

disposition for later reactions. To be sure, many of these discharges lead finally to muscle contractions which bring with them centripetal sensations from the joints, the muscles, the tendons, and these muscle and joint sensations themselves then become a part in the idea, for instance, of time, of space, of feeling. But the new part only reinforces the general tone which is given in the general discharge, and gives to it only the exact detail which gets its character just through the blending of these sensations of completed reactions with the accompaniments of the central discharge.

A consistent psychology thus may start with the following principles: It considers all variations of mental life as variations of the content of consciousness, and this content as a complex object, including in this first presupposition a complicated transformation of the real inner life, a transformation by which the subjectifying view of real life is denied for the causal psychological system. Every content of consciousness is further considered as a complex of sensations, that is, of possible elements of perceptive ideas. Every sensation is considered as having a fourfold manifoldness, varying in kind, in strength, in vividness and in value. The physiological basis of every sensation, and thus of every psychical element, is the physical process by which a centripetal stimulation becomes transformed into a centrifugal impulse, the kind depending upon the locality of the centripetal channel, the strength upon the quantity of the stimulus, the value upon the locality of the centrifugal channel, and the vividness upon the quantity of the discharge.

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SOPHUS LIE.

ON the eighteenth of February, 1899, the greatest mathematician in the world, Sophus Lie, died at Christiania in Norway.

He was essentially a geometer, though applying his splendid powers of space creation to questions of analysis. From Lie comes the idea that every system of geometry is characterized by its group. In ordinary geometry a surface is a locus of points; in Lie's *Kugel-geometrie* it is the aggregate of spheres touching this surface. By a simple correlation of this sphere-geometry with Pluecker's line-geometry, Lie reached results as unexpected as elegant. The transition from this line-geometry to this sphere-geometry was an example of contact-transformations.

Now contact-transformations find application in the theory of partial differential equations, whereby this theory is vastly clarified. Old problems were settled as sweepingly as new problems were created and solved.

Again, with his *Theorie der Transformationsgruppen*, Lie changed the very face and fashion of modern mathematics.

A magnificent application of his theory of continuous groups is to the general problem of non-Euclidean geometry as formulated by Helmholtz. To this was awarded the great Lobachévski Prize. Not even this award could sufficiently emphasize the epoch-making importance of Lie's work in the evolution of geometry.

Moreover, the foundations of all philosophy are involved. To know the non-Euclidean geometry involves abandonment of the position that axioms as to their concrete content are necessities of the inner intuition; likewise abandonment of the position that axioms are derivable from experience alone.

Lie said that in the whole of modern mathematics the weightiest part is the theory of differential equations, and, true to this conviction, it has always been his aim to deepen and advance this theory. Now it may justly be maintained that in his theory of transformation groups Lie has

himself created the most important of the newer departments of mathematics.

By the introduction of his concept of continuous groups of transformations he put the isolated integration theories of former mathematicians upon a common basis. The masterly reach of Lie's genius is illustrated by his encompassment of the fundamentally important theory of differential invariants associated with the English names Cayley, Cockle, Sylvester, Forsyth.

Thirteen years ago Sylvester announced his conception of 'Reciprocants,' a body of differential invariants not for a group, but for a mere interchange of variables. A number of Englishmen thereupon took up investigations about orthogonal, linear and projective groups, groups in whose transformations interchanges of variables occur as particular cases, and whose differential invariants are consequently classes of reciprocants, and of the analogues of reciprocants, when more variables than two are considered.

Now all these investigations were long subsequent to Lie's consideration of the groups in question as leading cases of a general conception. Thus they were merely secondary investigations!

Again, the theory of complex numbers appears as a part of the great 'Theorie der Transformationsgruppen.' Indeed, this continent of 'transformations' opened up and penetrated with such giant steps by Lie represents the most remarkable advance which mathematics in all its entirety has made in this latter part of the century.

Sophus Lie it was who made prominent the importance of the notion of group, and gave the present form to the theory of continuous groups. This idea, like a brilliant dye, has now so permeated the whole fabric of mathematics that Poincaré actually finds that in Euclid 'the idea of the group was potentially pre-existent,' and that he had 'some obscure instinct for it, without reach-

ing a distinct notion of it.' Thus the last shall be first, and the first last.

In personal character Lie was our ideal of a genius, approachable, outspoken, unconventional, yet at times fierce, intractable. His work is cut short; his influence, his fame, will broaden, will tower from day to day.

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SCIENTIFIC BOOKS.

Colour in Nature: A Study in Biology. By MARION J. NEWBIGIN. London, John Murray. 1898. Pp. 344.

On page 300 of this work we read: "We have now completed our general survey of the colours and colouring-matters of organisms. * * * That the survey as a whole is halting and incomplete must be obvious to all. We have seen that it is as yet impossible to classify pigments in a logical manner; that most of the problems connected with the subject are entirely unsolved." These statements are indeed true; and yet the book is an interesting and valuable one, and will be of real assistance to the working biologist.

The whole subject of color in animals and plants has suffered from the fact that it concerns the chemist and physicist as well as the biologist, and in these days of intense specialization it is hard to find anyone competent to treat the matter in all its aspects. Dr. Newbigin has endeavored, with some success, to take all the more important facts into consideration; but it is practically impossible for any one individual to have that intimate acquaintance with the vital phenomena of every group of living organisms which is necessary for a satisfactory discussion of their coloration. It was Darwin's method to seek the assistance of numerous specialists in different branches, who supplied him with information which he brought together and interpreted in a masterly manner. It may be that Dr. Newbigin has not yet felt justified in asking for such help, but now that she has fairly won her spurs (if one may use such a phrase in regard to a lady) it is not unreasonable to hope that she will adopt the Darwinian system, and eventually provide us