

The rather tedious treatment of the transit instrument in 45 pages contains no reference to two innovations, the most important since the invention of the chronograph, which have been successfully introduced into modern European practice with this instrument. The invention of the transit micrometer has furnished a simple and effective means of almost perfectly eliminating the influence of personal equation in transit observations, and the practice which has come into vogue in connection with this micrometer, of reversing the instrument upon every star observed, equatorial as well as polar, is revolutionary in its effect upon work with a portable transit. This reversal may be employed equally well with any good form of transit, and furnishes the very great advantage of automatically eliminating from the observation of each star a host of errors, such as the effects of collimation, flexure, inequality of pivots, etc., and the further signal advantage that the number of unknown quantities in the observation equation furnished by the star is reduced from the three or four recommended by the author to two. A work in which these advances are ignored is of doubtful service in 'illustration of the best modern practice' with the transit instrument.

The most serious general criticism to be brought against Professor Campbell's treatment of his subject is illustrated above; that he has not chosen methods and formulæ with sufficient reference to economizing the time and labor of the computer, although for the guidance of the latter, in matters left to his own judgment, there is furnished in Appendix A an excellent series of hints on computing.

Other points at which the author nods in varying degree from obscurity of statement to absolute error are the foot-note to p. 207 relative to projecting the sun's image upon a screen by 'focusing the eye-piece so that the images of the sun and wire are seen on the paper' and the statement, p. 75, that in the determination of the value of a revolution of a micrometer screw from transits of a star 'the effect of refraction is inappreciable if the observations are made near the meridian.' The first quotation is technically correct, but few students would infer from it that two distinct operations are to be

performed, one of which in the ordinary type of instrument consists in moving the objective. The second quotation is quite wrong if more than three significant figures are required in the result and in the illustrative example given by the author, by neglecting the refraction he has vitiated the final result to an amount twice as great as the probable error which he assigns to it.

An error made with all the emphasis of italics requires that an altitude measured from the sea horizon shall be corrected for refraction before the dip of the horizon is taken into account, and another error occurs at p. 160 (and also in the first edition of the work) where the rate of a chronometer is represented as a linear function of the temperature, although experience and theory alike indicate that the relation between these quantities must be expressed by an equation of at least the second degree.

It is very doubtful if a consensus of astronomical opinion could be brought to sanction the method of reduction of zenith telescope latitudes recommended by the author, viz: a least-square solution in which the value of a level division is introduced as an unknown quantity. Under all ordinary conditions the observations should be so conducted that the direct determination of this quantity shall far outweigh any value which can be derived from the latitude observations.

The mechanical execution of the work is excellent; it is provided with an adequate index and illustrated by cuts which are in the main well chosen, although here we regret that the author has selected as 'an excellent form of the prismatic (broken) transit' an instrument which is a complete failure and has been consigned to oblivion by the government bureau for which it was constructed.

G. C. C.

*Infinitesimal Analysis.* Vol. I., Elementary: Real Variables. By WILLIAM BENJAMIN SMITH, Professor of Mathematics in Tulane University. New York, The Macmillan Company. 1898. 8vo. Pp. xvi + 352. Price, \$3.25.

The book in hand is the initial volume of a treatise in course of composition which is to

consist of three volumes. Concerning the merits of this first part in so far as these may ultimately depend on its relations to the rest of the work, it would be premature to form an opinion. Apart, however, from this contingent and inchoate character of the volume, it has a unity and maturity of its own, being avowedly written as an introduction to the calculus, and as such is properly before the public for review.

The author's aim has been "to penetrate as far as possible, and in as many directions, into the subject—that the student should attain as wide knowledge of the matter, as full comprehension of the methods, and as clear consciousness of the spirit and power of this analysis as the nature of the case would admit." It is not easy to realize so high and composite an ideal. The nature of the case, it is well known, presents some grave difficulties. Of these the most obstinate inheres in the combination of doctrine and applications, of the general and abstract with the particular and concrete, in securing, despite the fragmentariness incident to illustration and example, the effect of unity and wholeness in the development of theory. French and German writers, such as Jordan, Harnack, Stolz, escape the difficulty of combining theory and practice by simply ignoring the latter. By this easy disregard of the needs of all students except specialists in graduate years, these authors are enabled to attain a coherency and symmetry of development which lend to their work, besides the scientific, something of an artistic character. The Englishman, on the other hand, is prone to lose both of these advantages by sinning in the opposite direction, by a distinct subordination of theory to practice, a collocation, however interesting and useful, of exercises for the ingenuity of students, being neither an æsthetic nor, in strictness, a scientific production.

The problem of overcoming instead of dodging the difficulty in question, of escaping the mentioned vices without losing their peculiar virtues, admits of only approximate solution. The necessary compromise has, as is well known, been skilfully effected in German in the deservedly much-praised treatise by Kiepert. In the book under review a notably similar success has been achieved in English. In fact,

these two works, though differing widely in method and detail, are closely allied in spirit and aim. The motive in both is to guide and inspire; both are honest, anxious not to deceive, faithful in indicating assumptions and limitations, and, while seeking first to be intelligible, are in general as rigorous as circumstances will allow. Neither author forgets that in last analysis his science resides in theory, which, therefore, properly receives the greater emphasis. Nevertheless, both works abound in concrete examples. These, curiously enough, are nearly all worked out in the German text, while in the English most of them are, as usual, left as exercises for the student.

In point of matter these works are not coincident nor coextensive either with one another or with their rivals, such as the treatises by Edwards, Williamson and Greenhill. For example, Kiepert gives a concise preliminary treatment of certain algebraic themes, as the binomial theorem, the potential and logarithmic series, convergency and divergency, determinants and others, while Smith has, for the sake of brevity, presumed knowledge of some of these, treatment of others being reserved for Vol. II. A like reservation is made in case of the complex variable, and, save for an elegant though very brief account, in case also of differential equations, to each of which topics Kiepert gives an introduction. On the other hand, Smith, like Williamson, deals with the gamma functions and inserts a helpful chapter on curve tracing, while Kiepert excludes the former subject and considers the latter but incidentally. The omission by the American, as by the German, of the theory of probability and the calculus of variations is a noticeable departure from British precedent.

The opening chapter of the volume before us is, in many respects, an admirable presentation of fundamental concepts and operations. The path pursued leads quickly into the heart of the subject. The student meets first things of first importance. The notion of limit is at once lifted into prominence, being carefully unfolded at the very outset, and employed without delay in definition and proof. The infinitesimal is correctly defined, and its *subjective* character is pointed out, the fact, namely, that

its essence consists not in any value it may assume, but in our power over it to make it small at will. The advantage of introducing the infinite in connection with the infinitesimal is not availed of; the former notion is, in fact, not defined at all. Similarly, the discussion of infinitesimals of higher order would have been enhanced by mention at least of the complementary topic. The author retains the entire respectable but obsolescent definition of algebraic function, the modern definition of such function as the root of an equation having coefficients rational in the independent variable, being apparently nowhere employed. Continuity is not adequately treated, and this preëminently important subject will doubtless be accorded suitable recognition in the next volume. Numerous examples of discontinuity, such as are given by Kiepert, are well-nigh indispensable aids to the student, whose attention, moreover, might with profit have been *explicitly* directed to the fact that the derivability of a function always implies, though is not implicit in, its continuity. The idea of uniform continuity is introduced, but only on occasion, as in the deduction of the theorem of total differential. On p. 11 the reader is warned against regarding  $\frac{dy}{dx}$  as a frac-

tion, and on p. 79, where the differential notation is explained, he is cautioned against 'attempting a magnitudinal interpretation' of  $du$  and  $dx$  in the 'symbolic equation  $du = u_x dx$ ,' which 'means that the derivative of  $u$  as to  $x$  is  $u_x$ .' The author's view of this critical matter, while not in full accord with that, for example, of Jordan's *Cours*, p. 61, is nevertheless intelligible, consistent and adequate.

The early introduction (Chapter II.) of the notions of integral and integration is attended with obvious advantages. The treatment is good scientifically and pedagogically. A specially commendable didactic feature is the calculation of several integrals by actually making the required subdivisions, forming the corresponding products, generalizing, and throwing the summation into a form suitable for perceiving its limit.

Space is wanting for briefest comment on many interesting sections as those dealing with illusory forms, maxima and minima, geomet-

ric interpretation of higher derivatives, change of variable, partial integration, Jacobians, multiple integrals, parametric derivation, and so on.

It remains to say that not the least praiseworthy quality of the book is found in its style. To be scientific it is not necessary to be vulgar. The volume affords another illustration of the compatibility of rigor and austerity of thought with a generous regard for the amenities of expression. To many the book will be distinctly the more attractive because of its human flavor, its dialectic color, its life, an occasional glance at the philosophic phases of the subject. A rare union of conciseness with precision and clearness is characteristic. For judicious accentuation little more could be desired. The reader is taken into confidence, invited to accompany rather than to follow. The work is not a compilation and not a mechanical structure; it is rather an organism, a growth, notable for its merits, though, of course, sharing in a measure the imperfections of its kind.

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*Defective Eyesight; The Principles of its Relief by Glasses.* By D. B. ST. JOHN ROOSA, M.D., LL.D., Professor Emeritus of Diseases of the Eye, New York Post-Graduate Medical School and Hospital; Surgeon to the Manhattan Eye and Ear Hospital; Consulting Surgeon to the Brooklyn Eye and Ear Hospital, etc. New York, The Macmillan Company. 1899. 8vo. Pp. 193.

This work is practically a revised edition of the author's little book 'On the Determination of the Necessity for Wearing Glasses,' published as one of the 'Physician's Leisure Library Series' in 1887, by George S. Davis, of Detroit, Michigan.

The volume has gained much by its revision, has had some excellent illustrated matter introduced and has been considerably enlarged.

The subject is divided into seven parts, all of which are written in the author's well-known easy style, making those who have had the pleasure of personally reading his writings more firmly convinced of his earnestness and erudition.

Considering the subject-matter in its given