as to time and mean range of tide for forty-nine other points on Chesapeake Bay, and for twenty-eight points along the Potomac River, to say nothing of sixty-one points on other tributaries of the bay. For such a region the investigator has an ample collection of facts to be used in proving or disproving any theory which he may formulate.

I am inclined to think that whoever successfully attacks this problem will use a graphic, or partially graphic method, plotting his results step by step upon the chart. In any wholly analytic method it will be especially difficult to take sufficiently into account the configuration of the bottom and shore.

In conclusion, I submit that to solve this boundary problem is to make an immense stride in our knowledge of the tides, a stride corresponding to a half century of ordinary progress; that it is in this line that our ignorance of the tides is most dense; that the facts are at hand for the investigation, and that, judging from the literature of the tides, this is, *comparatively speaking*, an unworked portion of the field. Along this line considerable pioneer work has been done, especially along purely mathematical lines, but the new comer will find neither a long series of failures to discourage him by indicating that the problem is intractable, nor a long series of successes to discourage him by making it appear that there is little opportunity to advance beyond what has already been done by others.

* * * * * *

It may seem that in this paper some attention should be paid to the fact that theory furnishes the relative amplitudes of certain harmonic components; that, in particular, theory indicates that certain relations exist between the relative amplitudes and the mass of the Moon, and that this theory has been born out by the fact that said mass has been computed with a high degree of accuracy from tidal observations.

It should, however, be kept clearly in mind that only the *relative* amplitudes are concerned in the computation of the Moon's mass. Further, the mass of the Moon as deducted from observations at a single tidal station is often largely in error. An accurate determination of the mass is obtained only when the results of observations at many stations are combined.

There is a decided significance, in the present connection, in the conclusions reached by two investigators who have carefully studied this phase of the tidal problem. Professor Ferrel, after a prolonged consideration of the matter, concludes that, to secure a better determination of the Moon's mass from the tides, a special study of 'shallow water components' should be made. In other words, the effects of friction due to the boundaries must be studied. Professor Harkness, in deriving the Moon's mass from tidal observations,* gives all stations equal weight, though the length of the series of observations varies at the different stations from one to nineteen years, on the ground that 'the accidental errors at any station are generally small as compared with those due to constant causes.' He indicates in the context that these 'constant courses' are constant for each point, but variable in passing from point to point along a coast; in other words, they are due to the local peculiarities of the boundaries.

John F. Hayford.

GEOMETRICAL OPTICAL ILLUSIONS.

During the last few years the subject of Optical Illusions has been receiving a degree of attention that may well be called remarkable. Both popular and scientific articles have been written, so that the general public, as well as the specialist, is well

* On the Solar Parallax and its Related Constants, Wm. Harkness, pp. 119–120.

informed about the simplest forms of the most interesting types of illusion. The signal for the scientific discussion of the subject seems to have been given in 1889 when Müller-Lyer first published the 'figures' which now bear his name. The investigations and discussions that grouped themselves about these particular figures soon spread to the whole field of optical illusions, until there grew up that body of technical literature which to-day has assumed very respectable dimensions. It is not so much that new forms of illusions have been devised or discovered—although more than one valuable contribution has been made—as that the heretofore well-known illusions have been subjected to a closer scrutiny than ever before. Like everything else accessible to experimentation, illusions have been taken into the laboratory. Variants, possessing characteristics that differ somewhat from the original form, have been devised. Each figure has been dissected and analyzed that it might be reduced to the lowest terms. And, above all, each figure has been subjected to a quantitative investigation, the amount of the illusion being accurately measured under the widest possible variety of conditions. These results in turn have been made the basis of theoretical considerations, and the end sought has, of course, always been some satisfactory *explanation* which shall furnish adequate grounds for the presence of an illusion in any given case. It is here that the war has waged. For while, in general, there has been sufficiently close agreement in reference to the results of experimental observation, there has been small uniformity in the theoretical conclusions reached. The one great attempt of to-day is, therefore, to bring harmony into this field; to establish, if possible, some single point of view which shall be applicable to all geometrical optical illusions alike, and which shall furnish that wished-for, comprehensive unity among the seemingly scattered and unrelated facts. That this attempt has met with complete success can hardly be asserted. Theories fundamentally antagonistic stand side by side with others that seek to combine and reconcile, and the day of perfect agreement seems yet distant enough. The splendid attempt of Wundt* to connect all illusions with actual or attempted *movements of the eyes;* the no less earnest attempt of Thiéry† and Filehne‡ to establish an explanation in terms of *perspective*; the classic attempt of Helmholtz and, more recently, of Heymans§ and Loeb to apply in one form or another the principle of *contrast*, and the very pretensious effort of Lipps¶ to define and utilize an *æsthetic* principle of unrestrained and victoriously striving activities, or their opposites, are all cases in point.

Perhaps the clearest way to give those readers of SCIENCE who, while interested in the subject, are unable to follow the technical literature closely and at first hand, some impression of the more recent work that has been done, will be to consider, in the first place, the discussions that have centered about some of the best known illusions, leaving until the end the account of the various explanatory principles that have been advanced and vigorously defended.

A.

1. *Zöllner's Pattern.*—In one or another of its many forms every one is familiar with the illusion of Fig. 1, in which a set of parallels is made to appear alternately convergent and divergent, by the addition to them of transverse cutting lines. But

* Wundt. Die geometrisch-optischen Täuschungen, 1898.

† Thiéry. Philosophische Studien., XI and XII.

‡ Filehne. Zeitsch. f. Psych., etc., XVII (1898): 15.

§ Heymans. Ditto., XIV., 118.

¶ Lipps. Raumæsthetik u. geometrisch-optischen Täuschungen, 1897.

familiar as this figure is, discussions in regard to the principles that should be applied to its explanation are by no means at an end. Two theories, especially, are at present contending for the primacy: the one, namely, falling back upon the supposedly

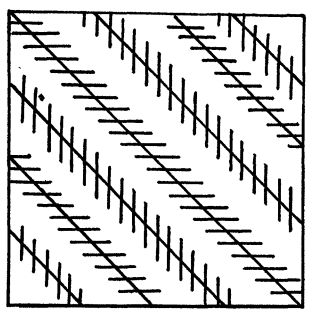

Fig. 1.

fundamental principle that when acute and obtuse angles come together in the field of view the former are relatively overestimated, the latter underestimated; the other appealing to perspective principles and calling attention to the fact that the above figure is to be seen, not as lying wholly in the plane of the paper, but as presenting elevations and depressions, projecting ends of lines and well-defined ridges. The first theory is well enough understood to render extended comment unnecessary. Suffice it to say that the main lines, rather than the short transversals, are affected by this false estimation of angles for the very obvious reason that they form in each case the common side of a multitude of angles. For a long time this theory was content to stand in this simple form. But the plastic phenomena of the figure, which we shall have to consider in a moment, are to-day too evident to be lightly disregarded. Hence the most recent statement of this theory* takes full account of the prospective phenomena present, but relegates them to a secondary position, making them dependent upon the already present deflections of the main lines.

In sharp contrast to this are the claims of the second theory. For it perspective influences are primary and all-sufficient. The observations, made long ago by Hering and Guye, to the effect that a careful attention to the figure will reveal unmistakable plastic characteristics, are here again emphasized. Especially if the above figure be drawn upon glass and viewed against a uniform background, the tri-dimensional properties become clearly apparent.* Not only do the ends of the main lines run alternately above and below the plane of the drawing, but, further, the transversals seem so to slope that if prolonged they would meet in ridges similarly above and below the plane. Accordingly, the illusion is due to the *interpretation* that we give the figure. We see the actual parallels projected, as it were, upon the surfaces of solid and hollow prisms which lean away from the vertical, and the lines being actually parallel the observer must interpret the more remote ends as diverging, as would actually be the case in ordinary perspective vision. In other and more general terms, the preponderatingly tri-dimensional character of all our visual experience compels us to interpret in the light of this every perspective motive that any linear drawing may contain. In the figure before us the arrangement of lines recalls by association certain real experiences with similar elements, and forthwith all the attributes that would be given to the lines and parts of a real seen object are given to the linear drawing, even though the observer be not consciously aware of anything beyond the final perception which turns out to be illusory.

Such in mere outline are the two opposing theories which to-day seem likely to

* Wundt., Op. cit.

* The observer should always remember that the perspective elements of any figure are most clearly seen when *one* eye only is used.

arouse discussion most seriously. To be sure, there are other theories in the field, but they may perhaps remain unmentioned here. How these two theories fare when they descend to details we must consider later.

Attention has been called recently in a very interesting way to certain phenomena of movement that are to be observed upon the heavy original Zöllner pattern shown in Fig. 2.* In his *Physiologische Optik*

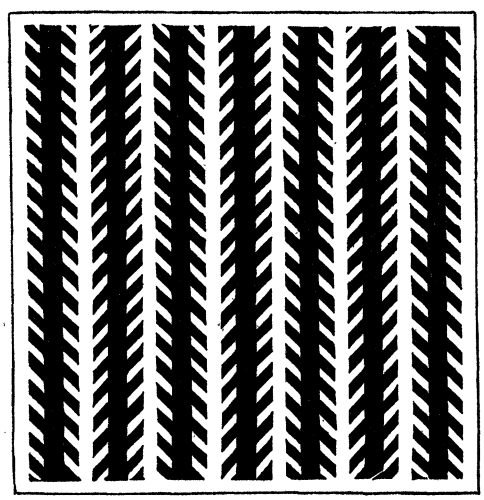

FIG. 2.

Helmholtz called attention to certain facts that appear to be less universally known than their importance would seem to justify. He noticed that if any small object, as the head of a pin, be steadily fixated while drawn horizontally across this diagram the heavy verticals appear to be set into motion in the direction of their own lengths, adjacent lines moving in opposite directions. The necessary rate of movement for the fixated point can be determined readily for each observer. If this motion be from left to right those lines bearing upward running transversals will seem to shoot upwards, those bearing downward running transversals downwards, and if the carefully fixated point be moved back and forth over the diagram a most startling unsteadiness is produced in it. Now the movements recently noticed by Filehne are very similar to these in appearance. They are to be seen thus: Placing the diagram so that the main lines lie horizontally, view it, not too fixedly, through a tube blackened within. Movements of diagram and tube being obviated in some way, a few moments' careful gazing will reveal the fact that the horizontals, two, three or more at a time, are darting about to the right and left, during a period of from one to three seconds. The motion of any particular horizontal is seen to be always *in the direction of the overhanging ends of the obliques.* Though it is affirmed that these phenomena may be seen even when the possibility of eye-movements is excluded, the writer finds slight movements absolutely essential, such, for example, as are produced by beginning to close the eye. Though most easily seen, perhaps, when the lines of the diagram are placed horizontally, these movements are none the less to be seen in the vertical position. Those lines that before darted to the left now pass upwards. If placed in an oblique position a peculiar fact may be noted. If the lines run upwards from right to left the observed motion will be upwards for those lines that before moved to the left and upwards respectively. But if the lines run upwards from left to right no motion is to be seen, since, while the tendencies to motion found in the horizontal and vertical positions have reinforced each other in the first oblique position, they are such as to cancel each other here. But in any case, whenever motion occurs, it takes place in the direction of the overhanging ends of the obliques. The reason for these phenomena Filehne claims to find in the consideration of 'memory-images of motion.' His theory is supplementary to the general perspective theory of which he is an earnest advocate. Many of our visual

* Filehne, Loc. Cit.

experiences are had while we are in motion. In such cases the angle made by any upright object that is perpendicular to the earth changes as we approach it or recede from it. This change is always of such a kind that, as we approach, the angle appears to diminish from an obtuse to a right angle, and as we reach and pass beyond the object the angle seems to increase, the perpendicular appearing, that is, to fall gradually away from the moving observer. But whether the observer be approaching or receding, the apparent movement—which may be regarded as a rotation about the point of contact with the ground—is always opposite to the direction of the observer's progress, and always towards that position where it shall seem to tip away from the observer as he has passed by. Most strikingly, possibly, is this seen in railway travel. The telegraph poles seem to rotate as they fly past, and always in the direction of that position where they shall appear to overhang. Well, countless experiences of this kind have stored up such a mass of memory-images that when, as in the Zöllner diagram, similarly overhanging obliques are viewed, these latent images are brought to the threshold of consciousness and the diagram itself becomes enlivened with an illusory motion, occurring in strict accord with actual objective experience. Psychologically considered, the language used is not wholly free from objection, but the meaning of the theory is on the whole clear enough. It has seemed well to report these new observations at some length, since their importance for the theory of optical illusions is evident. Filehne asserts the complete lack of connection between the illusory movements described by himself and those mentioned by Helmholtz. To the writer, however, it would seem that the two sets of phenomena are very closely related. Whether this be so, only a careful and more extended examination of the matter can determine. The alleged movements, or their lack, are too difficult of observation and too elusive of rigid verification to admit of any positive statements at present. Nevertheless the question may be asked very pertinently, how any visual impression, of whatever characteristics, should be capable of causing illusory perceptions of movement at a moment when every actual movement of the eyes is excluded. Certainly, if the observations recorded be true, we have something novel in the realm of psychology—a perception of motion, but a motionless object and a motionless eye.

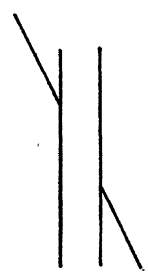

Fig. 3.

2. *The Poggendorff Illusion.*—The secondary illusion to be seen on the oblique lines of the heavy Zöllner pattern, the more usual form of which is shown in Fig. 3, still continues to be the object of experimental and theoretical inquiry. We meet here the same contrasted theories as before, though we find them somewhat less sharply stated.

The explanation that rests upon the overestimation of acute angles has, as usual, little difficulty with the matter. The free ends of the transversal are simply rotated about the points of contact with the verticals, with the result that a new line may be drawn at either end in apparent continuation of the other. It would seem also, as Wundt has pointed out, that other factors cooperate to produce the illusion, since its amount is much diminished by giving the figure a horizontal position. One such factor is doubtless the universal tendency to overestimate the upper as opposed to the lower half of any vertical dimension. Another factor may well be the likewise universal tendency to underestimate empty as opposed to filled spaces. Underestimating the open space between the inner ends of

the transversal would result in an apparent narrowing of the vertical strip and a consequent increase of the illusion. But this latter factor is probably effective here in a minimum degree.

Somewhat more interesting is the perspective explanation. This we meet under two different forms. The first employs the usual perspective argument. Carefully viewed, the ends of the transversal may be seen to issue from the plane. This becomes especially evident if the figure be made of wire and suspended before a uniform background. In consequence of this perspective quality of the figure the visual angle made by the transversal and the vertical is interpreted as *representing* an angle of greater magnitude than that seen, just as in the case of all angles seen perspectively in objective vision, and the illusion results. This, if I rightly understand him, is the argument of Thiéry. The second form of the perspective explanation differs quite materially from this. It is the explanation of Filehne. According to this the vertical strip of the Poggendorff figure serves principally to sunder the two ends of the transversal to such a degree that there is no longer any sufficient reason for regarding them as belonging together. Now, remembering that, in accordance with the perspective theory in general, the lines of a plane geometrical figure act chiefly as the means of suggesting real objects of actual experience, we can easily see the line of thought. For there is not the remotest necessity that two detached portions of a straight line represent objects whose bounding edges should appear continuous, merely because they would meet and form a continuous line in the linear drawing that represents them. It can be most readily and graphically shown by straight-line drawings of objects that two detached portions of one and the same line may represent objects in totally different planes of space, so that if the objects represented were to be prolonged in their own direction they might never meet at all, or at best only at an oblique angle. In the figure before us, consequently, it is highly probable that the sundered portions of the oblique recall some real experience, or set of experiences, in which the objects represented are absolutely unconnected. Such an experience may be suggested by a finger-post, an arm upon one side pointing obliquely towards the observer, an arm upon the other side—lower or higher, as the case may be—pointing obliquely away. Herr Filehne finds great support for this view in the alleged observation that every trace of the illusion vanishes in the above figure if only the two verticals be somehow united, or if some indications be present to show that the ends of the transversal are portions of a continuous whole, the missing part of which is hidden behind the vertical strip. The first condition can be secured by drawing within the latter a short line which shall be oblique to the transversal and meet the edges of the strip at points opposite those in which the ends of the transversal terminate. The second condition can be readily secured by making the transversal represent a pointed stick, or by placing at the outer ends of the transversal the drawing of some such device as weights and pulleys, which shall make it clear that the two ends are really acting in unity. But though these conditions be fulfilled to perfection, the illusion simply does *not* vanish, despite the assertion of Filehne to the contrary, nor would one expect it to do so. For what these particular devices are expected to accomplish is the closer approach to the actual conditions of tri-dimensional vision, where only one interpretation of the lines is possible. The best conditions for testing the theory would be found, therefore, in normal objective experience. Stretch a rope obliquely behind a tree trunk and at a distance of

some feet from it. The illusion persists, and yet there is no possible attempt to give an independent perspective interpretation to either end of the rope. Still more conclusive, perhaps, is the consideration of the so-called 'Illusion of the Gothic Arch,' a representation of which is given in Fig. 4.

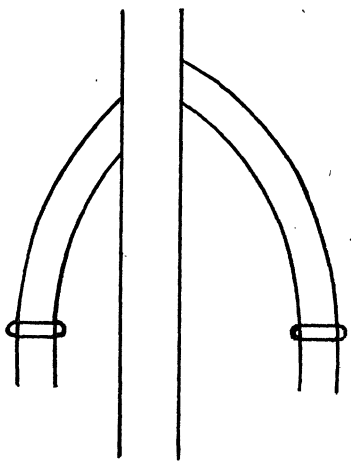

Fig. 4.

This illusion, many times independently observed, is manifestly but a variant of the more simple rectangular form, and its striking quality is not destroyed even though the observer be most intensely conscious that he is in the presence of actual objects, seen under the conditions of normal perspective. It seems most necessary, therefore, to look askance at this most recent attempt to apply the perspective interpretation to the Poggendorff figure.

One point seems to have been universally overlooked in the quantitative investigations made upon this figure. All measurements, namely, so far as one can judge from the literature of the subject, have proceeded upon the assumption that the amount of the illusory displacement is to be discovered by moving one end of the transversal vertically along the strip, the moving line to be kept always *parallel to itself*, until the point is reached where the two parts seem continuous. It would seem, however, that an unprejudiced approach to the problem should lead one to make room for any possible *angular* displacement that might be required to bring the moving end into a position of satisfactory apparent continuation with the fixed end. The writer recently constructed an apparatus which allows the determination of both vertical and rotatory displacement, but the meager results thus far obtained give no basis for any conclusion in the matter. Still the point seems well worthy of more extended attention.

3. *The Müller-Lyer Figure.*—Figure 5 presents in its typical form the much-discussed

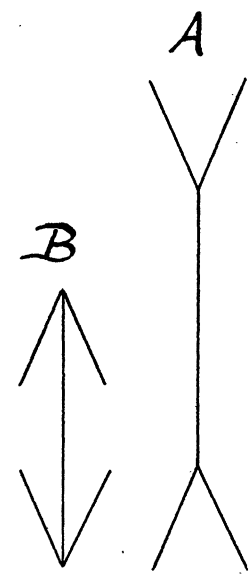

Fig. 5.

'optical paradox,' first published by Müller-Lyer in 1889, in the *Archiv für Physiologie*. No less than eight different explanations for this illusion have been propounded and warmly defended by the various writers. This number has been so far reduced by the reciprocal overthrow of the contesting parties that only three attempts at an explanation need detain us here. (*a*) The *perspective* theory is less fortunate here than usual.

According to it the principal line of *A* in the above figure is seen to be more distant from the observer than the principal line of *B*. The tiny obliques, that is, run forward from the point of contact in the former and backward in the latter. In consequence of this difference in apparent distance between the two lines, the dimensions of the figure must be apprehended in accordance with the universal law, that of two objects subtending the same visual angle that will be perceived as the greater which is projected to the greater distance from the eye. The perspective form, particularly of *A*, can be most easily seen where the ends of the obliques have been connected by straight lines; that is, when the whole figure has been enclosed in a rectangle. But now immediately the *equivocal* character of the figure becomes manifest. It may be seen, namely, either as an oblong hip-roof in miniature or as a hollowed out, crib-like object. In other words, the principal line may be made to appear now nearer, now more remote from the observer. Though less apparent, the same equivocal character is to be observed in the normal figure, *B*. But, whether nearer or more remote, *the apparent length of the principal line does not change*, and, since there are no compelling grounds to determine once and for all that *A* shall be seen in the distance and *B* in the foreground, the perspective explanation ignominiously fails, its whole structure being based, as we have seen, upon an *unequivocal* perspective reference.

The other theories mentioned are the 'confluxion-contrast' theory of Müller-Lyer, and the 'muscular energy' theory of Delbœuf and Wundt. (*b*) *Confluxion* is the term used to designate a class of facts where the estimated lengths of lines partake of the nature of the surrounding space in so far as this is indicated by other lines lying immediately adjacent. That is, the principal line of *A* is estimated as longer than that of *B*, because the total space inclosed by *A* is greater than that inclosed by *B*. Confluxion differs from contrast in that for the former any line *shares in* the characteristics of its surroundings, while for the latter any line assumes characteristics *opposite to* those of its surroundings. Here both motives are influential, confluxion as indicated, and contrast in so far as the principal line comes into comparison with the short obliques. This, in a word, is the 'confluxion-contrast' theory. Confluxion must evidently play the more important rôle in the present case. But an examination of this principle of explanation reveals the fact that its unreliability makes it very dangerous of acceptance. Cases can readily be found where, if true, the principle should be, but is not, effective. The most simple case is perhaps that given by Wundt (Fig. 6), where according to the

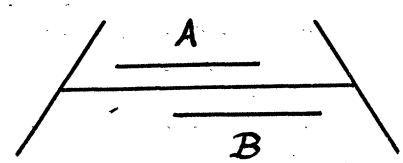

FIG. 6.

theory *B should* appear longer than *A*. Some further theory seems requisite, therefore, to satisfactorily account for the illusion.

(*c*) Though the forms given to the 'muscular energy' theory by Delbœuf and Wundt are by no means identical in respect to details, yet for purposes of description they may be brought together. Wundt has given by far the best statement of the theory. Its essence is that our visual spatial estimates are always influenced, if not first made possible at all, by the amount of energy expended by the muscles of the eye in running the point of regard over the figure viewed. The strength and range of this theory we can first see when later we examine the several principles of explanation. Meanwhile the statement will suffice that the tendencies to move the eyes beyond the

end of the principal line of A, and similar tendencies to fall short of the ends of the line in B, bring it about that the vertical of A is perceived as the longer. This will be true whether the eyes move freely along the line or remain fixated upon some point placed midway between the two figures lying side by side. In the latter case the illusion will be diminished in amount, though still existing, by reason of the fact that there can still be weakened impulses of the same kind as when the eyes are in motion.

A quantitative study of this illusion has shown that for each angle made by the two obliques a maximum of illusion is reached with a particular length of oblique, on either side of which the illusion diminishes. Thus, if the principal line be of 75 mm. and the angle between the obliques be 60° the maximum illusion occurs when the length of the oblique equals 30 mm.*

Innumerable forms can be given to the Müller-Lyer figures. The obliques may be replaced by fork-like ends with parallel prongs, or by circles and semicircles. Or A and B of Fig. 5 may be placed end to end in such a way that the outward pointing obliques of A become the inward pointing obliques of B, and in this condition various

FIG. 7.

combinations of obliques may be omitted without destroying the illusion. Many more complex figures may be constructed which display illusions due to the presence of Müller-Lyer motives. Fig. 7 shows two of these, the side A appearing longer than that of B, though they are of equal length. Again, the point of bisection of the altitude of an isosceles triangle seems placed too high, the angle of the vertex acting apparently in the sense of B, of Fig. 5. Type founders have taken account of this in placing the horizontal of the letter A far below the middle.

4. *Münsterberg's Illusion of the 'Shifted Checker-board Figure.'* The illusion of Fig. 8* differs essentially from all the foregoing, for while it resembles the Zöllner pattern in the converging and diverging character of the vertical lines its explanation rests upon a totally different principle, namely, that of *irradiation*. Fig. 8 may be called 'the illusion of the kindergarten patterns,' since it reproduces in black and white the type of patterns used in the occupation of mat-weaving. A single element of the illusion may be obtained by taking from the figure one of the vertical lines and the several pairs of overlapping rectangles that lie along it. If, further, the rectangles be changed to squares we have the form of the illusion first published by Professor Münsterberg only a few years ago in the Milton Bradley collection entitled 'Pseud-optics.' The brief assertion was there made that the illusion is due to irradiation. This statement, however, German writers have shown themselves singularly disinclined to adopt. Heymans† and Lipps‡

* Heymans. Zeitsch. f. Psych., etc., IX., 227.

* The illusion may be best seen by holding the diagram somewhat beyond the range of most distinct vision, or by viewing it as reflected in a mirror.

† Heymans. Loc. cit.

‡ Lipps. Op. cit., p. 319.

have brought it into relation with the Zöllner patterns and have made use of it to show that some other principle than the overestimation of acute angles must be employed in explaining the latter, since in the new variant no acute angles are present. Filelhne* has even attempted a perspective interpretation, according to which an element of the illusion, held horizontally, represents a bench, one end of which recedes into the background. As to the latter explanation, not only does it involve the arbitrary procedure of drawing additional lines to represent the seat of the bench, or of making the squares gradually smaller in order to suggest the greater distance of one end; but, more than all, it can give no satisfactory account of the illusion when the line of squares, or rectangles, lies vertically. There is, to be sure, a secondary illusion in the figure, to which several observers have called attention, which might lead one to suspect the presence of perspective elements. Hold Fig. 8 so that the plane of the paper makes a small angle with the line of vision. Then turn the diagram until the angle between the line of vision and the verticals equals about 30°. In this position the lines of rectangles that run away from the observer seem to form each a series of low steps. Running the eye along any vertical reveals this very clearly. The reason is evident. The background, as it were, for any rectangle viewed in this way is partly a white area, partly a similar black area. These areas are so distributed that when dark area is followed by dark area the middle portion of these joined areas must seem somewhat darker than the outlying parts, since the latter have received a grayish tinge from the white areas beyond. This darker portion can be interpreted only as a part lying in shadow, and hence the illusory perception of a low step, the 'riser' being the shadowed portion.

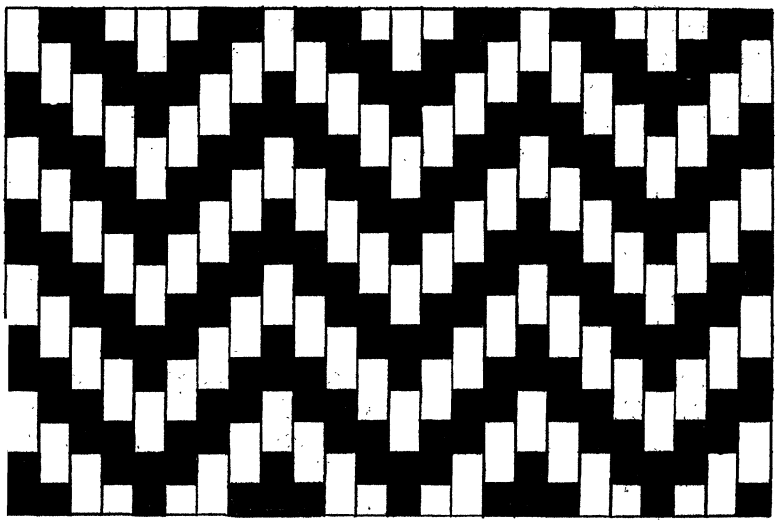

FIG. 8.

But as surely as this secondary illusion rests upon one of the accessory criteria of perspective vision, just as little can it furnish any basis for the perspective explanation of the primary illusion. That this is due to irradiation cannot now be doubted. The present writer endeavored recently* to

* Loc. cit, p. 42.

* *Psychological Review*, V. (1898), 233

show by a qualitative and quantitative study of this illusion that no factor other than irradiation need be appealed to for a thoroughly satisfactory explanation. The hesitation shown in accepting this explanation has been partly due, no doubt, to the fact that in the usual cases of encroachment by irradiation the diminished dark areas have retained outlines that are everywhere *parallel* to the original outlines of the figure. One has only to think of the dark square or circle on the light background. In the present case, however, the effective

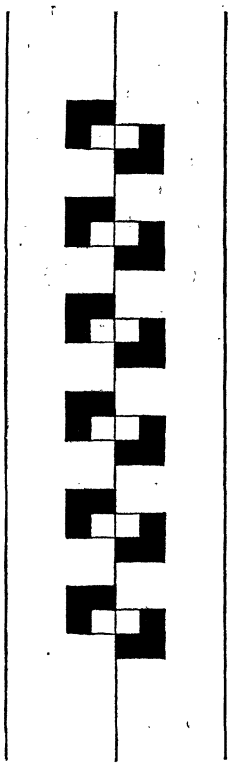

FIG. 9.

point of the irradiation is in the *corners* formed by the adjacent rectangles. The white areas *bore into* the dark corners, as it were. In this way those portions of any vertical that run along the sides of the various rectangles are deflected towards the corners, both above and below, and the deflections of these several portions give the tilted character to the line as a whole. This irradiation in corners is strikingly shown in Fig. 9, where the point of effective irradiation has been shifted to the centers of the incomplete squares. Here there is no longer a deflection of the vertical, but instead the bars of white that cross the line seem to slope slightly downwards to the right. The result of the qualitative and quantitative investigation above referred to showed clearly that the illusion vanishes whenever there can be secured the impossibility of irradiation in the corners situated along the line. The figures devised secured the latter condition while still retaining any factors that this figure may have in common with the Zöllner patterns. If, further, the character of the illusion in the regular figure was altered by substituting colors for the blacks and whites, or if the character of the illumination was changed by the use of the electric spark, or by the interposition of colored media between the diagram and the eye, the measurements of the illusion disclosed varying changes in the amount of apparent deflection. These results seemed to show the entire sufficiency of the explanation in terms of irradiation, at the same time rendering superfluous the appeal to, or the search for, any further explanatory principle.

5. *Loeb's Illusion.*—In 1895 Professor Loeb, of Chicago, called attention to the following interesting illusion.* Let *M*, Fig. 10, be a fixed vertical line and *a* a shorter line parallel with *M* and lying to the right. Placing *M* in the median plane and steadily fixating some point in it, place a second line *b* in such a position that it shall be continuous with *a*. This attempt will probably succeed very well. But now being a third line in the position occupied by *c* in the figure *b* will seem to lie too far to the right. That is, *b* must now be brought to

* Jacques Loeb. Pflüger's Archiv, LX., 516.

the position indicated in the figure in order to appear continuous with *a*. The lines employed may be narrow strips of black

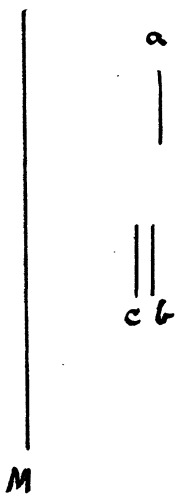

FIG. 10.

cardboard, or the lines may be replaced by coins or other objects of a similar nature. The discoverer's explanation is given in terms of *contrast*. If the space value of the impression *b* be the apparent distance of *b* from *M* we may say that this has been increased by the presence of *c*, since the resulting retinal impressions come now into the relation of 'contrast,' or of mutual repulsion, whereby the space value of *b* is increased. Accordingly *b* must be moved nearer to *M*, that its apparent space value may be equal to that of *a*. This explanation has not met with universal favor, and Heymans, Filehne and Wundt have each sought other solutions. The first attempts to unite this to the Zöllner illusion, the second to explain in terms of perspective. Wundt, in opposition to all previous attempts, points out the fact that this is an illusion of indirect vision, to be explained, therefore, only by reference to some known facts in that field. These facts he finds in the well known illusion of von Recklinghausen, in accordance with which rows of apparently horizontal and vertical points, placed farther and farther outwards from the point of fixation and in apparent parallelism with the real horizontal and vertical passing through this point, must be made to curve slightly with the convex side towards the point of fixation. In the illusion of Loeb *b* alone can be placed correctly in line with *a*, since the impression made by the lines is sufficiently strong to overcome the tendency to the Recklinghausen illusion. The addition of *c* restores the normal conditions somewhat, however, perhaps through the impression of imaginary lines drawn from *a* to *b* and *c*; and the expected inward inclination from *a* to *b* now takes place. That Wundt's explanation is wholly clear can by no means be asserted. Still, the full recognition of the fact that this is an illusion of indirect vision, and the attempt to subsume this under phenomena already known, are long steps towards a possible explanation that may prove more satisfactory.

6 *and* 7. *The Illusions of Baldwin and Judd.*—At the last meeting of the American Psychological Association, held during the Christmas holidays, two reports were made in reference to recently observed optical illusions. Professor Baldwin gave some new observations made upon the illusion of Fig. 11, with which the readers of SCIENCE were made acquainted through these colums in 1896. The point actually midway between the circumferences of the two circles seems nearer the larger. So far as the writer is aware, no final explanation has as yet been proposed. In the report referred to we are simply told that *perspective* 'has probably little influence,' and that the principle of 'equilibrium' cannot account for it, since the placing of the apparent middle point is in the contrary direction to that demanded by this principle. The announcement of further experimental results is awaited with interest. Dr. Judd has called attention to an interesting illu-

sion that seems to throw some light upon the general problem of visual space-perception. Two threads are so placed in a box that they cross each other at an acute angle while lying at different depths. If one of the points of crossing be properly fixated, two phantom threads will be seen passing between the main threads and making with each other a figure which, if viewed from the side, would resemble an X. The directions necessary for satisfactorily securing the proper conditions for this illusion are too detailed to find a place here. The interested reader is, accordingly, referred to the article designated in the foot-note.*

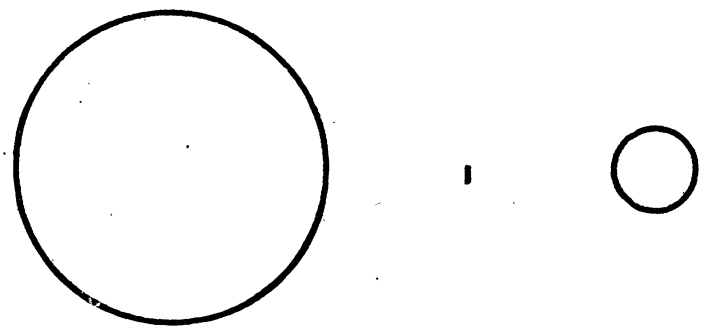

Fig. 11.

Such are the principal illusions that are being ardently discussed at the present time. To mention the numberless variants and minor illusions of form and magnitude that have come to light in the course of these discussions would be far beyond the scope of this article. The reader who cares to pursue the subject further is referred to the literature of the subject, particularly to Sanford's 'Laboratory Course in Psychology,' where may be found an excellent bibliography practically complete to the beginning of the current year.

B.

In attempting a brief summary of the discussions relative to the principles of explanation applied to optical illusions the following theories only will be considered: namely, the *contrast* theory, the *perspective* theory and the *physiological* theory. Lipps' æsthetic theory must remain unconsidered here, its unique form demanding rather a particular treatment by itself. Let us see how each of the three mentioned deals with the overestimation of acute angles. The *contrast* theory says that the two legs of an acute angle are in a relation of mutual antagonism, each point of one exerting a repelling influence, as it were, upon a point of the other. The consequence is, of course, that the whole angle appears larger than it really is. The *perspective* theory asserts that acute angles are not overestimated when unaccompanied with accessory lines formed by the prolongation of the legs of the angle, or otherwise. Then, and only then, the lines are regarded as *perpendicular* lines seen in perspective, and the acute angle gains therefrom an increment of magnitude. The *physiological* theory, by which that of Wundt is meant, claims that the relative magnitudes of angles depend upon the relative intensities of the muscular sensations gained by sweeping the eyes over the angle; and since for acute angles there is relatively more energy involved in the starting and stopping of the movements of the eyes, an acute angle, as compared with an obtuse, must be relatively overestimated.

*C. H. Judd, *Psych. Rev.*, V. (1898), 286.

As to *contrast*, the illustration here given does not exhaustively express the many phases under which this principle appears. Helmholtz has a theory of direction-contrast, Heymans one of movement-contrast, and Loeb one well illustrated by the case just treated. Since, however, no one of them contains in itself any reason for its particular way of working, it becomes in each case a mere name, a convenient expression only for the fact in hand. The only legitimate application of the principle of contrast is in those cases, well illustrated by the circles of Ebbinghaus (Fig. 12),

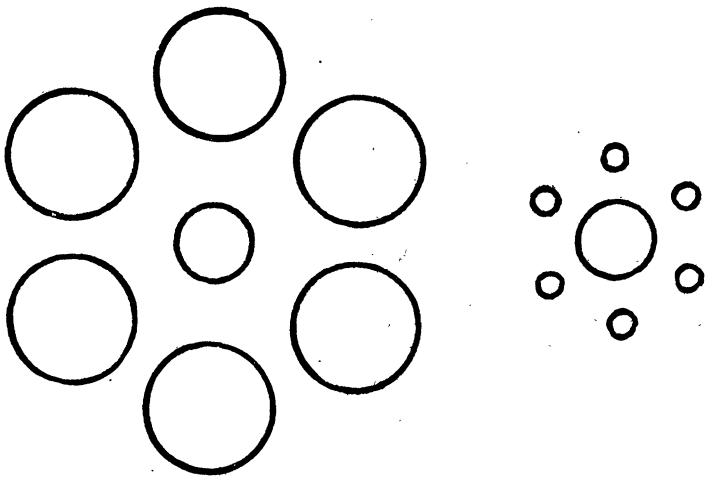

FIG. 12.

where two objectively-equal areas become apparently diminished or enlarged respectively, according as they are brought into approximation with larger or smaller areas of the same nature. In this sense of the term the fact to which the principle is applied is brought into range with a multitude of facts in every department of mental life, the general law of which is that when any mental state with certain prominent characteristics is brought into comparison with a second state of opposite characteristics the peculiar quality of each is intensified, just as a season of joy is more joyful when immediately following a season of pain.

In the monograph already referred to more than once Wundt powerfully emphasizes the fact that the principle problem of *perspective* is to determine whether its position is primary or secondary; whether, that is, it is the *cause* of the illusion in a given case or the *effect* of an already present illusion. To determine this figures are found with no accompanying perspective phenomena, though the nature of the illusion is analogous to that presented by figures *with* perspective phenomena. It would seem, therefore, that perspective were wholly *secondary* to some more fundamental factor. And this presumption is strongly fortified by the fact that the perspective phenomenon is always *unequivocal*, that is, that dimension perceived as greater is always projected to the greater distance, and cannot by any effort of 'imagination' or 'will' be brought nearer. These few words give but a faint hint of the force of the argumentation in detail.

The *physiological* theory is the outcome of an attempt to discover some principle that shall be fundamental and hence capable of universal application—valid, that is, not only for 'variable' illusions of magnitude and direction, but also for such 'constant' illu-

sion as the overestimation of the upper half of a vertical. Such a theory Wundt long ago propounded. His recent work is only an especially thoroughgoing attempt, from a novel standpoint, to defend the old thesis. Especial emphasis is placed upon the consideration of equivocal figures and upon the secondary character of 'perspective.' And everywhere attention is called to the effect of particular positions and movements of the eyes. The essence of the theory is that every visual spatial perception is a complex formed by the assimilation of visual qualities with sensations coming from the muscles of the eye. Whatever, therefore, increases the intensity of the muscular sensations that enter into the complex occasions the perception of a greater spatial magnitude. The particular conditions necessary for this increase of muscular intensity are to be found both in the assymmetry of the eye-muscles and in those cases to which the general mechanical principle can be applied, that brief movements require relatively more energy than those of longer duration, since it is harder to start and stop a movement than to maintain one already under way. This theory is called 'physiological,' to call attention to the fact that the conditioning factors are of physiological rather than psychological origin. With the exception of a few cases, such as the illusion of contrast shown in Fig. 12, this principle of muscular energy finds universal applicability. One may be unwilling to accept the wide-reaching implications that this theory has for the general doctrine of space perception. Yet one must frankly admire Wundt's masterly effort at unification and acknowledge the compelling power of his argumentation, especially as it appears in this new form.

In conclusion, attention may be called to the illusion of Fig. 13, in which the oblique line *ab* appears to curve slightly at its point of intersection with the vertical. The illusion is not marked, but it can usually be seen by all observers. For some it may be more distinct if the three figures be held

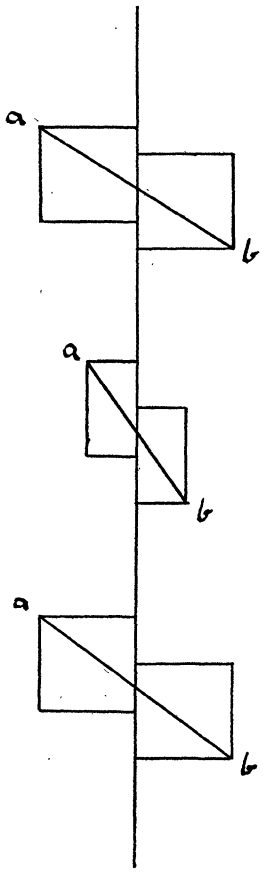

Fig. 13.

horizontally, and it may be more apparent in some one of the figures than in the other two. But if the eye carefully follow the line from *a* to *b* the line will probably be seen to bulge on either side of the point of intersection in such a way that it forms an extremely attenuated *S*. Each of the three principles of explanation considered above is applicable here. The first would say that the points of the oblique and the vertical are mutually antagonistic in the immediate vicinity of the point of intersec-

tion; the second, that perspective motives are operative in the neighborhood of the vertical, their further influence being prevented by the fact that the ends of the oblique are tied to the points *a* and *b*; the third, that the eye, in passing along the oblique, is solicited by the vertical, and the more resolute effort requisite to keep to the original path causes an apparent increase of the angle, the curving of the line being due to a conflict between the increase of the acute angles and the fixity of the outer ends of the oblique.

Which of these explanations shall we accept?

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SOME RECENT AND IMPORTANT EXPERIMENTS WITH THE EGGS OF THE SEA URCHIN.

THE well-known experiments of Boveri in which egg fragments were fertilized apparently gave evidence that the union of female cytoplasm with a spermatozoan may be followed by segmentation and development, but the proof is very inconclusive. It was left for Yves Delage to complete the evidence.

In a late communication to the French Acadamy* Delage states that he has succeeded in dividing the egg of *Strongylocentrotus lividus*, not *en masse* by shaking, as has been done heretofore, but by hand beneath the microscope and in such a way that there can be no doubt as to the fragments obtained being parts of the same egg. He was able to see that the nucleus was contained in one part and not in the other, which was, therefore, composed of ovulary cytoplasm. A whole or uninjured egg was placed beside the fragments and spermatozoa introduced into the drop of water in which the experiments were performed.

Sexual attraction manifested itself with equal energy by all objects. The controle egg and the two fragments were fecundated. A little later segmentation began, appearing first in the controle, a little later in the nucleated and still later in the nonnucleated fragment. The rapidity of segmentation was greatest in the controle and least in the nonnucleated, so that when controle was in the stage 8 or 16 the nucleated fragment had developed to stage 4 and the nonnucleated to stage 2. In the drop of water the development could not be of long duration, but in one case it was successfully carried through three days. At the end of this time the controle formed a typical gastrula. The nucleated fragment had developed so that the only difference apparent was its smaller size. The nonnucleated fragment also developed into a gastrula, but with the enteric and blastocœlic cavities very much reduced, owing, no doubt, to the smaller size of the fragment. In all cases a vitelline membrane appeared about the blastomeres. Some of the larvæ were fixed and stained, and the nuclei and nucleoli found in the cells from the nonnucleated to be no smaller than those in the cells from the nucleated fragment.

From these experiments Delage deduces the following very important conclusions:

1. The ordinary definition of fecundation must be rejected as being too strict. The union of the female and the male pronuclei certainly takes place, but it is not essential to development.

2. Fol's conclusions as to the union of the two pronunclei and of the demi-ovocenters with the demi-spermocenters must be cast aside. For, as the experiments show, the absence of an ovocenter is not an obstacle to development.

3. The theories in which fecundation is explained as the saturation of a female nuclear polarity by a male nuclear polarity must likewise be dismissed, and also those theories regarding the formation of the polar gobules as for the purpose of ridding the female nucleus of all male elements.

* *Comptes Rendus*, CXXVII., 15 pp., 528–31.