results of test in foreign navies also. Nearly all naval vessels, at home and abroad, now include in their specifications the water-tube boiler. "The tactical importance of water-tube boilers has been emphasized by the conditions which obtained in the blockade at Santiago and the great victory of July 3d. It was necessary, for a long period, that our ships should be ready to develop maximum power at a few minutes' notice; and with cylindrical boilers this involved keeping all the boilers under steam, with heavily banked fires and a large attendant consumption of coal. Water-tube boilers of the proper kind, which admit of the rapid raising of steam with safety, remove this difficulty and give the commanding officer a more complete command of his fighting machine." Giving great power with small weight, this modern apparatus of power-production is coming into use on all torpedo boats and, as a rule, on even the heavy battle-ships.

The steam turbine is referred to, but with the statement that it is not yet certain that it will find permanent place in the naval service.

The use of oil-fuels is pronounced promising in some naval work where costs of fuel are not of prime importance. Success is met with in the use of an oil of S. G. 0.85 to 0.87, a flash-point of 315° Fahr., and a burning point of 350° Fahr. This oil is entirely safe on shipboard.

Referring to the marvellous performance of the *Oregon*, her long voyage, perfect condition at its end, and later effective action with the squadron off Santiago, a record 'which has never been equaled in the history of navies,' and attributing this fact to the admirable work of designers and constructors, and still more, if possible, to the splendid character of the personnel of the engineer department of the ship, the Engineer-in-Chief says: "For the past ten years it has been my duty and a sad one to call attention to the urgent need of a reorganization of the personnel of the Engineer Corps." Its enlargement, provision for proper selection of its officers and professional training, and of suitable inducements to men of talent and genius in this branch to enter the corps, are vitally important and necessary amendments to existing provisions for its support. An efficient Engineer Corps is as essential to the efficiency of the navy as good war-engines. The engineer and his war-engine together make victories like those of Manila and of Santiago possible, with the no less essential aid of good 'men behind the guns.' The whole department is one of applied science of the most extensive and imposing character and an Engineer Corps, scientifically educated and systematically trained to its peculiarly exacting and responsible work, is the most pressing need of the 'new navy.' The report, and its conclusion regarding the lessons of the war, is most instructive from both scientific and the political standpoints.

R. H. THURSTON.

REPORT ON THE STATE OF THE MATHEMATICAL THEORY OF ELECTRICITY AND MAGNETISM.

IN considering the state of the mathematical theory of electricity and magnetism at the end of the first half-century of the existence of this Association, it seems hardly possible to avoid a comparison with the state of affairs at the beginning of that period. In 1848, strange as the statement may seem, most of the great discoveries in electricity had been made. Coulomb had by his remarkable quantitative experiments with the torsion-balance and the proof-plane set the law of inverse squares once for all upon its feet, and thus opened the way for the wonderful applications of the analysis of Laplace, Poisson, Gauss and Green. The experimental discovery by Oersted of the action of the electric current upon the magnetic needle had aroused the enthusiasm of Ampère, with the result of the swift production of his discovery of the laws of electrodynamics and their representation in mathematical form, by a process of reasoning characterized by Maxwell as 'perfect in form and unassailable in accuracy.' Within the next decade Faraday and Henry had independently discovered the phenomena of induction, and thus completed, with two exceptions, the list of epoch-making discoveries in the science. We may remark in parenthesis that the dynamo and the electric motor, which have wrought such a change in our city and Association in very recent years, were thus possible before the birth of the Association.

Not only had the quantitative laws of induction of currents been formulated by Faraday, but they had been obtained by a remarkable mathematical process by Neumann, using the then hardly recognized principle of the Conservation of Energy. The mathematical theory of the subject was accordingly in a well advanced state fifty years ago. It is to be noticed, however, that all the work then done had been on the basis of action at a distance, the existence of which was then unquestioned by mathematical physicists.

Not so, however, by that prince of experimental philosophers, Michael Faraday. Not less important for the theory of electricity than his discovery of current induction were his electro-statical researches by which he first showed that the forces between electrically charged bodies were not independent of the surrounding medium. Thus Faraday was led to concentrate his attention upon the medium instead of upon the charges themselves, and daringly to attack the notion of action at a distance. In order to clothe his ideas in an intuitional geometrical form, Faraday introduced the idea of physical lines of force, an idea that was long in having its fruitfulness recognized by others.

It was not until 1861 that the note was struck which has produced such a remarkable change in the theory of fifty years since. In Maxwell's papers on 'Physical Lines of Force' in the *Philosophical Magazine* of that year he gave Faraday's ideas a mathematical garb, and introduced to the mathematical world the theory that the energy was resident in the medium, rather than in the charged bodies. In 1865 appeared in the *Philosophical Transactions* Maxwell's chef-d'œuvre, the elaborate development of his ideas in his paper on 'A Dynamical Theory of the Electromagnetic Field.' Here we find for the first time the application of Lagrange's dynamical equations to obtain in a systematic and logical way the laws of electricity and its connection with magnetism. Besides the notion of the localization of energy, both electric and magnetic, in the medium, we find the other idea, foreign to the old theory, of the magnetic action due to time-variations of the electric field, these variations being termed by Maxwell displacement currents. It is not a little remarkable that Faraday had considered the possible changes taking place in the electrical state of the dielectric medium by changes of the magnetic field, and had attempted to make them experimentally evident, but without success. Whether Faraday contemplated the effect of the changes of the electrical state on the magnetic field I am not able to state. At any rate the introduction of this hypothetical magnetic effect of the displacement currents by Maxwell gave rise to a hitherto unlooked-for possibility, namely, the establishment of an electro-magnetic theory of light. This theory not only enjoyed the advantage of novelty, but was free from the fundamental difficulties of the previous dynamical theories of light, in that in it no longitudinal wave appeared.

If Maxwell's theory was true, then experimenters should discover the magnetic effect of the displacement current. We may well imagine that for many years investigators were devising means to accomplish this result; but if this was so, they were not rewarded by success. However, a crucial question had been agitated, for, according to Maxwell, electrical and magnetic disturbances must be propagated with a finite velocity, and the theory of action at a distance must be doomed.

Before passing to the post-Maxwellian, or, as we may call it, the modern era, it may be convenient to state Maxwell's theory as he left it.

Electrical changes are related, not to the so-called field intensity, whose components are X, Y, Z, derivable from a potential Ψ,

$$(1) \qquad X = -\frac{\partial \psi}{\partial x}, \; Y = -\frac{\partial \psi}{\partial y}, \; Z = -\frac{\partial \psi}{\partial z},$$

but to a new vector, called the electric displacement, and denoting the state of polarization discovered by Faraday. The components of this, f, g, h, are proportional to the components of the field,

$$(2) \qquad f = \partial \frac{K}{4\pi} X, \; g = \frac{K}{4\pi} Y, \; h = \frac{K}{4\pi} Z,$$

where K measures a physical property of the medium, Faraday's specific inductive capacity.

The density of the charge then is derived from the displacement by the equation

$$(3) \qquad \rho = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}.$$

Similarly in magnetism we are to consider two vectors, the field α, β, γ, derivable from a potential Ω,

$$(4) \qquad \alpha = -\frac{\partial \Omega}{\partial x}, \; \beta = -\frac{\partial \Omega}{\partial y}, \; \gamma = -\frac{\partial \Omega}{\partial z},$$

and the induction, proportional to it,

$$(5) \qquad a = \mu\alpha, \quad b = \mu\beta, \quad c = \mu\gamma,$$

where μ measures another physical property of the medium. Maxwell here puts however

$$(6) \qquad \frac{\partial a}{\partial x} + \frac{\partial b}{\partial y} + \frac{\partial c}{\partial z} = 0$$

so that the analogy with the electrical equation (3) is not quite perfect.

To the equations previously accepted giving the relations between the current density u, v, w, and the magnetic field produced by it,

$$(7) \qquad \begin{aligned} 4\pi u &= \frac{\partial \gamma}{\partial y} - \frac{\partial \beta}{\partial z}, \\ 4\pi v &= \frac{\partial \alpha}{\partial z} - \frac{\partial \gamma}{\partial x}, \\ 4\pi w &= \frac{\partial \beta}{\partial x} - \frac{\partial \alpha}{\partial y}, \end{aligned}$$

Maxwell adds the effect of the displacement currents, so that he has

$$(8) \qquad \begin{aligned} 4\pi \left(\frac{\partial f}{\partial t} + u \right) &= \frac{\partial \gamma}{\partial y} - \frac{\partial \beta}{\partial z} \\ 4\pi \left(\frac{\partial g}{\partial t} + v \right) &= \frac{\partial \alpha}{\partial z} - \frac{\partial \gamma}{\partial x} \\ 4\pi \left(\frac{\partial h}{\partial t} + w \right) &= \frac{\partial \beta}{\partial x} - \frac{\partial \alpha}{\partial y}. \end{aligned}$$

The induced electromotive-forces due to changes in the magnetic field are represented by Maxwell in a somewhat peculiar manner, as the negative derivatives of a new vector called the vector-potential, so that

$$(9) \qquad P = -\frac{\partial F}{\partial t}, \quad Q = -\frac{\partial G}{\partial t}, \quad R = -\frac{\partial H}{\partial t},$$

The vector-potential is related to the magnetic induction by the equations

$$(10) \qquad \begin{aligned} a &= \frac{\partial H}{\partial y} - \frac{\partial G}{\partial z}, \\ b &= \frac{\partial F}{\partial z} - \frac{\partial H}{\partial x}, \\ c &= \frac{\partial G}{\partial x} - \frac{\partial F}{\partial y}. \end{aligned}$$

Thus the vector-potential itself does not appear, but only its time-derivatives.

The vector-potential was introduced by Maxwell to denote what Farraday termed the electro-tonic state of a body undergoing induction of current by magnetic changes.

Strangely, as it now seems, the ideas of

Maxwell were slow in gaining acceptation. We must not omit to notice that in 1867 an electro-magnetic theory of light was developed by Lorentz, but it was deduced from different considerations. It was not until the appearance of Maxwell's treatise, in 1873, that the attention of Continental thinkers seems to have been drawn to the new views. Already, however, had begun the appearance of that series of papers by the master hand of Helmholtz, which, beginning with a powerful exposition of the old electrodynamical theories, led by successive steps to the development of a theory similar to that of Maxwell, which, as the life of the great German drew to a close, became completely adopted by him. Otherwise, however, there is little to chronicle on the Continent until the appearance of the treatise of Mascart and Joubert, over a decade later than Maxwell's, showed that the seed had not fallen upon stony ground. Let us accordingly return to England, whither the center of gravity of the mathematical development of the subject was now transferred.

The first paper to appear in the *Philosophical Transactions* on Maxwell's theory was fifteen years later, by Fitzgerald, in 1880, on the 'Electro-magnetic Theory of the Reflection and Refraction of Light.' Here we find an extension of the suggestion made by Maxwell that the magnetic energy of the field contains terms depending on vortical motions about the lines of magnetic force, and thus an attempt is made to explain the phenomena of reflection of light from the surface of magnets, discovered by Kerr.

In 1881 followed a paper by Niven on 'The Induction of Currents in Infinite Plates and Spherical Shells,' the former being a subject which had been investigated by Maxwell. In 1883 the subject of electrical motions in spherical conductors was treated by Lamb, who, however, makes the simplifying hypothesis that the velocity of propagation of the inducing effect is infinite, that hypothesis not materially conflicting with any experiments then made. In 1884 appeared a paper by Larmor on the same subject.

In 1884 appeared a very important paper by Poynting on 'The Transfer of Energy in the Electro-magnetic Field,' where, by a direct application of Maxwell's theory, it was shown how the route taken by the energy in its passage from one plate to another during variations of the field could be described by the statement that the energy flowed everywhere perpendicularly to both the electric and magnetic field-vectors, at a rate proportional to the area of the parallelogram constructed on their geometric representatives. Thus, according to Poynting, the energy passes from a dynamo to a motor not through the wires connecting the two, but through the air, being guided in its course, however, by the wires. The ideas of Poynting were extended by Wien in two papers in *Wiedemann's Annalen* in 1892, as well as in a paper by J. J. Thomson, on 'Faraday Tubes of Force.'

In 1885 we have a still more important paper by J. J. Thomson, on 'The Application of Dynamics to Physical Phenomena,' in which, not especially the electrical ideas of Maxwell are developed, but rather the method introduced by him of applying Legrange's equations is extended to a great number of phenomena. Singularly enough the same sort of methods were soon to be used by Helmholtz quite independently for similar purposes.

In 1887 Lamb attacked the more difficult problem than that of the sphere of ellipsoidal current sheets, treating as a special case the flow of currents in a circular disc, and in 1888 the problem of induction of currents in shells of small thickness was treated by Burbury. The flow of current in cylindrical conductors has been treated by J. J. Thomson and Lord Rayleigh. In all these researches upon current-flow, interesting questions regarding partial differential equations were treated, but much is still left for the mathematician to do. In fact, it was in this same year, 1888, that a striking sensation was produced in the scientific world by the publication of the experimental researches of a new genius, Heinrich Hertz, who produced in the laboratory the electrical waves conceived by Maxwell, and for the first time confirmed the theory by demonstrating the finite velocity of propagation. Experimental papers now succeeded each other with astonishing rapidity, confirming one point after another of the theory, and the experiments of Hertz immediately obtained a vogue which has not yet subsided. The energies of mathematicians were now taxed anew, for the question of the nature and period of the electrical motions in the curiously shaped 'oscillators' used by Hertz and his followers to produce the waves became very important. The previous investigations already described did not cover the case even for spheres and ellipsoids, for, in the case of vibrations of the rapidity now experimentally realized, the effect of the displacement currents could no longer be neglected nor could the velocity of propagation be considered infinite, while the phenomenon of radiation of energy from the conductor into space demanded mathematical recognition and treatment. This it failed to get, except in an approximate manner, for anything except the simplest case, that of the sphere. This was treated by J. J. Thomson and Poincaré. Nevertheless, the theory has not been experimentally verified for the sphere, because the sphere is a badly-shaped conductor for experimental purposes. In order to retain energy enough to maintain the vibrations for a number of oscillations, the conductor should be long or even dumb-bell-shaped, and not at all like a sphere.

The long spheroid is then next to be attacked, and then other surfaces, perhaps those obtained by the revolution of the curves known as cyclides. The introduction of suitable curvilinear coordinates into the partial differential equation concerned,

$$\frac{\partial^2\phi}{\partial t^2}=a^2\Delta\,\phi$$

will lead, even in the case of the spheroid, to new linear differential equations analogous to but more complicated than Lamé's, and will necessitate the investigation of new functions and developments in series.

The remarkable experimental skill shown by Hertz is not his only title to our admiration. His inaugural dissertation had been a treatment of the flow of electricity in spheres, and his electrical researches received a fitting completion in two mathematical articles in which Maxwell's theory was systematized and stated in an extremely clear and symmetrical manner. The equations of Maxwell, stated above, are unfortunately lacking in symmetry, and certain questions that have since arisen were not contemplated by him. These improvements were made in a very satisfactory manner by Hertz, as we shall describe later. We can not, however, award to Hertz the credit of priority in this matter, for the work had already been done by another writer, of whom I must now speak at length—I mean that extraordinary Englishman, Mr. Oliver Heaviside. Of this undoubted genius I feel that it is no exaggeration to say that he understands the theory of electricity probably better than anyone else now living. Of a decidedly eccentric personality and mode of expression, unknown to and unseen by most of his scientific countrymen, this self-taught luminary appeared on the horizon over twenty years ago, and slowly but surely approached the brilliancy of a star of the first magnitude. Unnoticed at first, he forced himself upon the attention of physicists by the sheer quantity of his productions, and it was then found that their quality was also exceptional. His writings have mainly dealt with a quite different sort of subject from those enumerated above, and have treated either the flow of variable currents in wires or the transmission of electro-magnetic waves in free space. As early as 1876, in a paper modestly entitled 'On the Extra-current,' he gives for the first time the partial differential equations for the propagation of current and potential along wires, and treats them by the methods of Fourier. Later he considers the most complicated questions caused by the terminal conditions involved by the introduction of various sorts of electrical apparatus, such as those used in telegraphy and telephony. These papers will well repay the attention of pure mathematicians, to whom they offer a host of questions for rigid treatment. For instance, to fix the ideas, we have the equation of propagation

$$a\frac{\partial^2\phi}{\partial t^2}+b\frac{\partial\phi}{\partial t}=c\frac{\partial^2\phi}{\partial x^2}.$$

with the condition that for two or more given values of x, φ is to satisfy given linear *ordinary* differential equations in t, and that for a given value of t, φ and $\frac{\partial\varphi}{\partial t}$ are to be given functions of x.

If we attempt to satisfy the equation by particular solutions which are trigonometric functions of the time we get an ordinary differential equation in x, and if we then make use of trigonometric functions of multiples of x the multiples allowable will be determined by certain transcendental equations according to the terminal conditions. The function φ is then to be developed in a series of such trigonometric terms. The nature of the proofs desired relating to the series may be readily inferred. The papers by Heaviside are extremely numerous and bulky, and it is desirable that the methods there used should receive critical attention from mathematicians, for it must be said that Heaviside uniformly disdains such things as existence-theorems, depending chiefly on his intuitions drawn from physical reasoning.

A large portion of Heaviside's labors has been devoted to the systematization and extension of Maxwell's theory and the attempt to disseminate a knowledge of that theory among the physical public. His results agree so nearly with those of Hertz that I shall give them in the notation and form used by the latter, which seem to me preferable. The attempt to bring out the symmetry or reciprocity between electrical and magnetic phenomena has been paramount with both Heaviside and Hertz. Accordingly, we have for the connections between the electric and magnetic field intensities, represented respectively by X, Y, Z and L, M, N, and the corresponding inductions or polarizations $\mathfrak{X}, \mathfrak{Y}, \mathfrak{Z}$ and $\mathfrak{L}$, $\mathfrak{M}$, $\mathfrak{N}$,

$$(11)\quad \mathfrak{X}=\varepsilon X,\quad \mathfrak{Y}=\varepsilon Y,\quad \mathfrak{Z}=\varepsilon Z \qquad\Big|\qquad \mathfrak{L}=\mu L,\quad \mathfrak{M}=\mu M,\quad \mathfrak{N}=\mu N \quad (12)$$

instead of equations (2) and (5), ε representing the specific inductive capacity and μ the magnetic permeability. For the electrical and magnetic densities ρ_e and ρ_m we have

$$(13)\qquad \rho_\epsilon=\frac{1}{4\pi}\left\{\frac{\partial\mathfrak{X}}{\partial x}+\frac{\partial\mathfrak{Y}}{\partial y}+\frac{\partial\mathfrak{Z}}{\partial z}\right\},$$

$$(14)\qquad \rho_m=\frac{1}{4\pi}\left\{\frac{\partial\mathfrak{L}}{\partial x}+\frac{\partial\mathfrak{M}}{\partial y}+\frac{\partial\mathfrak{N}}{\partial z}\right\}.$$

For the mutual connections of the two fields we have

$$(15)\qquad \begin{aligned} 4\pi u+\frac{\partial\mathfrak{X}}{\partial t}&=\frac{\partial N}{\partial y}-\frac{\partial M}{\partial z}, \\ 4\pi v+\frac{\partial\mathfrak{Y}}{\partial t}&=\frac{\partial L}{\partial z}-\frac{\partial N}{\partial x}, \\ 4\pi w+\frac{\partial\mathfrak{Z}}{\partial t}&=\frac{\partial M}{\partial x}-\frac{\partial L}{\partial y}, \\ -\frac{\partial\mathfrak{L}}{\partial t}&=\frac{\partial Z}{\partial y}-\frac{\partial Y}{\partial z}, \end{aligned}$$

$$(16)\qquad -\frac{\partial\mathfrak{M}}{\partial t}=\frac{\partial X}{\partial z}-\frac{\partial Z}{\partial x},$$
$$-\frac{\partial\mathfrak{N}}{\partial t}=\frac{\partial Y}{\partial x}-\frac{\partial X}{\partial y}.$$

Thus, with the exception of the electrical currents u, v, w on the left of equation (15) and the negative sign in (16), we have complete analogy between the electrical and magnetic equations. In these equations neither the electric nor magnetic potentials nor the vector potentials appear, and we are concerned only with the two field intensities, which have a more tangible existence than the potentials. While equations (15) are identical with (8), equations (16) take the place of (9) and (10), for if we differentiate equations (10) according to the time, and substitute on the right for the time-derivatives of F, G, H, their values from (9), we obtain (16).

In order to obtain the result of propagation with finite velocity, let us consider a non-conductor, where u, v, w are zero, and let us suppose ε, μ to be constants. Differentiating the third of equations (16) according to y and subtracting from the second differentiated according to z gives

$$\frac{\partial}{\partial t}\left\{\frac{\partial\mathfrak{N}}{\partial y}-\frac{\partial\mathfrak{M}}{\partial z}\right\}=\frac{\partial^2 X}{\partial x^2}+\frac{\partial^2 X}{\partial y^2}+\frac{\partial^2 X}{\partial z^2}$$
$$-\frac{\partial}{\partial x}\left\{\frac{\partial X}{\partial x}+\frac{\partial Y}{\partial y}+\frac{\partial Z}{\partial z}\right\}.$$

Substituting the value of the parenthesis in the left from the first of equation (15), after making use of equations (11) and (12) and assuming that the parenthesis on the right vanishes, since there is no free electricity, gives

$$\varepsilon\mu\frac{\partial^2 X}{\partial t^2}=\frac{\partial^2 X}{\partial x^2}+\frac{\partial^2 \gamma}{\partial y^2}+\frac{\partial^2 Z}{\partial z^2},$$

which is the equation for the propagation of waves, as we have it in the theories of sound and light. The velocity of propagation is $\dfrac{1}{\sqrt{\varepsilon\mu}}$.

The last paper of the series of Hertz treats of the equations to be used in connection with moving bodies, and besides introducing terms accounting for the remarkable discovery by Rowland of the magnetic effect of a moving charge of electricity suggests several matters not yet verified by experiment. Still deeper into the theory goes an elaborate paper by Heaviside on the 'Forces, Stresses and Fluxes of Energy in the Electro-magnetic Field,' published in the *Philosophical Transactions* of 1892. In this paper Heaviside deals with matters which are still open to controversy.

The last contributions of Helmholtz to the theory were his paper on the 'Principle of Least Action in Electrodynamics,' published in 1893, in which he seeks to deduce all the electrical equations from this fundamental mechanical principle, and his paper on the 'Electro-magnetic Theory of Dispersion,' in which he adapts his beautiful explanation of this complicated optical phenomenon to the electro-magnetic theory.

I have time here only to mention researches on electro- and magneto-striction, or change of form and shape of bodies in electric or magnetic fields, to which contributions have been made by Helmholtz, Boltzmann, Kirchhoff, Stefan, Lorberg, Adler, Cantone and Duhem, and on the magnetic effects produced by the motion of electrical charges, upon which subject papers have appeared by J. J. Thomson, Heaviside and Searle.

Before closing I must, however, mention several elaborate papers by Larmor, begun in the *Philosophical Transactions* for 1894, in which the attempt is made to propound a dynamical theory of the ether, which shall not only give a suitable explanation of light, but also a dynamical theory of all electric and magnetic phenomena, including the electro-magnetic theory of light. For this purpose the old theory of McCullagh is found to be available and is developed with extremely interesting results, while a great variety of phenomena are dealt with.

I have here made no mention of the work that has been done on the various theories of the mutual actions at a distance of current elements, as these are thoroughly dealt with in J. J. Thomson's admirable British Association report on electrical theories in 1885. I have thus, in a very brief and unsatisfactory manner, merely touched upon some of the principal points of the development of the theory of electricity, and traced its gradual but unceasing progress from the hands of the giants of the old days to those of the new.

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THE LIMITATIONS OF THE PRESENT SOLUTION OF THE TIDAL PROBLEM.

THAT which is new in science is always interesting. But it is well at times to let the old and well-tried pass in review before us, to plan renewed attacks upon the unknown, in the light of the elements of strength or weakness found in different portions of the army of known facts and principles, and with respect to the stubborness of the resistance which has been encountered by attacks upon different parts of the unknown. A helter-skelter attack may perhaps produce more interesting and more surprising results than a well planned campaign, but the latter would be expected to furnish the more important results. The purpose of this paper is not to state anything new, but to point out a very weak point in tidal theory, a point which it is important to have strengthened, and of which the strengthening is apt to lead to a decided advance in our knowledge of the subject.

The thesis which I submit is that the present theory of the tides upon the earth when used to explain those tides, or to predict their occurrence at a particular point, furnishes very little except the *periods* of the separate harmonic, or invariable, components of the tide. It does not furnish the times of occurrence of the tides, that is, the epochs of the components, nor does it furnish the range of the tide as defined by the amplitudes of the harmonic components.

This thesis may be exhibited in concise form by writing the algebraic expression for the height of the tide referred to mean sea level at any instant at a given point

$h = A_1 \cos(\alpha_1 t + \beta_1) + A_2 \cos(\alpha_2 t + \beta_2)\ldots\ldots$

Each term of this expression indicates one of the harmonic components of the tide. From pure theory, reasoning from the known motions of the Moon, Sun and Earth and the Newtonian law of gravitation, it has been shown that if certain definite values be assigned to the quantities $\alpha_1, \alpha_2, \ldots$ (fixing the periods of the separate terms), that each term truly represents one of the invariable components of the tide. Here, after *merely* fixing the periods of the separate components the contribution of tidal theory ends, and the work of direct observation at the particular station under consideration begins. The values of A_1, A_2, defining the amplitudes of the separate components and the range of the composite tide, and of $\beta_1\ \beta_2\ldots$ fixing the epochs of the separate components and the time of the tide, must be derived directly from observations at the particular station in hand. Their values cannot, at present, be assigned even approximately from theory.

If we look at the *actual* tidal problem presented to us by Nature the reason why theory furnishes so little will become evident. In approaching the actual problem let us begin by considering a simple ideal problem which can be and has been successfully handled, and then introduce into this problem, one by one, the *actual* conditions which modify it. In doing so we may, to a certain extent, follow the historic order of tidal research.

There are tides produced by both the Moon and the Sun. To avoid circumlocu-