It is not a number ∞ we are ever concerned with. When we say $w(a) = \infty$ we really mean $\lim |w(z)| = \infty$, $x = a$. Again when we ask how does $w(z)$ behave for $z = \infty$ we really mean how does w $(1/\varsigma)$ behave in the *vicinity* of $\varsigma = o$ where $\varsigma = 1/z$. Thereby ς is never required to assume the value of o. On using the sphere instead of the plane we get the punktierte Kugel. The missing point we can supply or not at our option. In any case no number shall correspond to it. We firmly believe that the easy intuitional way of treating ∞ in the function theory of a complex variable must be modified as here indicated.

The last chapter is devoted to a brief *aperçu* of the function theory from the standpoint of Cauchy and Riemann. We cannot appreciate the difficulties mentioned in § 164 as underlying the definition of a function from the Cauchy-Riemann standpoint. They seem to us to be due to the belief on the part of the authors that we must take the whole z-plane into our definition from this point of view. Such is not the case. As a domain D for the variable z we take any point multiplicity consisting only of *interior* points. If it be possible to pass from any point of D to any other of it along a continuous curve $x = \phi(t)$, $y = \psi(t)$ we say D is a simple domain. Otherwise D is composed of simple domains $D = D_1 + D_2 + \ldots$ To get a synectic function $w(z)$ for D we take two single valued functions $u(x,y)$, $v(x,y)$ defined over D and such that for every point in D they have a total differential and satisfy the equation.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

In any one of these simple regions as R_α, $w(z)$ can be developed into an integral positive power series. The analytic function $f(z)$ obtained from one of these elements is identical with $w(z)$. There certainly is no reason to suppose that $f(z)$ when continued into another region R_β should be identical with $w(z)$ in this region. This seems to answer all the objections in I and II of this article. Indeed, the advantage seems to be decidedly on the side of Cauchy, for exactly one of the points urged against Cauchy's theory is now without force, while it is, indeed, an important matter from Weierstrass' standpoint. This, in the author's words, is: "That Cauchy's definition implies in various ways a considerable preliminary grasp of the logical possibilities attached to the study of singular points." From our standpoint we fix in advance the domain D; it has no more singular points than we choose to assign. Not so with the analytic function. Here an element is given, one singular point must lie on its circle of convergence. Where the others are is a subject of further study.

We cannot see the difficulty mentioned under III. It is, ineeed, an interesting matter to know 'the irreducible minimum of conditions to impose on $w(z)$,' but it seems to us nowise necessary. It suffices that we know the necessary and sufficient conditions in order that $w(z)$ can be developed according to Taylor's Theorem. This we know and we have taken them into our definition of $w(z)$. It may be interesting to remark, however, that these conditions are already known, as will appear in a remarkable paper of E. Goursat shortly to be published.

We close, congratulating the authors for writing a work which we believe will prove an excellent aid to acquire some of the essentials of the theory of function. We should have preferred to see the two theories of Cauchy and Weierstrass blended together into an organic and indivisible whole. Although these two theories grew up quite distinct, they have already been welded into *one* greater and more powerful theory. It is only the purist who still tenaciously clings to the methods of Weierstrass. It seems, therefore, very desirable to us that an introductory work should be written more in accordance with this fact.

James Pierpont.
Yale University, March, 1899.

A Handbook of Metallurgy. By Dr. Carl Schnabel. Translated by Henry Louis. New York, The Macmillan Company. Two volumes, medium 8vo. Total pages, 1608. Illustrated. Volume 1, copper, lead, silver, gold. Volume 2, zinc, cadmium, mercury, bismuth, tin, antimony, arsenic, nickel, cobalt, platinum, aluminum. Price, $10.00.

The author states in the preface that, while many exhaustive works have appeared on the

metallurgy of individual metals, the few books on general metallurgy were arranged as text-books and made no pretence of thoroughness of detail or treatment. With these facts in mind the present work was compiled, with the stated object of giving a complete account of the metallurgical treatment of every one of the metals ordinarily employed, together with the recent improvements in the art, stating the underlying scientific principles and illustrating by actual practice.

This object is highly commendable, but the statement is rather misleading, as iron and steel have been entirely omitted and no mention made of the omission or of a subsequent volume upon this all-important branch of metallurgy. This fact should have been stated plainly by the author in the preface and by the publisher in the advertisements.

Dr. Percy's historic work was selected as the basis, and on this are grouped many facts from the works of modern writers, notably Hofman, on lead; Peters, on copper; Egleston, on gold and silver, and Borchers on electro-metallurgy. The work is quite exhaustive in character, as the grand total of 1608 pages indicates, but, unfortunately, the exhaustion is not limited to the subject-matter of the book and ofttimes extends to the reader, as much of the material is vague and unnecssarily verbose. The work lacks that clearness of description, lucidity of arrangement and concisement of statement so needful in the treatment of a large subject and so appreciated by American readers with whom time is an object.

It is to be regretted that much ancient material is perpetuated in excruciating detail, particularly as it is so interwoven with modern practice that the general reader is left in doubt what *is* in use at the present time. To illustrate this, under the chapter on silver, barrel amalgamation is quoted as now in use at the Pelican Mill, Georgetown, Colo., while, as a matter of fact, it was there abandoned twenty years ago. Another instance, under the chapter on zinc, the furnace used in the old English process—that rare bird of antiquity—shows forth resplendent in full detailed illustration. As to this furnace, Dr. Percy, in 1869, failed to find even the ruins of its foundation.

The large amount of material collected in these two volumes contains much of value to the specialist, but it is too encyclopedic in character to be of any marked assistance to the general reader. Its main value is for reference in a scientific or technical library.

A few minor errors, such as the location of Boston in Vermont (Vol. 1, p. 115) and Orford in New Jersey (Vol. 2, p. 104), may be overlooked in a work of this large size.

The criticism of this work may be considered harsh, but the eminent position occupied by Dr. Schnabel leads one to expect the highest standard of work and to be disappointed if it is not attained.

J. STRUTHERS.

BOOKS RECEIVED.

Organic Chemistry. Edited by R. ANSCHÜTZ. Authorized translation by EDGAR F. SMITH. Vol. I., Chemistry of the Aliphatic Series. Philadelphia. P. Blakiston's Son & Co. 1899. Pp. xviii + 625. $3.00.

Commercial Organic Analysis. ALFRED H. ALLEN, Philadelphia, P. Blakiston's Son & Co. 1899. Vol. II., Part I. Pp. x + 337. $3.50.

The Spirit of Organic Chemistry. ARTHUR LACHMAN. With an introduction by PAUL C. FREER. New York, The Macmillan Company. 1899. Pp. xviii + 299. $1.50.

The Arithmetic of Chemistry. JOHN WADDELL. New York and London, The Macmillan Company. 1899. Pp. viii + 133. 90 cents.

Algemeine Erdkunde. J. HANN, ED. BRÜCKNER and A. KIRCHHOFF. III., Abteilung Pflanzen-und Tierverbreitung. ALFRED KIRCHHOFF. Prague. Wien und Leipzig, F. Tempsky. 1899. Pp. xi + 327.

SCIENTIFIC JOURNALS AND ARTICLES.

The Botanical Gazette for March contains the following papers: D. H. Campbell: 'Notes on the structure of the embryo-sac in *Sparganium* and *Lysichiton*,' pp. 153–166, with one plate. This is a continuation of the author's studies of the primitive monocotyledons. The discovery of special interest is the extraordinary development of the antipodal cells in Sparganium, another evidence of the variable nature of the antipodal region. H. C. Cowles: 'The ecological relations of the vegetation on the sand