tain assumptions deduced theoretically. The second was then derived theoretically and was found to satisfy the data.

An abstract of No. 13 was read by the Secretary. It follows the theory of perturbations in the problems of mechanics in the order of its historical development from Lagrange to Lie with a view to the final presentation of the theory in its just position as one phase of Lie's theory of contact-transformations.

No. 14 was presented by the Secretary. It gave a bibliography of the Pythagorean proposition, followed by a general solution of the equation $x^2 + y^2 = z^2$, and concluded with an extensive numerical table of the sides of rational right-angled triangles.

No. 15 expressed in terms of the tabulated E and F functions a number of integrals, many of which had apparently never been completely worked out.

No. 16 showed how to express the effect of a small difference in the equatorial moments of inertia of the earth on the period of revolution of the instantaneous axis of rotation around the axis of figure. A remarkable value is obtained for the average angular velocity of that revolution, and a formula is deduced for the difference in the equatorial moments essential to explain the discrepancy between the observed and computed value of the Eulerian cycle.

No. 17 considered the case of no applied forces, or that in which there is conservation of moment of momentum. The problem is of practical interest in its application to the question of variation of latitudes on the earth. Several new theorems with respect to the motions of the mass were derived.

No. 18 obtained some general properties of a class of functions of which the spherical harmonics are special cases.

No. 19 described the present annoying state of the subject-matter of notation and coordinate systems, and advocated the adoption of certain standards. It is expected that this question will be further discussed at the Boston meeting with a view to obtaining a consensus on symbols and fundamental conventions.

No. 20 called attention to a certain quadrilateral whose properties throw new light on the theory of the Pascal lines of a hexagon inscribed in a conic.

In presenting No. 21 Professor Greenhill gave stereoscopic views of certain interesting curves in space and pointed out the bearing of some of them on certain parts of the theory of elliptic functions.

The section is also indebted to Professor Greenhill for interesting contributions to the discussions on many of the other papers.

The officers elected for the Boston meeting are Professors E. E. Barnard, of Yerkes Observatory, and Alexander Ziwet, of the University of Michigan.

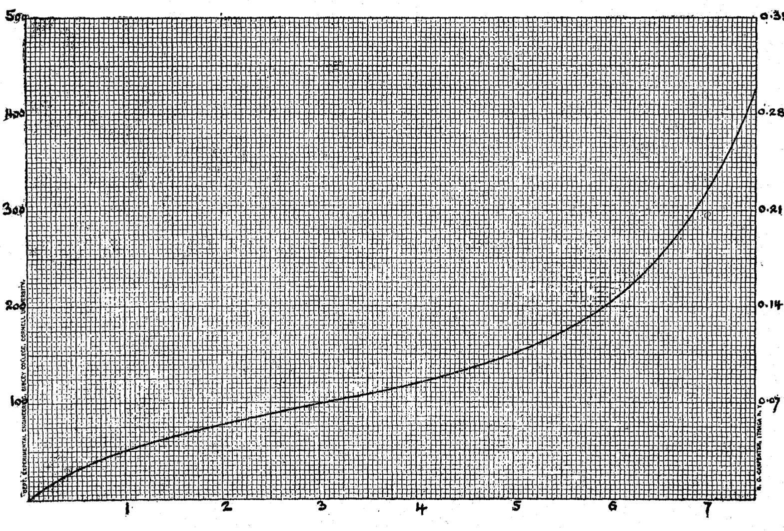
JAMES MCMAHON.

SINGULAR STRESS-STRAIN RELATIONS OF INDIA RUBBER.

THE curious and unaccountable behavior of India rubber in thermodynamic transformation of energy under load has long been familiar ; it is, perhaps, even more generally known that it exhibits a peculiar relation of elongation to load when approaching its limit of tenacity, but I am not aware that this later phenomenon has ever been exhibited by formal test or by graphic representation of the results of such tests. It is a matter of common observation that, when this substance is subjected to a pull of steadily increasing intensity, its resistance increases as does that of any elastic and ductile material; but that, at the end, instead of suddenly losing power of resistance, or even snapping without observable decrease of load, its resistance for a time rapidly and largely increases up to the point of rupture. This can be readily felt in even the breaking of one of the small bands of partially vulcanized rubber now so universally employed for filing papers and other purposes. At the end of the period of extension the resistance rises so rapidly as to produce the sensation of bringing the hand up against a rigid obstacle, resisting further elongation.

Figure 1 is the stress-strain diagram of a strip of rubber, partially vulcanized, but not sufficiently to disguise the peculiar characteristics of the material.

Studying this diagram, it will be observed that the substance behaves precisely like other familiar materials, up to a point which, in this case, is found at a load of thirty per cent. of the maximum, the breaking load, and at an extension one half the maximum. At this point there exists a reversal of the line, and the curvature is thence maintained convex to the axis of X, up to the point of rupture; fracture taking place, at the end, sharply and without any indication of that method of flow of the mass which, in the case of the irons and softer steels, for example, permits a falling-off of resistance after passing a point of maximum tenacity well within the breaking limit. The ratio of increase of load to increase of elongation steadily increases from the zero point, as with all substances, other than iron and steel, so far as known, up to this point of contrary flexure on the diagram; at which place the ratio is inverted and resistance increases in greater proportion than extension, finally assuming a comparatively high value.



Strain per Unit Length.

FIG. 1.—Stress-Strain Diagram of India Rubber.

The singularity of this action will be brought into relief by examining the diagrams produced with other materials employed in the arts.* Swedish or Norway Iron, Common Merchant Iron, 'Mild' Steel, and Tool-Steel have been subjected to loads increasing to the point of rupture, with occasional removal of the load.

*Thurston's Materials of Engineering, Vol. II., p. 611.

The nearly-vertical lines on each curve, at intervals, from the origin, are the 'elasticity lines,' as they have been called, showing the action of the material when the load is gradually relieved. These substances do not restore themselves to the original dimensions, but remain permanently distorted, taking more or less set and, usually at least, maintaining a constant value of the modulus of elascity. The slight droop in each line, near its outer extremity, shows the behavior of the substance when the load is left unchanged and elongation is maintained constant. This 'exaltation of the elastic limit' was discovered by me many years ago, and announced to the American Society of Civil Engineers in 1873.* India rubber exhibits none of the phenomena giving the characteristic form of the diagrams of the irons and steels. Even when stretched to the point of rupture it restores itself very nearly to its original dimensions and gradually recovers a part of the loss of form at that instant observable. Its almost complete stability of form when relieved from load, and especially when in the shape of springs such as are used on railway trucks, constitutes one of its most valuable properties. Like cork, when confined laterally, it is practically indestructible and incapable of distortion when used as a spring. The singular stress-strain relations of the substance may probably be found to give it peculiar value for many other purposes. The sample illustrated by our diagram was of the kind employed for springs and rubber bands and, as usual, slightly vulcanized.

The equation of the curve of this character described by the materials of construction generally may be taken, without important error, usually, as of the form

$$E=a\sqrt{T};$$

and with elongation, E, and T, tenacity, respectively, in per cent. of total length of specimen and in pounds per square-inch, the constant, a, is usually not far from 0.1 for good irons, 0.05 for tool-steels, and for good soft copper it is about 1.2. For this specimen of india rubber the equation is obviously so entirely different in class and form, and indicates so evidently an entirely different method of molecular action in resistance of strain, that it must constitute a problem by itself. The peculiarities of this form may, however, prove a key to the corresponding singularities of its internal structure and forces. So far as known, no other substance gives such unique relations of forces holding the substance in stable form to the variations of form to which the external application of force gives rise. The volume of the mass remains, so far as can be seen, constant, or nearly so, and the expenditure of work upon the substance results simply in changing the intermolecular distances of adjacent particles. This fact will probably be found to simplify the process of experimental interpretation of this curious case.

The cause of the singular behavior of this thermodynamically unique material remains to be discovered. It is probably safe to assume that its mechanical and physical properties will have some close relationship, and that the investigation, to be complete, must comprehend both lines of research. The gum must evidently possess either some very strange molecular force-relations or it must be the fact that, in the case of the ordinary materials of engineering and construction, these apparently peculiar characteristics are, in them, as yet unobserved because of their minute effects. The study of the subject cannot fail to lead to new and interesting facts relative to the molecular constitution of this and probably of other substances.

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* Trans. Am. Soc. C. E., November, 1873.