under the same latitudes, while favored by the low elevation of the country, is less advantageously situated in that it does not usually receive the greatest possible force of the sun's rays during the hottest weeks of summer.

2. *Soil.* — The soil preferred by the great majority of Austro-riparian plants which are met with in the mountains, especially those of our first category, which are assumed to be of neo-tropical origin, is light, sandy and poor in organic matter ; consequently readily permeable to water and becoming quickly and strongly heated. It is very similar to the soils which cover a great part of the Coastal Plain. In a substratum of this character, whether on the lower slopes or in the river bottoms, we invariably find established the larger colonies of Lower Austral species. In consonance with their environment, most of them are xerophytic or hemi-xerophytic in structure, as is the case with a great portion of the vegetation of the coastwise pine-barrens.

On the heavier and consequently colder and wetter soils, and on slope exposures other than southern, the flora is always of predominately Transition character, at the same elevation or even, in places, descending to lower altitudes than are often reached on the opposite slope by Carolinian and Austro-riparian forms.

Unfortunately no investigations have yet been made in this mountain region which afford us exact data as to the amount of isolation received by plants growing in the situations described; nor have we the measurements of soil temperature which are necessary to the further prosecution of the present inquiry. A comparative study of this question in various parts of the Appalachian region and of the Coastal Plain, coupled with an investigation of the ecology of the vegetation along anatomical-physiological lines, would beyond all doubt yield results of the greatest interest and value. It is earnestly hoped that such an investigation can be undertaken in the nea future.

THOS. H. KEARNEY, JR.

SCIENTIFIC BOOKS.

Gauss and the non-Euclidean Geometry. CARL FRIEDRICH GAUSS WERKE. Band VIII. Göttingen. 1900. 4to. Pp. 458.

We are so accustomed to the German professor who does, we hardly expect the German professor who does not.

Such, however, was Schering of Göttingen, who so long held possession of the papers left by Gauss.

Schering had planned and promised to publish a supplementary volume, but never did, and only left behind him at his death certain preparatory attempts thereto, consisting chiefly of excerpts copied from the manuscripts and letters left by Gauss. Meantime these papers for all these years were kept secret and even the learned denied all access to them.

Schering dead, his work has been quickly and ably done, and here we have a stately quarto of matter supplemental to the first three volumes, and to the fourth volume with exception of the geodetic part.

Of chief interest for us is the geometric portion, pp. 159–452, edited by just the right man, Professor Staeckel of Kiel.

One of the very greatest discoveries in mathematics since ever the world began is, beyond peradventure, the non-Euclidean geometry.

By whom was this given to the world in print?

By a Hungarian, John Bolyai, who made the discovery in 1823, and by a Russian, Lobachévski, who had made the discovery by 1826.

Were either of these men prompted, helped, or incited by Gauss, or by any suggestion emanating from Gauss?

No, quite the contrary.

Our warrant for saying this with final and overwhelming authority is this very eighth volume of Gauss's works, just now at last put in evidence, published to the world.

The geometric part opens, p. 159, with

Gauss's letter of 1779 to Bolyai Farkas the father of John (Bolyai János), which I gave years ago in my Bolyai as demonstrative evidence that in 1799 Gauss was still trying to prove Euclid's the only non-contradictory system of geometry, and also the system of objective space.

The first is false; the second can never be proven.

But both these friends kept right on working away at this impossibility, and the more hot-headed of the two, Farkas, finally thought he had succeeded with it, and in 1804 sent to Gauss his 'Göttingen Theory of Parallels.' Gauss's judgment on this is the next thing given (pp. 160–162). He shows the weak spot. "Could you *prove*, that $dkc = ckf = fkg$, etc., then were the thing perfect. However, this theorem is indeed true, only difficult, without already presupposing the theory of parallels, to prove rigorously." Thus in 1804 instead of having or giving any light, Gauss throws his friend into despair by intimating that the link missing in his labored attempt is true enough, but difficult to prove without petitio principii.

Of course we now know it is *impossible* to prove.

Anything is impossible to prove which is the equivalent of the parallel postulate.

Yet both the friends continue their strivings after this impossibility.

In this very letter Gauss says: "I have indeed yet ever the hope that those rocks sometime, and indeed before my end, will allow a through passage."

Farkas on December 27, 1808, writes to Gauss: "Oft thought I, gladly would I, as Jacob for Rachel serve, in order to know the parallels founded even if by another.

"Now just as I thought it out on Christmas night, while the Catholics were celebrating the birth of the Saviour in the neighboring church, yesterday wrote it down, I send it to you enclosed herewith.

"To-morrow must I journey out to my land, have no time to revise, neglect I it now, may be a year is lost, or indeed find I the fault, and send it not, as has already happened with hundreds, which I as I found them took for genuine. Yet it did not come to writing those down, probably because they were too long, too difficult, too artificial, but the present I wrote off at once. As soon as you can, write me your real judgment."

This letter Gauss never answered, and never wrote again until 1832, a quarter of a century later, when the non-Euclidean geometry had been published by both Lobachévski and Bolyai János.

This settles now forever all question of Gauss having been of the slightest or remotest help or aid to young János, who in 1823 announced to his father Farkas in a letter still extant, which I saw at the Reformed College in Maros-Vásárhely, where Farkas was professor of mathematics, his discovery of the non-Euclidean geometry as something undreamed of in the world before.

This immortal letter, a charming and glorious outpouring of pure young genius, speaks as follows:

"Temesvár 3 Nov., 1823.

"My dear and good father,

"I have so much to write of my new creations, that it is at the moment impossible for me to enter into great detail, so I write you only on a quarter of a sheet. I await your answer to my letter of two sheets; and perhaps I would not have written you before receiving it, if I had not wished to address to you the letter I am writing to the Baroness, which letter I pray you to send her.

"First of all I reply to you in regard to the binominal.

* * * * * *

"Now to something else, so far as space permits. I intend to write, as soon as I have put it into order, and when possible to publish, a work on parallels. At this moment it is not yet finished, but the way which I have hit upon promises me with certainty the attainment of the goal, if it in general is attainable. It is not yet attained, but I have discovered such magnificent things that I am myself astonished at them.

"It would be damage eternal if they were lost. When you see them, my father, you yourself will acknowledge it. Now I cannot say more of them, only so much: *that from nothing I have created another wholly new world.*

All that I have hitherto sent you compares to this only as a house of cards to a castle.

"P. S. I dare to judge absolutely and with conviction of these works of my spirit before you, my father; I do not fear from you any false interpretation (that certainly I would not merit), which signifies that, in certain regards, I consider you as a second self."

In his autobiography János says: "First in the year 1823 did I completely penetrate through the problem according to its essential nature, though also afterward further completions came thereto. I communicated in the year 1825 to my former teacher, Herrn Johann Walter von Eckwehr (later imperial-royal general), a written paper, which is still in his hands. On the prompting of my father I translated my paper into Latin, in which it appeared as *Appendix* to the *Tentamen* in 1832."

So much for Bolyai.

The equally complete freedom of Lobachévski from the slightest idea that Gauss had ever meditated anything different from the rest of the world on the matter of parallels I showed in SCIENCE, Vol. IX., No. 232, pp. 813–817.

Passing on to the next section, pp. 163–164, in the new volume of Gauss, we find it important as showing that in 1805 Gauss was still a baby on this subject. It is an erroneous pseudo-proof of the impossibility of what in 1733 Saccheri had called 'hypothesis anguli obtusi.' To be sure Saccheri himself thought he had proved this hypothesis inadmissible, so that Gauss blundered in good company; but his pupil Riemann in 1854 showed that this hypothesis gives a beautiful non-Euclidean geometry, a new universal space, now justly called the space of Riemann.

Passing on, we find that in 1808, Schumacher writes: "Gauss has led back the theory of parallels to this, that if the accepted theory were not true, there must be a constant *à priori* line given in length, which is absurd. Yet he himself considers this work still not conclusive."

Again, with the date April 27, 1813, we read: "In the theory of parallels we are even now not farther than Euclid was. This is the partie honteuse (shameful part) of mathematics, which soon or late must receive a wholly different form."

Thus in 1813 there is still no light.

In April, 1816, Wachter, on a visit to Göttingen, had a conversation with Gauss whose subject was what he calls the anti-Euclidean geometry. On December 12, 1816, he writes to Gauss a letter which shows that this anti-Euclidean geometry, as he understands it, was far from being the non-Euclidean geometry of Lobachévski and Bolyai János.

The letter as here given by Staeckel, pp. 175–176, is as follows:

* * * "Consequently the anti-Euclidean or your geometry would be true. However, the *constant* in it remains undetermined: why? may perhaps be made comprehensible by the following:

"* * * The result of the foregoing may consequently be so expressed:

"The Euclidean geometry is false; but nevertheless the true geometry must begin with the same eleventh Euclidean axiom or with the assumption of lines and surfaces which have the property presumed in that axiom.

"Only instead of the straight line and plane are to be put the great circle of that sphere described with infinite radius together with its surface.

"From this comes indeed the one inconvenience, that the parts of this surface are merely symmetric, not, as with the plane, congruent; or that the radius out on the one side is infinite, on the other imaginary. Only it is clear how that inconvenience is again overbalanced by many other advantages which the construction on a spherical surface offers; *so that probably also then even*, if the Euclidean geometry were true, the necessity no longer indeed exists to consider the plane as an infinite spherical surface, though still the fruitfulness of this view might recommend it.

"Only, as I thought through all this, as I had already fully satisfied myself about the result, in part since I believed I had recognized the ground (la métaphysique) of that indeterminateness necessarily inherent in geometry—also even the complete indecision in this matter, then, if that proof against the Euclidean geometry, as I could not expect, were not to be considered as stringent; in part, so not to consider as lost all the many previous researches in plane geometry, but to be used with a few modifications, and that still also the theorems of solid geometry and mechanics might have approximate validity, at least to a quite wide limit, which perhaps yet could be more nearly determined; I found this evening, just while busied with an attempt to find an entrance to your transcendental trigonometry, and while I could not find in the plane sufficing determinate functions thereto, going on to space constructions, to my no small delight the following *demonstration for the Euclidean parallel theory.* * * *

"* * * Just in the idea to conclude I remark still, that the above proof for the Euclidean parallel-theory is fallacious. * * * Consequently has here also the hope vanished, to come to a fully decided result, and I must content myself again with the above cited. Withal I believe I have made upon that way at least a step toward your transcendental trigonometry, since I, with aid of the spherical trigonometry, can give the ratios of all constants, at least *by construction of the right-angled triangle.* I yet lack the actual reckoning of the base of an isosceles triangle from the side, to which I will seek to go from the equilateral triangle."

As to Gauss's transcendental trigonometry, nothing was ever given about it but its name. *Requiescat in pace.*

Yet Gauss writes, April 28, 1817:

"Wachter has printed a little piece on the foundations of geometry.

"Though Wachter has penetrated farther into the essence of the matter than his predecessors, yet is his proof not more valid than all others."

We come now to an immortal epoch, that of the discovery of the real non-Euclidean geometry by Schweikart, and his publication of it under the name of Astral-Geometry.

On the 25th of January, 1819, Gerling writes to Gauss:

"Apropos of parallel-theory I must tell you something, and execute a commission. I learned last year that my colleague Schweikart (prof. juris, now Prorector) formerly occupied himself much with mathematics and particularly also had written on parallels.

"So I asked him to lend me his book. While he promised me this, he said to me that now indeed he perceived how errors were present in his book (1808) (he had, for example, used quadrilaterals with equal angles as a primary idea), however that he had not ceased to occupy himself with the matter, and was now about convinced that without some datum the Euclidean postulate could not be proved, also that it was not improbable to him that our geometry is only a chapter of a more general geometry.

"Then I told him how you some years ago had openly said that since Euclid's time we had not in this really progressed; yes, that you had often told me how you through manifold occupation with this matter had not attained to the proof of the absurdity of such a supposition. Then when he sent me the book asked for, the enclosed paper accompanied it, and shortly after (end of December) he asked me orally, when convenient, to enclose to you this paper of his, and to ask you in his name to let him know, when convenient, your judgment on these ideas of his.

"The book itself has, apart from all else, the advantage that it contains a copious bibliography of the subject; which he also, as he tells me, has not ceased still further to add to."

Now comes, pp. 180–181, the precious enclosure, dated Marburg, December, 1818, which, though so brief, may fairly be considered the first *published* (not printed) treatise on non-Euclidean geometry.

It is a pleasure to give this here in English for the first time.

The non-Euclidean Geometry of 1818: By SCHWEIKART.

"There is a two-fold geometry—a geometry in the narrower sense—the Euclidean, and an astral science of magnitude."

The triangles of the latter have the peculiarity, that the sum of the three angles is not equal to two right angles.

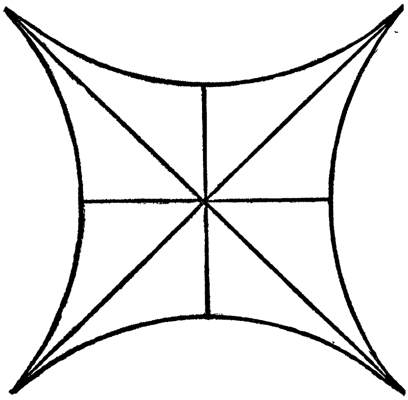
This presumed, it can be most rigorously proven:

(*a*) That the sum of the three angles in the triangle is *less* than two right angles;

(*b*) That this sum becomes ever smaller, the more content the triangle encloses;

(*c*) That the altitude of an isosceles right-angled triangle indeed ever increases, the more one lengthens the side; that it, however, cannot surpass a certain line, which I call the *constant.*

Squares have consequently the following form :



Is this constant *for us* half the earth's axis (as a consequence of which each line drawn in the universe from one fixed star to another, which are ninety degrees apart from one another, would be a tangent of the earth-sphere), so is it in relation to the spaces occurring in daily life infinitely great.

The Euclidean geometry holds good only under the presupposition that the constant is infinitely great. Only then is it true that the three angles of every triangle are equal to two right angles ; also this can be easily proved if one takes as given the proposition that the constant is infinitely great.

Such is the brief declaration of independence of this hero.

Nor were Schweikart's courage and independence without farther issue. Under his direct influence his own nephew, Taurinus, developed the real non-Euclidean trigonometry and published it in 1825, with successful applications to a number of problems.

Moreover, this teaching of Schweikart's made converts in high places.

In the letter of Bessel to Gauss of 10 Feb., 1829 (p. 201), he says : "Through that which Lambert said, and what Schweikart disclosed orally, it has become clear to me that our geometry is incomplete, and should receive a correction, which is hypothetical and, if the sum of the angles of the plane triangle is equal to a hundred and eighty degrees, vanishes.

"That were the *true* geometry, the Euclidean the *practical*, at least for figures on the earth."

The complete originality and independence of Schweikart and of Lobachévski are recognized as a matter of course in the correspondence between Gauss and Gerling, who writes, p. 238 : "The Russian steppes seem, therefore, indeed a proper soil for these speculations, for Schweikart (now in Königsberg) invented his 'Astral-Geometry' while he was in Charkow."

This fixes the date of the first conscious creation and naming of the non-Euclidean geometry as between 1812 and 1816.

Gauss adopts and uses for himself this first name, Astral-Geometry (1832, p. 226 ; 1841, p. 232).

At length the true prince comes. On February 14, 1832, Gauss receives the profound treatise of the young Bolyai János, the most marvellous two dozen pages in the history of thought. Under the first impression Gauss writes privately to his pupil and friend Gerling of the ideas and results as 'mit grosser Eleganz entwickelt.' He even says 'I hold this young geometer von Bolyai to be a genius of the first magnitude.'

Now was Gauss's chance to connect himself honorably with the non-Euclidean geometry, already independently discovered by Schweikart, by Lobachévski, by Bolyai János.

Of two utterly worthless theories of parallels Gauss had already given extended notices in in the *Göttingische gelehrte Anzeigen* (this volume, pp. 170–174 and 183–185).

To this marvel of János, Gauss vouchsafed never one printed word.

As Staeckel gently remarks, this certainly contributed thereto, that the worth of this mathematical gem was first recognized when John had long since finished his earthly career.

The 15th of December, 1902, will be the centenary of the birth of Bolyai János.

Should not the learned world endeavor to arouse the Magyars to honor Hungary by honoring then this truest genius her son?

GEORGE BRUCE HALSTED.

AUSTIN, TEXAS.

SCIENTIFIC JOURNALS AND ARTICLES.

IN the July number of the *American Journal of Insanity*, Dr. J. G. Rogers, of Indiana, pre-