physical research in our own generation were stricken out!

The future should look bright, indeed, as we view it to-day. Munificence and skill have provided this splendid observatory with means for promoting knowledge in both the older and the newer branches of the sublime science to which it is dedicated. Its magnificent equipment will be used by men who have won merited distinction in both the older and the newer methods of research. It has the cooperation and support of a great institution of learning. From this happy union of ability and opportunity we have reason to expect results of the highest import to the new astromony, and to its allied branches of physical science.

But, lest any words of mine should give rise to expectations that may not be fulfilled, I wish to say once more that important results are not necessarily of a striking or surprising character. We can hardly assume that every increase in telescopic power will be followed by the discovery of new planets or satellites. Such discoveries, if they come, will be welcome ; but they should not be expected. There may be no more planets or satellites, yet undiscovered, in the solar system. But we may confidently expect from the work of this observatory those results which throw light on the dark places in nature, and which, therefore, though they may not stimulate the popular imagination, are of the very highest importance, for they are indispensable to true scientific progress.

JAMES E. KEELER.

ALLEGHENY OBSERVATORY.

MATHEMATICS AND ASTRONOMY AT THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.

THE officers of the Section of Mathematics and Astronomy were as follows: Chairman, W. W. Beman; Secretary, J. McMahon; Press Secretary, P. A. Lambert; Councillor, E. W. Hyde; Sectional Committee, W. W. Beman, J. McMahon, A. Macfarlane, W. F. Durand, J. E. Kershner, W. S. Pritchett; Member of Nominating Committee, A. Ziwet; Committee to nominate officers of Section, W. W. Beman, J. McMahon, A. Hall, Jr., R. S. Woodward, A. Macfarlane.

The Chairman's address was on 'A Chapter in the History of Mathematics,' which has already been published in this JOURNAL.

The following papers were presented to the Section:

1. A Problem in Substitution-groups. By Dr. G. A. Miller, Rosette, Kan.

2. Continuous Groups of Spherical Transformations in Space. By Professor H. B. Newson, Lawrence, Kan.

3. The Treatment of Differential Equations by Approximate Methods. By Professor W. F. Durand, Ithaca, N. Y.

4. Commutative Matrices. By Professor J. B. Shaw, Jacksonville, Ill.

5. On the Theory of the Quadratic Equation. By Professor A. Macfarlane, Lehigh University, South Bethlehem, Pa.

6. A New Principle in Solving Certain Linear Differential Equations that occur in Mathematical Physics. By Professor A. Macfarlane, Lehigh University, South Bethlehem, Pa.

7. Condition that the Line Common to $n-1$ Planes in an n-space may Pierce a Given Quadric Surface in the Same Space. By Dr. Virgil Snyder, Ithaca, N. Y.

8. The Psychology of the Personal Equation. By Professor T. H. Safford, Williamstown, Mass.

9. Compound Determinants. (Preliminary communication.) By Professor W. H. Metzler, Syracuse, N. Y.

10. Waters within the Earth and Laws of Rainflow. By W. S. Auchincloss, C.E., Philadelphia, Pa.

11. On the Secular Motion of the Earth's Magnetic Axis. By Dr. L. A. Bauer, University of Cincinnati, Cincinnati, O.

12. Simple expressions for the Diurnal range of the Magnetic Declination and of the Magnetic Inclination. By Dr. L. A. Bauer, University of Cincinnati, Cincinnati, Ohio.

13. The Theory of Perturbations and Lie's Theory of Contact-transformations. By Dr. E. O. Lovett, Princeton, N. J.

14. On Rational Right Triangles. No. I. By Dr. Artemas Martin, U. S. Coast Survey, Washington, D. C.

15. Some Results in Integration expressed by the Elliptic Integrals. By Professor James McMahon, Cornell University, Ithaca, N. Y.

16. Modification of the Eulerian Cycle due to Inequality of the Equatorial Moments of Inertia of the Earth. By Professor R. S. Woodward, Columbia University, New York.

17. Integration of the Equations of Rotation of a Non-rigid Mass for the case of Equal Principal Moments of Inertia. By Professor R. S. Woodward, Columbia University, New York.

18. General Theorems concerning a certain class of Functions deduced from the properties of the Newtonian Potential Function. By Dr. J. W. Glover, Ann Arbor, Mich.

19. The Importance of adopting Standard Systems of Notation and Coordinates in Mathematics and Physics. By Professor Frank H. Bigelow, U. S. Weather Bureau, Washington, D. D.

20. A Remarkable Complete Quadrilateral among the Pascal Lines of an Inscribed Six-point of a Conic. By Professor R. D. Bohannan, Columbus, Ohio.

21. Stereoscopic Views of Spherical Catenaries and Gyroscopic Curves. By Professor A. G. Greenhill, Royal Artillery College, Woolwich.

No. 1 pointed out that to every simple isomorphism of a group to itself corresponds some substitution of its operators; and that to all such isomorphisms corresponds a substitution group, which has been called the *group of isomorphisms* of the given group. A new and simple proof was given of the following theorem of Jordan's: When a regular group (R) of order n is transformed into itself by the largest possible group (L) of its own degree, the subgroup of L which includes all its substitutions that do not contain a given element is the group of isomorphisms of R. Other theorems were proved regarding those isomorphisms which may be derived from a given one by means of real transforming operators.

In No. 2, of which a brief abstract was read by the Secretary, the general group is that of Lie's Kugelgeometrie. All infinity in space is regarded as a single point and all planes as spheres through the point at infinity. The general group is tenfold. All transformations leaving a point invariant form a sevenfold sub-group. There is a sixfold subgroup which leaves a sphere invariant, and which is identical with the sixfold group of circular transformations on the Neumann sphere or in the complex plane.

No. 3 was read in joint session of Sections A and B. It showed how to compute successive values of a function defined by a differential equation without solving the equation analytically. Using Newton's notation for derivatives of y as to x, and indicating successive values of y by subscripts, the simple trapezoidal rule would serve to express approximately the difference between $\dot{y}_0$ and $\dot{y}_1$ in terms of $\ddot{y}_0$ and $\ddot{y}_1$, and similarly that between y_0 and y_1 in terms of $\dot{y}_0$ and $\dot{y}_1$. Hence $\dot{y}_1$ and y_1 may be ultimately expressed in terms of y_0, $\dot{y}_0$, $\ddot{y}_0$, $\ddot{y}_1$. Substituting their values in the differential eqnation, the latter will give the value of $\ddot{y}_1$. This value substituted back will give values of $\dot{y}_1$ and y_1, and hence the quantities for the point 1 are completely known when those for the zero point are assigned; and so on from point to point. The trapezoidal rule may be replaced by more accurate rules if desired. The method is applicable to equations of any order, and also to simultaneous equations.

No. 4 showed that if two matrices are commutative, *i. e.*, if $\varphi\psi = \psi\varphi$, then there is no latent system of any root of the one which lies in the extension composed of two or more latent regions of a root of the other unless it includes the entirety of these regions. The case when one of the matrices has equal roots was developed.

No. 5 presented a new theory of the quadratic equation $x^2 + 2bx + c = o$; stating that when b^2 is greater than c the roots may be either real numbers or 'hyperbolic complexes,' and that when b^2 is less than c the roots may be either 'circular complexes' or scalar numbers. In this view the square root of negative unity can in certain cases be interpreted as a scalar instead of a versor.

No. 6, which was read in joint session, showed by an example that when one term in the differential equation is the orthogonal projection of a plane motion, it is in some cases easier to pass to the auxiliary motion by means of 'planar algebra' than it is to proceed with the given equation directly. (See Trans. A. I. E. E., Vol. X., p. 186.)

In No. 7, of which the Secretary presented a brief abstract, there were $n - 1$ linear equations and one quadratic equation in the same n variables, and the problem was to determine when the simultaneous values of the variables that satisfy these equations are all real. The criterion obtained has applications for $n = 2, 3, 4$ or 5, whether the generating element be a point, a plane, a line or a sphere.

The object of No. 8, which was read by the Secretary, was to awaken an interest among astronomers and psychologists such as to induce them to pay more attention to the work of each other and thus improve their own methods where necessary.

In No. 9, which was presented in abstract by Dr. Shaw, the idea of obtaining the value of $A(m)$ the m^{th} compound of the determinant A, as a power of A by multiplying it by its adjugate $A(n-m)$, the $(n-m)^{\text{th}}$ compound of A, is extended to finding the value of certain minors of $A(m)$ in terms of A and its minors. By making use of a comprehensive notation the whole subject is unified, the laws of vanishing minors set forth, and such well-known theorems as Sylvester's and others are easily established.

In the absence of the author No. 10 was read by title, and a printed pamphlet was distributed in Sections A and B.

An abstract of No. 11 was presented by the Secretary. It stated that about 70 per cent. of the total magnetization of the earth can be referred to a homogeneous magnetization about a diameter inclined to the earth's rotation axis by an angular amount of about 12°. This axis has been termed by Gauss the earth's magnetic axis. It is an interesting question to determine the motion of this axis during the past two or three centuries, and Dr. Bauer's paper was an attempt to solve this problem as far as is possible with the data at present at command.

No. 12 was also read in brief abstract by the Secretary. It stated that as yet no formulæ had been found by which the diurnal range of the magnetic declination, for example, could be computed for various portions of the earth. The author has found the following simple formulæ to hold true within the fluctuations to which the quantities themselves are subject: Diurnal range of declination $= 2'.58 \sec^2 \varphi$; diurnal range of inclination $= 6'.1 \div 1 + 3 \sin^2 \varphi$; wherein φ, the magnetic latitude, is found from the equation $\tan \varphi = \frac{1}{2} \tan \mathrm{I}$, in which I is the magnetic dip. The first formula was discovered empirically, then under certain assumptions deduced theoretically. The second was then derived theoretically and was found to satisfy the data.

An abstract of No. 13 was read by the Secretary. It follows the theory of perturbations in the problems of mechanics in the order of its historical development from Lagrange to Lie with a view to the final presentation of the theory in its just position as one phase of Lie's theory of contact-transformations.

No. 14 was presented by the Secretary. It gave a bibliography of the Pythagorean proposition, followed by a general solution of the equation $x^2 + y^2 = z^2$, and concluded with an extensive numerical table of the sides of rational right-angled triangles.

No. 15 expressed in terms of the tabulated E and F functions a number of integrals, many of which had apparently never been completely worked out.

No. 16 showed how to express the effect of a small difference in the equatorial moments of inertia of the earth on the period of revolution of the instantaneous axis of rotation around the axis of figure. A remarkable value is obtained for the average angular velocity of that revolution, and a formula is deduced for the difference in the equatorial moments essential to explain the discrepancy between the observed and computed value of the Eulerian cycle.

No. 17 considered the case of no applied forces, or that in which there is conservation of moment of momentum. The problem is of practical interest in its application to the question of variation of latitudes on the earth. Several new theorems with respect to the motions of the mass were derived.

No. 18 obtained some general properties of a class of functions of which the spherical harmonics are special cases.

No. 19 described the present annoying state of the subject-matter of notation and coordinate systems, and advocated the adoption of certain standards. It is expected that this question will be further discussed at the Boston meeting with a view to obtaining a consensus on symbols and fundamental conventions.

No. 20 called attention to a certain quadrilateral whose properties throw new light on the theory of the Pascal lines of a hexagon inscribed in a conic.

In presenting No. 21 Professor Greenhill gave stereoscopic views of certain interesting curves in space and pointed out the bearing of some of them on certain parts of the theory of elliptic functions.

The section is also indebted to Professor Greenhill for interesting contributions to the discussions on many of the other papers.

The officers elected for the Boston meeting are Professors E. E. Barnard, of Yerkes Observatory, and Alexander Ziwet, of the University of Michigan.

JAMES MCMAHON.

SINGULAR STRESS-STRAIN RELATIONS OF INDIA RUBBER.

THE curious and unaccountable behavior of India rubber in thermodynamic transformation of energy under load has long been familiar; it is, perhaps, even more generally known that it exhibits a peculiar relation of elongation to load when approaching its limit of tenacity, but I am not aware that this later phenomenon has ever been exhibited by formal test or by graphic representation of the results of such tests. It is a matter of common observation that, when this substance is subjected to a pull of steadily increasing intensity, its resistance increases as does that of any elastic and ductile material; but that, at the end, instead of suddenly losing power of resistance, or even snapping without observable decrease of load, its resistance for a time rapidly and largely increases up to the point