ments of a branchial arch to disappear, not being developed even in tailed amphibians, and it will be safe to call the lower portion of the posterior arch of the hyoid a ceratobranchial, and the adjoining segment an epibranchial. The anterior pair of bones, in the body of the tongue, C, are naturally ceratohyals.

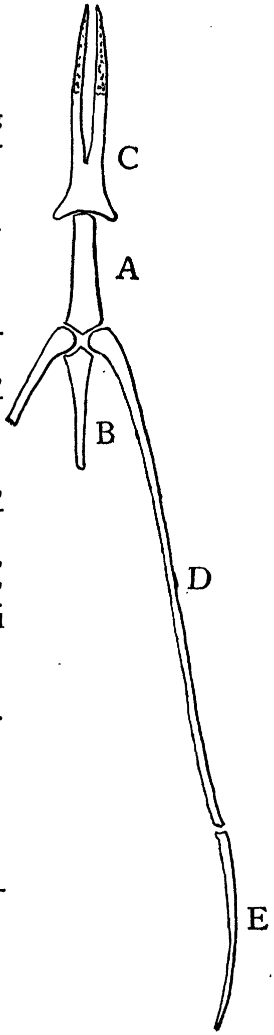

FIG. 1.

Hyoid of Grebe showing component bones, much enlarged.

A First basibranchial.
B Second basibranchial.
C Ceratohyals.
D Ceratobranchials.
E Epibranchials.

A { Basibranchial: Parker. Basihyal: Mivart, Gadow, Beddard-figure.

B { Basibranchial 2: Parker. Urohyal: Mivart, Gadow, Beddard.

C { Ceratohyal: Parker, Mivart, Gadow, Beddard in part. Basihyal: Beddard in part.

D { Ceratobranchial: Parker. Thyrohyal: Mivart. Basibranchial: Gadow. Hypobranchial: Beddard.

E { Epibranchial: Parker. Ceratobranchial: Gadow, Beddard.

The next question, that of the proper name for the anterior basal bone of the hyoid, A, calls for some reflection, since it involves not only the nomenclature of the hyoid in birds, but in mammals as well. This bone is called basibranchial by Parker and basihyal by Gadow, this latter name being ordinarily used for the basal bone of the mammalian hyoid.

A true basihyal, or as it is better called from its relations, glossohyal, is found in fishes at the upper, anterior portion of the hyoid apparatus. It is also present in turtles, where it has the same relation to the tongue as in fishes, and where it ossifies some little time after the first basibranchial, with which it soon becomes confluent. It seems a little doubtful if a true basihyal occurs among birds, although the median piece of cartilage contained in the fleshy portion of the tongue and articulating with the fused ceratohyals in such birds as ducks may represent this bone. The question is one which the embryologist can readily answer. As pointed out by Parker, the true basihyal does not occur in mammals, the term being given to a bone that is morphologically the first basibranchial. It would seem that exact morphological nomenclature should reject the term basihyal for the first basal median bone in the hyoid of birds and mammals, including, of course, man, as there is no reason why human anatomy should stand as a stumbling block in the way of the student of comparative anatomy, although it has often done so.

F. A. LUCAS.

SCIENTIFIC BOOKS.

A Treatise on Universal Algebra. By A. N. WHITEHEAD, M.A., Fellow and Lecturer of Trinity College, Cambridge. Cambridge, University Press; New York, The Macmillan Co. Vol. I. Pp. xxvii+586. Price, $7.50.

By 'Universal Algebra' is meant the various systems of symbolic reasoning allied to ordinary algebra, the chief examples being Hamilton's Quaternions, Grassmann's Calculus of Extension and Boole's Symbolic Logic. The present volume contains an exposition of the general principles of universal algebra, followed by a separate detailed study of the Algebra of Logic and of the Calculus of Extension; the second volume will contain a separate detailed study of Quaternions and Matrices and a detailed comparison of the symbolic structures of the several algebras. The main idea of the work is not unification of the several methods, nor generalization of ordinary algebra so as to include them, but rather the comparative study of their several structures. But, it may be asked, if the branches of universal algebra are essentially distinct from ordinary algebra and from one another, what bond is there to connect them into one whole? A connecting bond is found in the generalized conception of space; the properties and operations involved in that conception are found capable of forming a uniform method of interpretation of the various algebras.

The work is well and clearly written and, when completed, will form an admirable presentation of the subject from the formal view of mathematical analysis. One excellent feature is conservatism in the use of symbols; by this means the author makes his pages easier reading to those who have already studied some of the special branches.

Another excellent feature of the volume consists in the Historical Notes appended to some of the chapters. In these Mr. Whitehead gives a brief history of the development of the special branch, so far as known to him, without making an exhaustive research. The importance of the Historical Notes probably calls for a more exhaustive research, as the work covers a great and growing province of mathematics and will, when completed, be considered one of the best authorities on its subject in the English language.

The feature which is most open to discussion is the view which the author takes of the fundamental nature of mathematics; and it is most important, for it determines the whole plan of the work. In the preface the author thus states his view, in very plain terms: "Mathematics is the development of all types of formal, necessary, deductive reasoning. The reasoning is formal in the sense that the meaning of propositions forms no part of the investigation. The sole concern of mathematics is the inference of proposition from proposition. The justification of the rules of inference in any branch of mathematics is not properly part of mathematics; it is the business of experience or philosophy. The business of mathematics is simply to follow the rules. In this sense all mathematical reasoning is necessary, namely, it has followed the rules. Mathematical reasoning is deductive in the sense that it is based upon definitions which, as far as the validity of the reasoning is concerned (apart from any existential import), need only the test of self-consistency. Thus no external verification of definitions is required by mathematics as long as it is considered merely as mathematics. Mathematical definitions either possess an existential import or are conventional. A mathematical definition with an existential import is the result of an act of pure abstraction. Such definitions are the starting points of applied mathematical sciences; and, in so far as they are given this existential import, they require for verification more than the mere test of self-consistency. Hence a branch of applied mathematics, in so far as it is applied, is not merely deductive, unless in some sense the definitions are held to be guaranteed *a priori* as being true in addition to being self-consistent. A conventional mathematical definition has no existential import. It sets before the mind, by an act of imagination, a set of things with fully-defined self-consistent types of relation. In order that a mathematical science of any importance may be founded upon conventional definitions, the entities created by them must have properties which bear some affinity to the properties of existing things. Thus the distinction between a mathematical definition with an existential import and a conventional definition is not always very obvious from the form in which they are stated. In such a case the definitions and resulting propositions can be construed either as referring to a world of ideas created by convention or as referring exactly or approximately to the world of existing things."

In reply, it may be asked: Is geometry a part of pure mathematics? Its definitions have a very existential import; its terms are not conventions, but denote true ideas; its propositions are more than self-consistent—they are true or false; and the axioms in accordance with which the reasoning is conducted correspond to universal properties of space. But suppose that we confine our attention to algebraical analysis—to what the treatise before us includes under the terms ordinary algebra and universal algebra. Are the definitions of ordinary algebra merely self-consistent conventions? Are its propositions merely formal without any objective truth? Are the rules according to which it proceeds arbitrary selections of the mind? If the definitions and rules are arbitrary, what is the chance of their applying to anything useful? The theory of probabilities informs us that the chance must be infinitesimal, and the author admits that the entities created by the conventions must have properties which bear some affinity to the properties of existing things, if the algebra so founded is to be of any importance. The author says 'some affinity;' it may be asked how much? Unless the affinity or correspondence is perfect, how can the one apply to the other? How can this perfect correspondence be secured, except by the conventions being real definitions, the equations true propositions, and the rules expressions of universal properties? In the last sentence quoted, Mr. Whitehead makes a large concession to the realist view; it is only necessary to change the sentence into—"In the case of any algebra worthy of scientific attention the definitions and propositions refer exactly or approximately to the world of existing things."

M. Laisant, in his recent work, 'La Mathematique,' refers to the formal view of mathematical science when discussing the theory of fractions, p. 35. He opposes it, as marching in the direction opposite to progress, and as a survival of the spirit of the sophist.

The realist view of mathematical science has commended itself to me ever since I made an exact analysis of Relationship and devised a calculus which provides a notation for any relationship, can express in the form of an equation the relationship existing between any two persons, and provides rules by means of which a single equation may be transformed, or a number of equations combined so as to yield any equation involved in their being true simultaneously. The notation is made to fit the subject, and the rules for manipulation are derived from universal physiological laws and the more arbitrary laws of marriage. A very real basis, yet the analysis has all the characteristics of a calculus, and throws light by comparison on several points in ordinary algebra.

But what is the subject of which ordinary algebra is the analysis? Quantity; and in space we have the most complex kind of quantity; so that if space can be analyzed, the analysis will serve for any less complex kind of quantity. Mr. Whitehead admits that, as a matter of history, mathematics has till recently been the science of number, quantity and the space of common experience. But "the introduction of the complex quantity of ordinary algebra, an entity which is evidently based upon conventional definitions, gave rise to the wider mathematical science of to-day. Ordinary algebra, in its modern development, is a large body of propositions interrelated by deductive reasoning and based upon conventional definitions which are generalizations of fundamental conceptions."

The imaginary quantity, more generally the complex quantity, of ordinary algebra is the foundation upon which the formalist builds his theory; if it can be shown that it is not an entity based upon conventional definitions, but corresponds to a reality, then his whole superstructure falls down. The complex quantity first arises in analysis in the solution of the quadratic equation. The general form of the root consists of a quantity independent of the radical sign and a quantity affected by the radical sign. When the quantity under the radical sign is negative the root is said to be imaginary, because it appears to be incapable of direct addition to the part independent of the radical sign. In certain papers recently published I have shown at length that the root of a quadratic equation may be versor in nature or scalar in nature. If it is versor in nature, then the part affected by the radical involves the axis perpendicular to the plane of reference, and this is so, whether the radical involves the square root of minus one or not. In the former case the versor is circular, in the latter hyperbolic. When the root is scalar in its nature the two parts add to form the final result, but in the case where the square root of minus one is present the sign must be preserved in the intermediate processes of calculation. A complex index (both terms involving a sign of direction) has its meaning in an angle which is partly circular, partly hyperbolic; and a scalar complex quantity expresses the cosine or sine of such complex angle. It follows that the functions of a complex quantity can be defined really. It has been the practice of writers to follow the formal view, and define, for instance, the cosine of a complex quantity as the sum of a certain infinite series. Let z denote a complex quantity, then, according to that view, by cos z is meant the sum of the series

$$1-\frac{z^2}{2\,!}+\frac{z^4}{4\,!}-\text{etc.}$$

But when the cosine of a complex angle is defined in the same manner as the cosine of a circular angle or of a hyperbolic angle, namely, as the ratio of the projection of the radius-vector to the initial line, then

$$\cos z=1-\frac{z^2}{2\,!}+\frac{z^4}{4\,!}-\text{etc.},$$

becomes not a dead convention, but a living truth.

In the first book the author states more fully the principles of universal algebra: "There are certain general definitions which hold for any process of addition and others which hold for any process of multiplication. These are the general principles of any branch of universal algebra. But beyond these general definitions there are other special definitions which define special kinds of addition or multiplication. The development and comparison of these special kinds of addition or of multiplication form special branches of universal algebra," p. 18. The general principles are as follows: Addition follows the commutative and associative laws, viz: $a+b=b+a$ and $(a+b)+c=a+(b+c)$. Multiplication follows the distributive law, viz: $a(c+d)=ac+ad$ and $(a+b)c=ac+bc$. Multiplication does not necessarily follow the commutative and associative laws, that is, $ab=ba$ and $(ab)c=a(bc)$ are laws of special branches only. It has been maintained by followers of Hamilton that the associative law is essential to multiplication. It is true of spherical quaternions, but is not true of the complementary branch of vector analysis. It is satisfactory to find that Mr. Whitehead adopts the latter view, and, indeed, it is involved in his detailed exposition of vector analysis in the concluding book of his first volume.

But one who looks upon algebraic analysis not as the sum of several correlated branches, but as one logical whole, must consider the above principles or so-called definitions as arbitrary. For let p and q denote two quaternions, then e^pe^q is not in general equal to e^qe^p; consequently e^{p+q} is not equal to e^{q+p}; hence the commutative law does not hold in the addition of these indices. Thus to define addition as necessarily following the commutative law, and multiplication as not necessarily following it, is an arbitrary procedure.

In expounding the algebra of logic the author follows largely the exposition of Dr. Schroeder in his learned treatise, 'Vorlesungen über die Algebra der Logik,' but he does not take up the most valuable part of that work, namely, the Algebra of Relatives. Symbolic Logic as expounded by Schroeder differs essentially from the calculus devised by Boole in his 'Laws of Thought.' It was Boole's aim to keep as close as possible to ordinary algebra, and to make his method the foundation of a calculus of probabilities. In fact, the full title of his famous book is 'An Investigation of the Laws of Thought on which are founded the mathematical theories of Logic and Probabilities.' According to Boole the special peculiarity of the algebra is that $x^2=x$, when x is an elementary elective symbol. Jevons is said to have introduced the further supposed law that $x+x=x$, which destroys the quantitative character of the calculus. Indeed, Mr. Whitehead says that the algebra is non-numerical, and in Dr. Schroeder's elaborate work no mention is made of probabilities. According to the more recent school $a-b$ supposes that b is included in a (p. 82), whereas Boole made no such limitation. It is a step backwards, just as it would be a step backwards in ordinary algebra to hold that $a-b$ carries the supposition that b is less than a.

The detailed exposition of Grassmann's system is excellent and will be welcomed by all who wish to assimilate the ideas of that great master of space-analysis. The last book of the present volume is on the application of the calculus of extension to geometry, and it is evident from the fourth chapter, entitled 'On Pure Vector Formulæ,' that the author considers vector analysis to be supplementary to quaternion analysis. They are not the same thing; and both gain when it is perceived that they are not redundant, but supplementary to one another.

In conclusion, the work reflects great credit on the author and on the Cambridge University Press; it is likely to lead to further advances in Universal Algebra, not only by what it lays down, but by the questions which it brings forward for discussion.

ALEXANDER MACFARLANE.

The Principles of Agriculture. By L. H. BAILEY. New York, The Macmillan Company. 1898. Pp. xx + 300.

'Principles of Agriculture,' by Professor L. H. Bailey and his associates in Cornell University, is a new volume in the Rural Science Series and in many respects is the most important one of the series, as it serves as an introduction to the others. The book is intended to be used as a text-book for schools and rural societies, but it will prove interesting and valuable for the agriculturally inclined who have had little or no training in the natural sciences. It is essentially a book for beginners, and as such serves its purpose better than any of the small handbooks which have attempted to treat of the elementary principles of agricultural science.

The volume is edited by Professor Bailey and some of the chapters are written by him; the remaining chapters are written by his associates, who are specialists in the departments of which they have written. At the end of each chapter are suggestions which serve to elucidate the text for readers whose knowledge of natural science or of rural affairs is scanty, and also give useful hints for teachers who may use the volume as a text-book.

In the introduction we are told that "agriculture is not itself a science, but a mosaic of many sciences, arts and activities, or, a composite of sciences and arts, much as medicine and surgery are. * * * But the prosecution of agriculture must be scientific." The aim of the book is to deal with 'fundamentals' rather than 'incidentals.' "The mistake is often made of teaching how to overcome obstacles before explaining why obstacles are obstacles. * * * The purpose of education is to improve the farmer and not the farm." Would that more of our farmers could see the truth contained in these statements.

The book opens with a brief treatment of the formation of the different kinds of soils. On page 27 the author says: "The profit in agriculture often lies in making the soil produce more abundantly than it is of itself able to do." On page 202: "In intensive and specialty farming manures may be bought." These statements are true, but do not consist well with what is said about ideal agriculture on page 2. Inorganic compounds are explained as those which are not produced by living organisms, and phosphoric acid is given as one example, notwithstanding that a large amount of phosphoric acid used in commercial fertilizers is made from bone. Although the chemists call it an inorganic compound, yet because it is found in the remains of animals the reader who has had no knowledge of chemistry might be puzzled until some further explanation was made.

The second chapter, which is written by Professor Spencer, shows what is meant by 'texture' of the soil, why good texture is important and how to obtain it. That "the texture or physical condition of the soil is nearly always more important than its mere richness in plant food" is a fact not recognized by some tillers of the soil.

The 'moisture of the soil' and 'tillage' are next treated in a brief and creditable manner. Several figures are given to illustrate the art of plowing and one of an 'ideal general purpose plow.' All plowmen will think that this implement might be improved upon, but the low handles should be appreciated by everyone. The handles of many plows are too far from the ground.

Chapters IV. and V. treat of enriching the