'Notes on regeneration and regulation in Planarians,' by F. R. Lillie.

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Secretary.

SCIENTIFIC BOOKS.

Leitfaden der Kartenentwurfslehre für Studierende der Erdkunde und deren Lehrer bearbeitet von PROF. DR. KARL ZÖPPRITZ *in zweiter neubearbeiteter und erweiterter Auflage herausgegeben von* DR. ALOIS BLUNDAU. Erster Theil: Die Projectionslehre. Mit 100 Figuren und zahlreichen Tabellen. Leipzig, B. G. Teubner. 1899.

The first edition of Zöppritz' 'Leitfaden der Kartenentwurfslehre,' a volume of 162 pages, appeared in 1884, and treated of projections, topographical drawing, plotting of itineraries and other matter more remotely connected with the construction of maps, such as the astronomical determination of geographical positions, constructions of geometrical curves, etc. The reputation of the author and the variety of contents secured a favorable reception to the volume, but it is a singular fact that its chief merit, that of opening a warfare upon the almost universal practice of misusing projections, should have been the least appreciated. Zöppritz was the first one in Germany who recognized the far-reaching importance of Tissot's investigations concerning distortions of projections and utilized them for his work, but the innovation was coldly received by German geographers. It was not until two years after Zöppritz' death, in 1887, that Hammer took up the fight and by his masterly translation of Tissot's mémoire succeeded in securing foothold for Tissot's ideas in the German scientific mind, and thus removed the last doubt about an ultimate victory of the principle "that the proper selection of a projection for a special purpose is not, like fashion, a matter of custom and taste, but dictated with analytical rigor," and thus prepared the way for a new edition of Zöppritz' 'Leitfaden.' The first part of this new edition, now in our hands, treats of projections exclusively and will be followed by a second volume, devoted to topographical drawing. No disparagement of the memory of Zöppritz is implied, but a tribute is paid to the sound judgment and industry of his successor, Blundau, for the proper assimilation of the new information and experience accumulated since Zöppritz' death, if I make the statement that the present edition is superior to the first one by a more exhaustive and systematic treatment of the subject in hand, by a general application of Tissot's tests and by the subordination of mere geometrical construction to computation.

The 'Leitfaden' was designed primarily as a guide to students and professors of German universities; and it is a significant indication of the conditions prevailing in these institutions, that Zöppritz should have deemed it necessary to apologize for the introduction of two or three formulas of spherical trigonometry, and that Blundau should make it a rule to avoid calculus; and in several instances, such as in giving the formula for equatorial distances in Mercator's projection, to rather omit the proof.

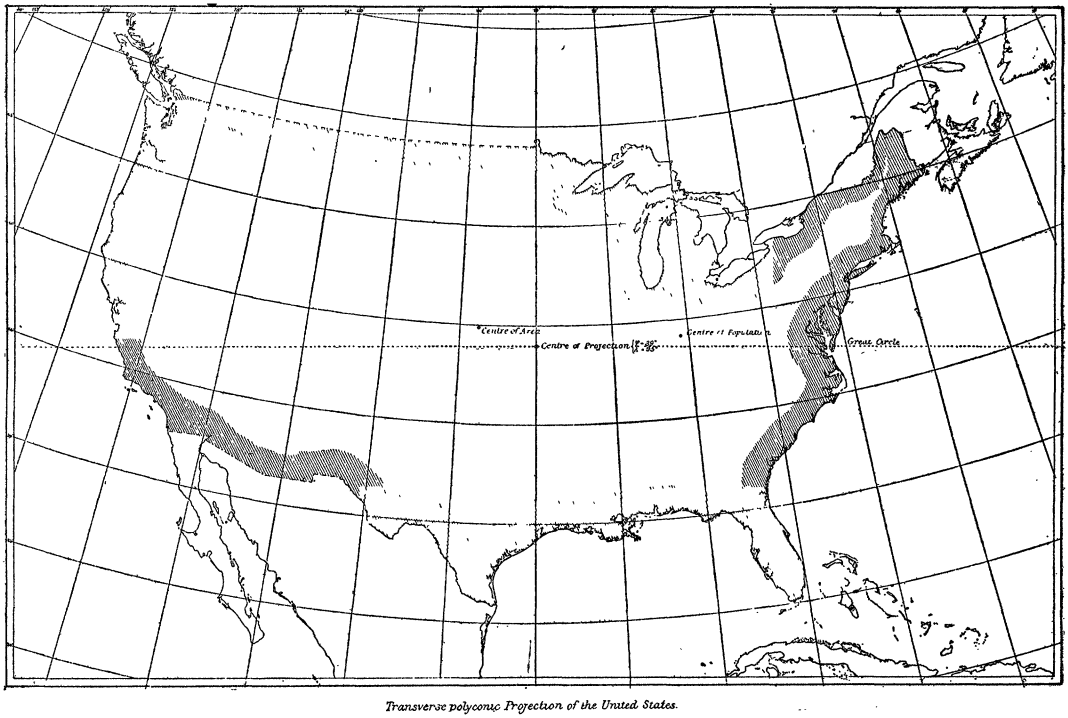
In passing over the contents of this volume in cursory review, I propose to pause only when meeting meritorious projections, which appear to have been neglected in this country, or such as recommend themselves to cartographers for special purposes.

Azimuthal projections.—This class of projections, although of considerable antiquity, has of late years become almost totally neglected. Many of them possess peculiar properties not found in any other projections which make them well suited for special purposes. Blundau introduces a very salutary departure from the usual practice of treating these projections separately by stating their common properties and the distinguishing features of each kind. It is a common property of all azimuthal projections that every point on the surface to be represented is shown in its true azimuth from a central point, and the distinguishing feature of each kind is the particular function of the spherical zenith distance of the point from the center of the map which is adopted as a measure of its distance, or $m = f(\delta)$, where m represents the radius or distance from the center of the map, and δ the zenith distance. The first projection coming under consideration is *Postel's*, in which the distances are given by the arcs

($m = \text{arc } \delta$). No projection can be devised which gives *all* distances correctly; Postel's gives the correct distance from one point, the center of the map; but for a limited area, even one as large as the United States which requires zenith distances of 21°, the error in distances is much less than in any of the other projections in common use, not excepting the polyconic, and it is to be greatly recommended for such maps as railroad and post-route maps. The most important one of these projections however is *Lambert's equivalent*, which gives distances by the chord ($m = 2 \sin \frac{1}{2}\delta$), because it gives true areas which for most ordinary purposes is the most desirable requisite. It would be most admirably suited for census purposes; for accuracy of distances it is inferior to the preceding one but superior to the polyconic. [For 20° zenith distance Blundau gives the elements of linear distortion for Postel's as follows: Tangential direction $a = 1.021$; central direction $b = 1.000$, and for Lambert's $a = 1.015$, $b = 0.985$.] In the *gnomonic* projection radial distances are given by the tangents of the zenith distances ($m = \text{tang } \delta$) it has the valuable property possessed by no other projection, that all great circles are represented by straight lines. For this reason it is a valuable adjunct to sailing charts, and the Hydrographic Office has published charts of all the great oceans on this projection. This is also a perspective projection with the point of view at the center of the earth. The *orthographic* projection, in which the sines of the zenith distances are taken as radii ($m = \sin \delta$) may also be regarded as a perspective one, it interests us only in so far as all lunar charts are constructed on it, in fact, cannot be constructed on any other. The *stereographic* projection which has the formula $m = 2 \text{ tang } \frac{1}{2}/\delta$ deserves mention on account of its antiquity, having been already used by Hipparchus (160–125 B. C.), and it is the only azimuthal projection which has no angular distortion or in which every circle is projected as a circle. It may also be treated as a perspective projection if the point of view is taken on the surface and the earth is assumed to be transparent; the map will then appear reversed like the type of a print. Amongst the conventional azimuthal projections I wish to call attention to one proposed by Hammer in Petermann's Mitt. of 1892, which consists of a Lambert's azimuthal hemisphere converted into a full sphere by a manipulation suggested by Aitow. This projection appears to be well adapted to replace the Mercator projections in atlases of Physical Geography, and has the advantage over Mollweide's, so often used for that purpose, that angular distortions are greatly reduced, besides being, like the latter, equivalent.

Conical Projections.—The transition of azimuthal projections into conical projections is effected by substituting the apex of the developing cone in the place of the center of the map, and by reducing the azimuthal angles in a common ratio, *i. e.*, multiplying them by a constant factor, which in the *ordinary conical projection* is the cosine of the polar distance of the tangent parallel. This *ordinary conical projection*, in which the central parallel and the distances between the parallels only preserve their relative values, while all other parallels and areas are exaggerated, should be used only where facility of construction is the principal, and accuracy a subordinate consideration. Two different methods are in use for compromising the exaggeration of the parallels. In the first one two parallels instead of one are given their true dimensions, one at half the distance between the lowest parallel and the middle one, the other at half the distance between the middle and highest parallel. The radii of the concentric parallels are prescribed by the condition that the latter shall retain their true distance from each other. This method is usually called that by an *intersecting cone*, which designation is misleading for the reason that the cone thus constructed is an ideal one which cannot be directly applied to the sphere; Blundau proposes to call it the *De L'Isle conic projection* after the French astronomer who made the first use of it. The second modification of the conical projection which is also frequently used in atlases, was devised by Mercator and should be called after him. Here also two parallels are given in their true dimensions, just as in the preceding projection, but the radii of the concentric parallels are not those of an ideal cone, but those furnished by a cone tangent to the middle parallel; the meridians no longer cross the parallels at right angles, nor do they meet in one point, the apex of the developed cone. By sacrificing the equidistance of the parallels, conical projections may be constructed which are either equivalent or conformal (without angular distortions) and yet retain either one or two true parallels. Lambert and Gauss have devoted considerable study to these projections; but there is one, devised by Albers in 1805, which has equivalence and two true parallels, qualities which should entitle it to special consideration, but hitherto it has not received any special trial. Amongst the pseudo or conventional conical projections, Bonne's occupies the first rank; although as long ago as 1880, Tissot exposed its glaring defects, it has nearly up to the present date retained almost undisputed possession of the principal atlases. This has sometimes been ascribed to *undue French influence*, but it may just as well be a consequence of custom and convenience. It is somewhat surprising that after all that has been said by Tissot and Zöppritz the 'Geodetic International Association' should persist in making use of Bonne's projection in their reports.

Regarding the *polyconic projection*, devised by the Coast Survey and very extensively used in this country, Blundau has not much to say that might be considered as very flattering. He says in substance that the distortion which in the equi-distant conic projections is most perceptible near the upper and lower borders is here shifted to the more distant parallels, and that it is of real advantage only when applied



Transverse polyconic Projection of the United States.
Drawn by A. Lindenkohl

to the representation of regions of predominating meridional dimensions. It is a fact that in maps of the United States on this projection, like those issued by the General Land Office, the Geological Survey and the Census, the exaggeration of the meridians and areas near the Atlantic and Pacific borders reaches fully $6\frac{1}{2}$ per cent., which is nearly three times as much as might be considered a fair allowance, and it seriously interferes with the use of these maps for the measurement of either distances or areas near these borders. But there are several methods by which the undoubted advantages of the polyconic projection can be preserved and its disadvantages greatly reduced, to which Blundau cannot be an entire stranger. One way would be not to adhere strictly to one central meridian, but in the case of an oblique map to shift the apices of the tangent cones in such a manner that the central meridians pass as nearly as may be through the middle of the map; the meridians would then assume a spiral shape. I have used this method on several occasions, but Hammer is the first one, I believe, who has called public attention to it. If the map should have a predominating east and west dimension, the developing cones may be applied in a transverse position; some great circle passing centrally through the map might be treated as a central meridian and the poles might be transferred to the equator. In the accompanying sketch I have constructed a projection of the United States on this principle; the 95° long. is substituted for the equator and the great circle, which in lat. 39°, is perpendicular to the meridian of 95°, is taken as central meridian. The distortion, which in an ordinary polyconic projection, accumulates near the right (east) and left (west) borders is here transferred to the vicinity of the upper (north) and lower (south) borders.* It may not be amiss to mention that for certain purposes Blundau has recommended the employment of abnormal conic projections, in which case the axis of the cone does not coincide with the axis of the earth and gives as illustration a map of Africa, in which the point in which the equator intersects the western coast of the continent is chosen as apex of the cone. On this projection the elements of distortion show very favorable conditions, but it has the serious defect of leaving a blank space by the complete development of the cone on a plane, and since Hammer has shown in Petermann's Mitt. of 1894, that just as favorable conditions may be attained by an equivalent azimuthal projection, the application of abnormal conic projections does not appear to deserve much encouragement. The *polyhedral* projection has a trapezoidal shape. It is now generally adopted in Europe for the single sheets of serial publications of government surveys on a large scale (between 1/20000 and 1/100000); it is similarly used by the U. S. Geological Survey, and has been proposed by Penck for the prospective map of the world on the one millionth scale. It is the shape which any part of the earth's surface, enclosed by two parallels and two meridians will assume in many kinds of projections now in use, provided the size of the section is small enough (not more than 15 or 30 minutes or one degree of latitude and longitude) to allow the substitution of the chord for the arc of the parallels. Consequently this so-called polyhedral projection, properly speaking, is no projection at all; the separate sheets may be joined in different ways, such as will conform to either a polyconic or to a simple tangent conical projection.

Cylindric Projections.—The transition from conic to cylindric projections takes place when the constant factor (n) of the azimuthal angle becomes 0. In this case the meridians become straight parallel lines and the contact occurs at the equator. This great circle as well as the parallels appear also as straight lines, intersecting the meridians at right angles. These conditions are common to all cylindric projections, and the only difference between the several varieties consists in the particular function of the latitude $y = f(\phi)$ which is adopted as measure for the distance of the parallels from the equator. If the meridional arcs are given in their true dimensions ($y = \text{arc}\ \phi$) we have the *square* projection which should not be used for more than 15° from the equator. For broader zones an 'intersecting' cylinder should be substituted (corresponding to the intersecting cone of the De Lisle's projection) which will transform the square into a rectangular projection. The cylin-

* In the polyconic projection the lines of equal linear (and areal) distortion are parallel to the central meridian, and the distortion for modern distances increases as the square of the distance from this line. [Distortion $= 0.01\left(\frac{\Delta}{8^\circ.1}\right)^2$ where $\Delta =$ distance from central meridian in degrees of arch of great circle.] Since the distance across the United States from north to south, is only about three-fifths of that from east to west, it follows that by the above manipulation the maximum of distortion is reduced from $6\frac{1}{2}$ per cent. to about $2\frac{1}{2}$ per cent.

der may also be made tangent to a meridian instead of the equator by which the square projection is changed into Cassini's, which, for moderate distances from the central meridian, greatly resembles the polyconic. Cylindric projections may also be made equivalent ($y = \sin \phi$), when we obtain Lambert's equivalent cylinder projection, but it is only when it assumes that shape (called *conformal* by Gauss and *autogonal* by Tissot), in which there is no angular distortion, or in which the elementary arcs of longitude and latitude preserve their relative dimensions, when $y = \log$ nat. tang $(45° + \frac{1}{2}\phi)$, that this projection, as the Mercator, has attained an importance which puts all others into the shade. The vexatious question of nearly fifty years standing whether the Mercator or polyconic projection offers greater advantages for hydrographic charts, does not appear to have been finally settled yet. Granting that sailing by the *orthodrome* is preferable to sailing by the *loxodrome*, and that the polyconic gives the *orthodrome* in a more nearly straight line than the Mercator, this departure from a straight line may assume sufficient proportions to render the polyconic chart unreliable while, with the positive knowledge that, with the exception of Meridians and the equator, all great circles are curves on the Mercator chart, no sailor will meet with any difficulty in laying down *orthodromes* on a Mercator chart without calculations with the assistance of a gnomonic chart; but the chief argument in favor of Mercator charts will always remain the facility of laying down positions and courses. For hydrographic charts which are not intended to be used as sailing charts, or which are on such large scales that the sea area occupies but a narrow margin, the employment of the Mercator projection should be avoided for the reason that the differences within the narrow limits between the two contending classes of projections are sufficiently small to allow the use of a polyconic for the same purpose as the Mercator chart, but they are great enough to render a Mercator chart unfit for most of the uses a map may be put to, such as the measurement of distances and areas. Another very extensive use to which the Mercator projection has been put is for planispheres in atlases, especially for the purpose of disseminating and illustrating information of a statistical or physical nature, and here Blundau is slightly mistaken if he assumes that in this capacity it could very properly be superseded by projections of less objectionable features. For many purposes, for meteorological charts for instance, it is of greater importance to have the cardinal directions, north and south, east and west always point the same way and remain parallel to the borders of the chart, than to have correct areas and to have these lines run in every possible way. The objection so frequently raised against the Mercator projection that it does not furnish any indications for the courses of great circles, may readily be overcome by constructing on transparent paper a system of great circles which intersect the equator in two opposite points. If, however, the Mercator chart is used to illustrate conditions in which correctness of area is more important than parallelism in identical bearings, like those showing density of population, or the distribution of animals and plants, it may very properly be superseded by others of an equivalent nature; if no other, by Mollweide's which is very easily constructed, and always available.

The closing chapter about projections treats of the selection of one with least distortion and gives a résumé of the results of Tissot's investigations with tables giving the relative values of the elements of 'deformation' for the principal projections in use, reducing the formerly often troublesome question about the relation of a projection for a special purpose to one of easy solution. But this applies only to maps embracing large areas, as those of continents; the question about the projection with least distortion for areas of restricted size, such as European countries, does not admit of a general solution, but has to be solved for each country separately and this solution is definite. In no such case would the projection be one admitting geometrical construction; it would be one entirely dependent upon analysis.

Having carefully perused the volume from beginning to end, I conclude that it is an eminently practical book and gives to the cartographer all the information he may possibly need regarding the nature of projections and their constructions; but it is because of this utilitarian tendency, together with Blundau's manifest aversion to cross the threshold of higher mathematics, that to the disciples of Gauss, Lagrange and Tissot (who care more for the theory than for the application of projections), the treatment of 'autogonal' or 'conformal' projections is not altogether satisfactory. He should have introduced the elements of the theory of functions without which a proper treatment of these projections is impossible. He should at least have said that the coördinates of a sphere (u, v) are connected with those (x, y) of Mercator's projection by the relation

$$x + iy = u + i \log \operatorname{tang} \left(\frac{\pi}{4} + \frac{v}{2}\right)$$

and that by suitably taking Φ, the coördinates of any other autogonal projection (X, Y) are given by the relation

$$X + iY = \Phi\,(x + iy).$$

If $\Phi(\ \ ) \equiv e^{\pm i(\ \ )}$ a stereographic projection is obtained in which the north or south pole is the center of the map. If to this stereographic projection we apply $\Phi(\ \ ) \equiv -K + \mathrm{cn}^{-1}(\ \ )$, we obtain Peirce's *quincuncial* projection, etc.

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The Evolution of General Ideas. By TH. RIBOT. Translated by FRANCES A. WELBY. Chicago, Open Court Publishing Company. 1899. Pp. 231.

The scope and mode of treatment of Professor Ribot's monographs are well known; and this one follows closely the general plan of those which have preceded it. The topic itself is a most interesting one; the genesis of the powers of abstraction and the evolution of the general ideas which represent the fruit of such abstraction. The material for the early forms of the process is to be found in the mental operations of animals, of children and savages, and of deaf-mutes before education. These have, in common, the absence of words and the dependence of the abstraction upon the generic images formed by sense-experiences. The intermediate stage involves the use of words and is reflected in the character and growth of language. The word fixes the material basis of the abstraction and aids the mind in focusing upon the 'abstracted' relation. In the highest stages of abstraction the element of representation has faded away, and the word practically constitutes its entire content. Following the description and illustration of these processes is a special consideration of the development of the special concepts of number, space, time, cause, law and species. The fundamental insistence upon experience as the basis of such development and the suggestiveness of the genetic point of view find apt application in this part of the thesis.

But in spite of a well-chosen theme and of a discerning utilization of the literature; in spite of much interesting material and suggestive modes of treatment, the general impression of the book is a rather unsatisfactory one. There is a judicious occupation of points of advantage; skirmish lines are thrown out in various directions, a campaign is carefully planned—and the planning is rather too freely discussed—but there is no vigorous nor successful attack upon the real stronghold of the situation. None the less, the monograph will be a helpful one to the student, who will appreciate the significance of the problem, as Professor Ribot outlines it, and who will be led by the interest of the exposition to assimilate the essential factors involved in the growth and functioning of the powers of abstraction. His attention may be specially directed to a point touched upon in the last chapter, but worthy of more extensive treatment; namely, that the criterion of the utility of abstraction is not to be sought merely in its products—such as the higher mathematics or metaphysics—but as well in the process itself, by which the individual learns to focus the attention at will upon any aspect of a complex experience which may become important. And, in the same chapter, he should not overlook the suggestive delineation of the parts played by theory and practice, by the incentive of genius and by gradual development, in the actual history of the sciences depending upon abstraction.

Of the translation, the best that can be said is that it is barely satisfactory. A good translation of a psychological work involves the absorption and re-expression of the author's perspective of ideas, not of his words alone;