(1) White Sands,	9 ears,	weighing	10.38	grams,	containing	283	grains,	weighing	7.310	grms.
(2) Larrea,	10 "	"	9.77	"	"	243	"	"	6.995	"
(3) Mesquite,	10 "	"	10.60	"	"	285	"	"	7.610	"
(4) Tornillo,	10 "	"	11.88	"	"	330	"	"	8.270	"
(5) Adobe,	10 "	"	10.24	"	"	254	"	"	7.335	"

PEAS (Measurements in cm.).

	Feb. 26.	Mar. 7.	Mar. 14.	Mar. 19.	Apr. 1.
White Sands,	12.	27.	38.	48.	57.
Larrea,	13½.	25½.	27½.	36.	51.
Mesquite,	11½.	25.	31.	38.	42.
Pluchea,	13½.	29½.	36.	47.	53.
Adobe,	12½.	23½.	26.	35¾.	54.

No. 1 and 4 were not perfectly ripe, and may owe a very little of their weight to the extra moisture they contain. It will be seen that the gypsum wheat weighs up well with the others, and when its green heads, above mentioned, are ripe the product will outweigh considerably all of the others.

It is seen from the table that the gypsum peas are decidedly the best. We could not measure the yield (the gypsum peas were the first to bloom, on March 19th), because certain girls of the class in horticulture saw fit to remove some of the pods when unobserved by their professor.

CONCLUSIONS.—It appears, from these preliminary researches, that nearly pure gypsum will nourish plants as well as ordinary soil, or even better. It is not apparent how the wheat, etc., come by their nitrogen in such a soil, though the peas may well get it by means of their root-tubercles. The absence of other elements is also noticeable, but it is not worth while at the present time to enter into a detailed discussion of causes and effects, as further researches will, it is hoped, make such a discussion more profitable at a later date.

T. D. A. COCKERELL.
FABIAN GARCIA.

N. M. AGR. EXP. STA.
MESILLA PARK, N. M., June 17, 1898.

*THE CROSS-RATIO GROUP OF 120 QUADRATIC CREMONA TRANSFORMATIONS OF THE PLANE.**

GROUPS of *linear* substitutions have long been studied with reference to (1) the geometric representation in the plane or on the sphere, (2) the rational integral functions left invariant under the operators of the group. These questions now prove to be of interest when investigated for groups of transformations of order higher than the first. The theory of birational transformations (quadratic and higher) has been given by Cremona, Cayley, Clebsch and others. Groups of such transformations have been enumerated by Autonne and S. Kantor. The cross-ratio Cremona transformation groups of order $n!$ were first given by Professor E. H. Moore in his lectures at the University of Chicago in the spring of 1895. These groups are found by determining for *each* permutation of n quantities a fundamental system of n–3 cross-ratios in terms of which the cross-ratios of *every* four out of the n quantities are expressible, and then setting up the transformation relations among these $n!$ fundamental systems.

* Abstract of a Dissertation submitted to the Faculties of the Graduate Schools of Arts, Literature and Science in the University of Chicago, April, 1898, in candidacy for the degree of Doctor of Philosophy (Department of Mathematics), by H. E. Slaught.

The case $n=5$ gives the group of 120 *quadratic* transformations of the plane which is the subject of the present study. This group contains a *linear* subgroup of order 244 ! which permutes in all possible ways the four fundamental points of the Cremona group. This subgroup is isomorphic with Klein's linear group of order 4 ! for which Professor Moore has shown a division of the plane to be given by a certain complete quadrangle (including its diagonals) whose vertices are the four points permuted.

I. This affords a means of finding a geometric representation for our *quadratic* group as follows:

(1) A linear fractional transformation is found which throws the complete quadrangle for the Klein group into another whose vertices are the four fundamental points of the *quadratic* group, and which, therefore, gives the division of the plane for the *linear* subgroup. (2) The linear subgroup is transformed by all the quadratic operators of G120, giving four quadratic subgroups conjugate with the linear subgroup. (3) The division of the plane for these quadratic subgroups differs from that of the linear subgroup only by replacing, each time, the three diagonal lines by certain three conics. (4) The division of the plane for the main group is then given by a composite of the five pictures belonging to these five conjugate subgroups, and consists of the original complete quadrangle together with its diagonals and twelve conics. A further study of the various subgroups shows the following conjugate systems of special lines or points: (1) A system of ten elements consisting of the six sides of the original quadrangle, which are fundamental lines, and the four pencils of 'directions' at the four fundamental points. (2) A system of fifteen lines consisting of the three diagonals and twelve conics. (3) A system of fifteen lines consisting of certain three conics not in the configuration and twelve 'direction' lines through the fundamental points. (4) Twelve real points at each of which five lines intersect. (5) Fifteen real points where four lines intersect. (6) Twenty imaginary points of three-fold intersection. (7) Thirty real points of two-fold intersection. (8) Twenty imaginary points lying by pairs on the six sides and four pencils. (9) Thirty imaginary points lying by pairs on the three diagonals and twelve conics.

II. The Klein linear group of order 4! also affords the means of finding the invariants of the quadratic group, as follows: (1) The complete form-system of the linear subgroup comes from the known system for the Klein group by the same transformation which throws the generators of the former group to those of the latter. (2) The most general invariant form of any given degree under the linear subgroup is then set up with arbitrary coefficients and operated upon by the quadratic generator which extends the linear subgroup to the main group. (3) This doubles the degree of the given form, and hence the only possibility for the existence of an invariant under the quadratic group is to so determine the arbitrary constants that a factor in the variables may divide out, leaving the original form. (4) Hence an invariant form under a quadratic operator must be a *rational fraction*, such that a common factor in the variables will cancel from numerator and denominator, leaving the original fraction. (5) It is found that the most general forms suitable for numerator and denominator of invariant fractions of the 6th, 12th and 18th degrees respectively are:

$$mA,\ m_1A^2 + m_2P^2,\ m_1A^3 + m_2AP^2 + m_3C$$

when the m's are arbitrary constants and

$$A = 2p^2q^2 - 6(p^3r + q^3) + 19pqr - 9r^2$$

$$P^2 = p^2q^2r^2 - 4(q^3r^2 + p^3r^3) + 18pqr^3 - 27r^4$$

$$C = 100p^4q^4r^2 - 1242r^6 + 560p^3q^3r^3 - 2150p^2q^2r^4$$
$$+ 2826pqr^5 - 286(p^2p^5q^5r^2 + p^5q^2r^3)$$

$$- 34(p^6r^4 + q^6r^2) - 292(q^3r^4 + p^3r^5)$$
$$+ p^2q^8 + p^8q^2r^2$$
$$+ 530(p^4qr^4 + pq^4r^3) + 50(p^7qr^3 + pq^7r)$$
$$- 4(q^9 + p^9r^3) - 12(p^6q^3r^2 + p^3q^6r)$$

in which p, q, r in terms of the homogeneous variables are

$$p = Z_1 + Z_2 + Z_3,\ q = Z_1Z_2 + Z_1Z_3 + Z_2Z_3,$$
$$r = Z_1Z_2Z_3.$$

It is to be noted that P itself is not expressible in terms of p, q, r, but

$$p = Z_1Z_2Z_3(Z_1 - Z_2)(Z_1 - Z_3)(Z_2 - Z_3).$$

As a remarkable coincidence it was found that the three invariants of the complete form-system of the binary quintic form, when written in terms of a fundamental system of two cross-ratios of the roots, are precisely these forms A, P^2, C, when similarly expressed in terms of the cross-ratios. It is shown that A, P^2 and C are the complete form-system of our quadratic group, G_{120}, by a series of theorems of which the most important are the following: (1) An invariant under a quadratic operator must be a *fraction* whose numerator and denominator throw off a common factor in the Z's. (2) The numerator and denominator of an invariant fraction must be absolute or relative invariants under the linear subgroup and hence rational integral functions of the known invariants in its complete form-system. (3) There can be no invariant fraction whose numerator and denominator are of odd degree or of unequal degree. (4) The most general invariant form suitable for numerator or denominator of an invariant fraction under G_{120} is of degree $6n$ and throws off the factor r^{2n} $(r = Z_1Z_2Z_3)$ under the quadratic generator; $Z'_1 : Z'_2 : Z'_3 = Z_2Z_3 : Z_1Z_3 : Z_1Z_2$. (5) The most general invariant form under $G_{120}(a)$ is of the form $R_{6n} = P^{2\mu}R_{6(n-2\mu)}$, where $\mu = 0$ or a positive integer and $R_{6(n-2\mu)}$ contains no factor of P; and (b) has at each of the four critical points a multiple point of order $2(n + \mu)$. (6) If α and β are two such ternary forms having the binary forms a and b, of degree λ and μ, respectively, as tangential quantics at one of the critical points, then $\alpha\beta$ has ab as its tangential quantic at the same multiple point, and $\alpha + \beta$ has a, b or $a + b$ according as λ is less than, greater than or equal to μ. (7) No ternary form can have a binary tangential quantic at any critical point of *odd* degree in *either* or *both* of the cubic invariants belonging to the dihedron subgroup which leaves the critical point fixed. (8) Two reduced ternary forms of the same degree which have the same tangential quantic at any critical point can differ only in such terms as involve P^2 as a factor.

By means of these theorems it is then shown, by a process of successive reduction, that the most general invariant form under G_{120} is expressible as a rational integral function of A, P^2, C, and thus a system of fundamental forms is established in terms of which all invariant fractions under the quadratic group can be expressed. The above forms are *absolute* invariants. The only *relative* invariant fractions are those expressible in terms of A, P and C, which are invariant except for change of sign.

THE CONFERENCE OF SCIENCE TEACHERS IN THE TRANS-MISSISSIPPI EDUCATIONAL CONVENTION.

A FEW months ago the undersigned was requested by the program committee to arrange a series of conferences of science teachers in connection with the Trans-Mississippi Educational Convention, to be held in Omaha, June 28th, 29th and 30th. As a result there were held seven conferences, namely, in Chemistry, Physics, Astronomy, Botany, Zoology, Geography and Geology, occupying the afternoon sessions of the 29th and 30th. The following abstracts of the principal papers will give some idea of these meetings. The attendance was not large,