*A DETERMINATION OF THE NATURE AND VELOCITY OF GRAVITATION.**

The present note is to be taken as a supplement to my previous paper on 'The Nature of the Electric and Magnetic Quantities.' † A fuller development of the theory of gravitation advanced by the writer is in course of preparation. This will, however, be delayed for some time, possibly for several years, as it is desired to investigate certain phenomena rather more accurately than has hitherto been done, and at present the writer is occupied with pressing work in another line.

It has seemed advisable, however, to publish this determination of the velocity of gravitation at the present time, and without waiting to complete the fuller treatment, for the reason that, as will be seen later, the value obtained clears up a number of perplexing optical problems, and removes a number of obstacles which have hitherto stood in the way of the development of that branch of physics.

On account of the fact that the writer's papers on these subjects have unavoidably been published in somewhat scattered form, it is considered best to give a brief résumé of the work which forms the basis of the method by which this velocity is deduced.

In 1893 the writer perceived that Fourier's 'Dimensions' could be developed into a very powerful agent of research, and one which should bear a relation to the usual methods similar to that which Qualitative chemistry bears to Quantitative. It was for this reason that the name 'Qualitative Mathematics' was given to this new branch.

As its name signifies, it is used, not for the exact determination of numerical quantities, but for the prediction and classification of phenomena.*

It was first shown † that the nature of electricity and magnetism was, at that time, indeterminate, as all the electric and magnetic phenomena which we were able to completely express dynamically could be comprised in three qualitative equations, whilst we had four unknown quantities.

Then, by several methods, Williams's result, that either specific inductive capacity, (k), or permeability (μ), must be a density, the other term being a compliancy, was confirmed.

It was then further deduced that the one which is a compliancy must decrease with the first power, whilst the one which is a density must decrease with the second power, of the corresponding intensities, *i. e.*, if F be the electric potential difference per unit length, and H be the magnetic potential difference per unit length, then if μ be

* Being a supplement to 'A Determination of the Nature of the Electric and Magnetic Quantities and of the Density and Elasticity of the Ether.'

† *Phys. Rev.*, January, 1900; and also of the earlier papers: 1891–2, on 'The Laws and Nature of Cohesion.'

* Such a branch of mathematics is absolutely necessary to supplement the work done by the other methods. For the latter can tell us nothing of the *nature* of the quantity involved. Their very greatest strength is their greatest weakness. The fact that a certain function, which gives us the state of things at the end of an organ pipe, also gives us the way a current of electricity distributes itself near the end of a wire dipping in a mercury cup is gratifying in its compre hensiveness, but disappointing in that when we meet that function, we do not know which of the many possible phenomena it represents. Take, for instance, our equations for light. They fit in with a simple elastic-solid wave, and we have fallen into the habit of speaking of light as really being such a wave, and some eminent physicists, even, as I have pointed out elsewhere, have fallen into the mistake of supposing that the magnetic rotation of light necessarily implies a rotation of the medium in a magnetic field, overlooking the fact that the whole proof is based on this unfounded, and, as we now know, certainly incorrect, supposition. All that the equations really mean is that light is some kind of periodic motion, but, if I remember rightly (as it was some years ago that I investigated the matter), there are eight kinds of periodic motion which can be equally well represented by the light equations.

† *Ibid.*, also *Elect. World*, May 18, 1895.

a compliancy, on increasing H, μ will decrease by an amount depending upon the first power of H, and on increasing F, k will decrease by an amount depending upon the second power of F. Also, in this latter case, the diminution of k must depend inversely upon the coefficient of volume elasticity.

On the other hand, if it is k which is the compliancy, these relations will be interchanged.

It was at once noticed that several of the empirical formulæ expressing the relation between H and μ gave a diminution depending upon the first power of H. A somewhat elaborate investigation was then undertaken, extending over the greater part of a year, and the fact was definitely established that the diminution did depend upon the first power of H accurately, the maximum amount of deviation from that called for being less than one-fourth of one per cent., which was about the limit of experimental accuracy.

This, of itself, would have sufficed to have settled the point, but in addition the other relation, which should exist if the theory were correct, *i. e.*, that the specific inductive capacity, k, should vary with the inverse second power of the slope of electric potential, and as the coefficient of volume elasticity, was also discovered. This was found to be the complete expression of Kerr's electrostatic phenomenon.

A prism of glass, one cm. thick and one cm. wide, stretched with a force of 30.10^6 dynes gave a change of density of nearly 3.10^{-5}. The change in the thickness of the glass was approximately $1.5.10^{-5}$. The change in velocity of the light which passed transversely through the glass, was approximately $.7 \times 10^{-5}$.

It was thus found that the actual mechanically produced change in density of the glass was sufficient to account for the observed change of velocity, though the agreement was not so close as it might have been, possibly owing to experimental difficulties.

From the observed change in velocity when placed in a strong electrostatic field, whose value was approximately determined by its sparkling distance, it was calculated that the value of the $F^2/8\pi k$ stress required to produce the same change of velocity as had been produced mechanically was nearly 25.10^6 dynes. The value of the purely mechanically applied stress, as given above, was 30.10^6. The close agreement is probably accidental, as the experimental error was considerably greater than the small difference observed. It is intended to repeat these experiments under conditions permitting of a much higher degree of accuracy.

The results obtained are however sufficient to show that Kerr's effect can be accounted for by purely mechanical stresses, electrically produced and resulting in a change of density.

Now it was pointed out above, that whichever of the medium coefficients, k or μ, varies as the square of the corresponding intensity, that one must be a density. Since, therefore, it has already been shown by Kerr that the change in velocity, and hence, as my experiments prove, the change in density,* is proportional to the square of the electric intensity, it follows that k is a density.

It still remained to be shown that Kerr's effect depended upon the volume elasticity. This was done by testing different glasses and noting that, the compensating pieces being made from the glass under test, the same force was always required to compensate, independent of the material tested.

We see, therefore, that the results deduced from the experiments on the relation between H and μ are completely confirmed

* Velocity is proportional to square root of density, but change of velocity is proportional to change in density, both being small.

by the results obtained on investigating the relations between F and k.

A number of additional pieces of corroborative evidence were also given, *i. e.*:

3. The relation between the magnetic constant α and the elasticity.

4. The relation between this constant and elastic strain.

5. The relation between this constant and permanent strain.

6. The relation between this constant and hysteresis.

7. The relation between k and the density of substances.

Several phenomena were also predicted, *i. e.*:

A. A change in the velocity of light, along a slope of electric potential.

B. A relation between refractive index and piezo-electric effect in doubly refracting substances.

These have not yet been confirmed, but arrangements are being made to investigate the former.

This same result, originally obtained by qualitative mathematics, can also be obtained by Lagrangian methods. By considering the way in which permeability and specific inductive capacity are affected in the case of stressed iron, and in the case of Kerr's phenomenon, as influenced by the elasticity of the material, it can be shown that a change in μ involves the first power of the magnetic intensity and that a change of k involves the second power of the electric intensity. This proof will be given later. It, however, in reality, adds nothing to the proof already given, which in the opinion of the writer is of such a character that we may say that the nature of electricity and magnetism is now definitely and finally determined, though no doubt it may be years before the absolutely decisive nature of the proof is generally appreciated.

Next, it follows from the writer's experiments on the relation between H and μ that the presence of matter does not alter the elasticity of the ether by as much as one-fourth of one per cent. Also, knowing now that k is a density, we are enabled to say that aberrational and other optical phenomena show that the density of the ether is not appreciably altered by the presence of matter, otherwise the $(n^2 - 1)$ and similar formulæ would not hold. From these facts we see that the actual volume of the atom, compared with the space occupied by it, must be quite small.

The diameter of the mercury atom I have shown to be $2.75\ (\pm 0.2) \times 10^{-8}$, and in 1899 I showed that the actual cross section of the space actually occupied by the atom must be less than one four-hundredth of the space occupied by the atom to the exclusion of other atoms, and that the atoms 'must have a configuration analogous, in its effects, to that of structures of thin platinum wire, suspended in oil.'

Later, J. J. Thomson, from his beautiful and wonderfully ingenious work on electric discharges in gases, was able to show that the atom is made up of a number of smaller bodies, which he calls corpuscles.

On comparing the results of Thomson, Ewers, Kaufmann, Lenard, Lorenz, Wiechert and Simon, we arrive at the conclusion that there are about 1,000 corpuscles in a hydrogen atom, and that the weight of a corpuscle is therefore about 1.5×10^{-27} gm.

Since, then, there are about 200,000 corpuscles in a mercury atom, and their cross section is less than one four-hundredth part of the cross section of the mercury atom, we find that the diameter of the corpuscle is certainly less than 2.10^{-11} cm.

From J. J. Thomson's formula for the electrically produced inertia of a charged sphere, we find, as was shown by Thomson (and independently by the writer), that if the diameter of the corpuscle is approximately 10^{-13} cms., the ionic charge which

it carries will account for its full quantity of inertia.

So long as we knew nothing of the size of the corpuscle, and since there might be a thousand corpuscles in a hydrogen atom, and yet each corpuscle be about 3.10^{-9} cms. in diameter, we were hardly justified in holding that inertia is an electric phenomenon. But when we take into consideration, in addition, the writer's proof that the diameter of the corpuscle must be less than $\frac{1}{1000}$ the diameter of the atom, and that this is the *superior* limit in size, we have a reasonable basis for holding, as the writer has done,* that the corpuscular charges are the cause of the inertia of matter.

Assuming this, we arrive at the result that the corpuscle is about $\frac{3}{4} \times 10^{-13}$ cms. in diameter. The ionic equivalent being about $4\ (\pm 1) \times 10^{-10}$ e. s. units, we find for the electrostatic tension and pressure at the surface of the corpuscle, about 2.10^{32} dynes.

One of the theorems immediately deducible by Qualitative Mathematics is that, "Whenever the electric or magnetic forces act in the presence of matter, the resultant effect is made up of two terms, one expressing the result of the action on the matter, the other that of the action on the ether."

We have seen that the electric stresses produce a change of volume in matter, and hence we must have also an effect of the same quality in the ether. Such a change of density in the ether would produce a gravitational attraction, and we may now calculate what value the ether constants must have in order to produce the observed amount of gravity which is associated with the corpuscle.

Taking Boys's value for the gravitational attraction of two masses, each of one gramme, and one cm. apart, *i. e.*, 6.65×10^{-8}, we get for the gravitational energy of the corpuscle about 10^{-48} ergs.

From this and the electrostatic stress we can calculate the volume elasticity, and we find it to be about 10^{78}.

But I have previously shown that the density of the ether is about 0.66 and its rigidity about 6.10^{20}.

Hence we can calculate the value of the compressional or gravitational wave, and find it to be approximately 5.10^{36} cms. per second.

It will be at once seen that this value agrees with our astronomical facts, and that it does away with a great many optical difficulties. For in the first place it makes the compressional wave vanish, and in addition, which is of the greatest importance, it makes the amount of energy in the compressional wave infinitesimally small.*

We may summarize our conclusions as follows:

The ether itself is a composite body, having a structure whose elastic properties are analogous to rubber. This is shown by the low value of the rigidity as compared with the compressibility, and by the form of the equation expressing the relation between H and μ.

This would immediately suggest a vortex theory, even if the quality of the ionic charge, *i. e.*, M/T, were not called to the attention. If we take Fitzgerald's vortex theory, and develop it along the lines indicated by my theory we have the vortices analogous to what, in the case of india rubber, I have called the 'skein material,' and the fluid in which the vortices form, which,

* *Elect. World*, May, 1900.

* In a paper on Comet's Tails, *Astrophysical Review*, January, 1897, the writer showed that all the phenomena so far noted in this connection, including the bridge of Biela's comet, the apparent retardation, the shape, etc., could be accounted for by supposing that the ultra-violet light of the sun acted on the surface of the nucleus of the comet to throw off negatively charged particles. It is possible that this compressive wave may be a factor in this discharge, though on the other hand it is possible that the light itself may be sufficiently effective.

until some one suggests a better name, we will call ethēron, taking the place of the 'filling-in material.'* The compressibility of the ethēron is very high, as we have seen, it being the thing which determines the velocity of the compressive wave. The vortex structure is what is concerned in transmitting the light waves and its modulus; the rigidity modulus is much smaller and of a different order, just as in the case of india rubber.

We do not need more than one vortex, 'the umbilical cord of the universe,' as one aspect of it suggested itself, stretching with its ends fixed on some free surface of the ethēron and itself forming one inextricable tangle. The circulation being the same everywhere simplifies matters. The parting of the vortex anywhere means the destruction of all matter.

Such a medium, as Fitzgerald has shown, gives an ether which can transmit light. Following up this theory, we conclude that corpuscles are vortex singularities, and that it is the hydrodynamic head of their flow which gives the ethēron density-variation round them. This change in density varies as the fourth power of the distance from the corpuscle. All the gravitational energy tends to that of compression, and if two corpuscles come together, their gravitational energy goes to increasing the compression energy of the ether. They do not come together because their approach brings into play forces which depend upon the energy of the vortices themselves. The fact that there is but one vortex, and consequently the circulation is the same everywhere, gives the atoms definite sizes and the corpuscles the same quantity of electricity, *i. e.*, the ionic charge.

A group of so many thousands of these corpuscles makes up the atom. The inertia of the atom is due to the electromagnetic inductance of the corpuscular charge, and gravity is due to the change of density of the ether surrounding the corpuscles, produced by the electrostatic stress of the corpuscular charge. Mass and gravity thus bear a constant ratio.

The cohesive force of the atoms, as I have shown elsewhere,* is due to the electrostatic attraction of the atoms for one another. Chemical force, as has been shown by Davy, Berzelius, Helmholtz, Ostwald and other workers, is due to the same cause.

It may here be noted that the idea of the ionic charge as an ever-present element of the atom is an interesting example of a theory, negatived absolutely, apparently, by fundamental principles, and yet developing in spite of its apparent incompatibility with facts in many other directions, with such success as to finally obtain a firm footing, although the arguments against it have never been answered.

The fact that the ionic charge is the agent in chemical action had been shown by the physicists just mentioned above. The presence of charged ions in electrolytes had also been firmly established, and J. J. Thomson had suggested that conduction in metals also took place through a breaking up of molecular groups, as in the case of electrolytes. But when in 1890 and 1891† I introduced the theory that the ionic charge is attached to the atom, *not only when it is concerned in chemical actions or formed part of a molecule, but in every case and always*, and is the cause of a number of physical phenomena, such as cohesion, rigidity, etc., a number of objections were made; that charges could not exist in the interior of a conductor; that the atoms of metals must be conducting, and so could not have equal charges of electricity; and others, as for example, the well founded

* Ether is the structure formed by the fluid and the vortices, ethēron the fluid alone.

**Elect. Soc.*, Newark, 1890; *Elect. World*, Aug. 8–22, 1891.

† *Ibid.*

criticism of Ostwald (under date Sept. 16, 1891): "The electrostatic theory of cohesion is new to me, * * * but for electrolytes there is the question to be answered, why stuffs like alcohol, etc., do not conduct? whilst according to *your* theory, *all* elements have electric charges."

These objections could not be met then, and have not been met up to the present time, in spite of the fact that this new concept (of the ionic charge being a fundamental part of the atom, apart from its chemical functions) has proved a most fertile one, and has been considerably developed by the orignator and by later workers, Richartz, Chattock, Lorentz, Larmor and others. Nor will these objections ever be met until we know the nature of metallic conduction.

This is one of the great outstanding problems. It has long been known that there is a relation between electric and heat conductivity. The writer has shown that there is a connection between the velocity of sound (and hence the elasticity and density) and the electric conductivity of wires. J. J. Thomson, as mentioned above, suggested that the current was carried by the electrolysis of molecular groupings, and his later work renders it probable that it is by means of the corpuscles. It is possible that the atoms of a metal are really dissociated and the negatively charged corpuscles are in a state similar to that of the ions of a solution, *i. e.*, the metallic atom is not a fixed combination of certain corpuscles, but is constantly changing in composition, the negative corpuscles being, as it were, in solution in the metal, and changing about freely.

Such an hypothesis would account for the relation between the velocity of sound and the electric conductivity. For the cohesion of the atoms would be due to these negative corpuscles acting, as the mortar between bricks, to bind together the positive groupings, and hence the greater the number of free corpuscles the greater the elasticity and the greater the conductivity, the conductivity being simply the number of free corpuscles per cubic centimeter. The greater the number of corpuscles in the positive groupings, *i. e.*, the greater the molecular mass, the less the conductivity.

In presenting this summary I am aware, of course, that much of it is in need of further experimental evidence, and I hope, in time, to supply at least a part of this. It is considered, however, that the scheme here presented has a weight apart from its experimental foundation, in that it is a whole and consistent theory by which for the first time all physical phenomena are reduced to the simplest possible elements.

REGINALD A. FESSENDEN.

ADDRESS OF THE PRESIDENT OF THE SECTION OF GEOLOGY OF THE BRITISH ASSOCIATION.

I.

EVOLUTIONAL GEOLOGY.

THE close of one century, the dawn of another, may naturally suggest some brief retrospective glance over the path along which our science has advanced, and some general survey of its present position from which we may gather hope of its future progress; but other connection with geology the beginnings and endings of centuries have none. The great periods of movement have hitherto begun, as it were, in the early twilight hours, long before the dawn. Thus the first step forward, since which there has been no retreat, was taken by Steno in the year 1669; more than a century elapsed before James Hutton (1785) gave fresh energy and better direction to the faltering steps of the young science; while it was less than a century later (1863) when Lord Kelvin brought to its aid the powers of the higher mathematics and instructed it in the teachings of mod-