

Emory McClintock in his article 'On the non-Euclidean geometry' in the Bulletin of the N. Y. (Amer.) Math. Soc., Vol. II., pp. 21-33, which reached the pitiful conclusion (p. 32): "The chief lesson to be obtained from all non-euclidian diversions (sic) is that the distinguishing mark of euclidian geometry is fixity of distance—measurement."

Mr. Russell, with equal deftness, puts in pillory the gross blunder made by Andrew W. Phillips and Irving Fisher, professors in Yale University, in the note on p. 23 of their Elements of Geometry, where they say: "Lobatchewsky in 1829 proved that we can never get rid of the parallel axiom without assuming the space in which we live to be very different from *what we know it to be through experience.*"

By experience, of course, we can never know or prove our space to be other than a non-Euclidean space with a comparatively large constant. How unexpected, then, the error of Professor H. Schubert, of Hamburg, in the *Monist*, Vol. VI., No. 2, p. 295, where he says:

"Let me recall the controversy which has been waged in this century regarding the eleventh axiom of Euclid, that only one line can be drawn through a point parallel to another straight line. The discussion merely touched the question whether the axiom was capable of demonstration solely by means of the other propositions, or whether it was not a special property, *apprehensible only by sense-experience*, of that space of three dimensions in which the organic world has been produced."

After 20 years' study of writers on the non-Euclidean geometry, the present reviewer cannot recall even one who was ever silly enough to think that the exact equality of the angle-sum of a rectilinear triangle to two right angles was apprehensible by sense-experience, or could ever be known through experience.

This new Yale geometry also makes the old *petitio principii* of defining a straight line as the *shortest* distance between two points. This our author treats in his third chapter, p. 167:

"We are accustomed to the definition of the straight line as the *shortest* distance between two points. * * * Unless we presuppose the straight line, we have no means of comparing the lengths of different curves and can, therefore, never discover the applicability of our definition."

In projective geometry any two points uniquely determine a line, *the straight*. But any two points and their straight are, in pure projective geometry, utterly indistinguishable from any other point-pair and their straight. It is of the essence of metric geometry that two points shall completely determine a spatial quantity, *the sect*. If our author had used for this fundamental spatial magnitude this name, introduced in 1881, his exposition would have gained wonderfully in clearness.

Both the accepted popular and the accepted mathematical definition of 'distance' make it always a number, as, *e. g.*, the Cayley-Klein definition: "The distance between two points is equal to a constant times the logarithm of the cross ratio in which the line joining the two points is divided by the fundamental quadric."

It is the misfortune of our author to use the already overworked and often misused word 'distance' as a confounding and confusing designation for a sect itself and also the measures of that sect, whether by superposition, ordinary ratio, indeterminate as depending on the choice of a unit, or projective metrics, indeterminate as depending on the fixing of the two points to be taken as constant in the varying cross ratios.

This whole book might be cited as an overwhelming vindication of the only American treatise on Projective Geometry against the attack on it made by a critic in *SCIENCE*, because, forsooth, it was founded and developed as *pure* projective geometry, without any quantitative ideas whatever.

Into the fourth and last chapter, 'Philosophical Consequences,' we will not here go. Suffice it to say that Projective and Metric Geometry, though eternally separate in essence, and each unable ever to absorb the other, are happily wedded, and expand joyfully ever after.

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Sight: An Exposition of the Principles of Monocular and Binocular Vision. JOSEPH LE CONTE. New York, D. Appleton & Co. 1897. Second edition, revised and enlarged. Pp. xvi + 318. \$1.50.

A revised and enlarged edition of Professor

Le Conte's book on 'Sight' is extremely welcome. It remains true now, as it was in 1880, when the first edition was published, that there is no other book in the English language covering this field. During the past year two excellent American contributions have, indeed, been made to the subject—Professor Bowditch's survey in the 'American Text-book of Physiology' and the various articles in the 'Systems of Diseases of the Eye,' by Norris and Oliver. But Professor Le Conte's book is the only treatise covering the subject of vision in English, and, in fact, in any language, with the exception of the more technical works of von Helmholtz and Aubert. While it is surprising, and not altogether creditable to the 'new psychology,' that we have only one book on the subject, it is fortunate that it is so excellent.

Professor Le Conte devotes two-thirds of his book to binocular vision. This is excessive for a text-book, but it is justified by the interest of the subject and by the important original contributions of the author. Following an introduction on the senses, the part on monocular vision includes sections on the general structure of the eye, the formation of the image, the perfect eye, defects of the eye, structure of the retina, space perception and color perception. In this part should, perhaps, be included the final chapter on the evolution of the eye, added in this edition, which, however, appears as the 7th chapter of the third part, entitled in the table of contents, 'Some abstruser points, especially in binocular vision,' and in the text, 'On some disputed points in binocular vision.'

The author has added some new matter to this first part, especially on color blindness and color perception, including a full statement of the Donders-Franklin theory. I regret the omission or brief treatment of subjects so important as the intensity of sensations, their time relations, the field of vision, illusions, the combination of colors,* etc. It is in any case evident that everything cannot be included in 100 pages, and it is a wonder that Professor Le Conte has been able to give with such clearness so much. There are several points on which I

*Fortunately some of these topics are fully treated in another admirable book in this same series, Professor Rood's 'Modern Chromatics.'

should differ with the author. For example, the explanation of 'upright vision,' which has been discussed in this JOURNAL. Professor Stratton, a colleague of Professor Le Conte's at the University of California, has since made the ingenious experiment of wearing, for several days, glasses that inverted the normal retinal image and shows how quickly adjustment is made. But it is still easier to stand on one's head and notice that the landscape is not seen inverted.

The two parts on binocular vision contain clear and concise accounts of experiments largely devised by Professor Le Conte. Making these experiments would, as the author says, be for any one an admirable culture in scientific method. Many of the experiments are valuable contributions to science, but the details are somewhat complex and cannot be made clear in a review. The reader must turn to Professor Le Conte's book, with its many new and ingenious illustrations, in order to appreciate the importance of a study of binocular vision and the great value of the author's contributions to the subject. I may note that I have recently been told by an eminent oculist that the conflicting results in the case of Listing's law found by von Helmholtz were due to unrecognized astigmatism in his eyes.

There are some points where Professor Le Conte's statements do not seem to me quite accurate. For example, he says, "We always see things double, except under certain conditions." This is scarcely correct psychology; we must learn by practice to see things double, and then usually see them double only while the experiment lasts. Professor Le Conte says, "I believe that the existence of the central spot is necessary to fixed, *thoughtful attention*, and this, again, in its turn, is necessary for the development of the higher faculties of the mind." But may not the mental faculties of those born blind be developed? There are further many subjects, such as the horopter, that I cannot regard as finally solved by Professor Le Conte, but his researches have accomplished much toward their solution and should be accepted as the basis of future work.

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