC JOURNALS AND ARTICLES.

In the January–February number of the *Journal of Geology*, Frank Dawson Adams gives an account of 'The Excursion to the Pyrenees in Connection with the Eighteenth International Geological Congress.' The interest centers largely around certain rocks supposed by Lecroix and some other eminent French geologists to illustrate the transformation of limestone into diorite and of shales into gneiss and granite by emanation accompanying granitic intrusions. Professor Adams does not regard the case as proved and suggests, among other things, chemical analysis as a means of testing the hypothesis. O. C. Farrington contributes a discussion of 'The Structure of Meteorites.' They are treated under three classes, iron, iron-stone and stone meteorites. Structures of the monogenic meteorites are discussed crystallographically those of the polygenic according to their mode of aggregation. In a paper entitled 'The Problem of the Monticuliporoidea,' F. W. Sardeson discusses these much neglected although important organisms of the Paleozoic faunas. Unlike many of the recent writers, he has considered them as corals rather than as bryozoans, and several of the commoner forms are described in such detail as to greatly assist students beginning the investigation of these fossils. 'Valleys of Solution in Northern Arkansas' are discussed by A. H. Purdue. They are described as steep and bilaterally symmetrical, with remarkable straightness, due, no doubt, to their connection with jointing planes.

The Botanical Gazette for February contains the first of two papers by Dr. H. C. Cowles upon 'The Physiographic Ecology of Chicago and Vicinity; a study of the origin, development and classification of the plant societies.' Dr. Cowles gives his views on the classification of plant societies, and proposes a method based on the laws of physiography. The general principles outlined in full have been noted already in Science (Vol. XII., p. 708, Nov. 9, 1900), and are here worked out in connection with the Chicago area. The plant societies are grouped in five genetic series, the first of which, that on rivers, is presented in the first instalment. The paper is illustrated by half-tone reproductions which show the various stages in the development of river-plant societies, from the ravine with its mesophytic slopes, through the xerophytic bluffs stages, and culminating in the mesophytic forest of the flood plain which is regarded as the climax phase of regional development. Mr. John Donnell Smith, publishes his 22nd paper on 'Undescribed Plants from Guatemala and other Central American Republics,' describing about thirty new species from this exceedingly prolific region. Miss Mary H. Smith, of Cornell University, publishes an account of some experiments which would indicate that nitrates are a source of nitrogen for saprophytic fungi. Carlton E. Preston, of Harvard University, publishes a second note on non-sexual propagation in *Opuntia*. Various book reviews, minor notices, notes for students, and news items complete the number.