phosphorus, silicon trichlorid and many other substances. It is a question whether this is justifiable. It seems irrational to put rhombic and monoclinic sulphur in one class and the two modifications of phosphorus in another; but it is certainly interesting, and the application of his theory to the point of reaction, to reaction velocities and to explosions deserves careful attention. It will interest many to note that Duhem's view of a mixture of hydrogen and oxygen as being actually in equilibrium at low temperatures is not reconcilable with the Ostwald-Nernst idea that it is not a case of equilibrium at all, but rather of immeasurably low reaction velocity. A fairly strong argument can be made out for either view, and the scientific world owes thanks to Duhem for making the question a live one.

WILDER D. BANCROFT.

Trigonometry for Beginners. REV. J. B. LOCK, M. A. Revised and enlarged by JOHN A. MILLER, A. M., Indiana University. New York, The Macmillan Company. 1896. 200 pp. Price, $1.10.

Trigonometry, of all elementary branches of mathematics, might easily substantiate its right to be considered the most congenial and popular subject that necessarily claims the attention of engineers and practical men, otherwise but little inclined to sympathize with the purest in the science. Led on by its numerous and interesting applications, many a student, without being aware of it, has taken his first step in the theory of functions, acknowledged the results to be fascinating as well as eminently practical, and gone his way to rail at the higher theory, quite unconscious of the spectacle he thereby makes of himself. The natural result of this favoritism has been a steady improvement in the quality of the text-books produced in trigonometry until such works as that of Chauvenet and, to mention a less ambitious book, that of Wells, challenge competition successfully for a series of years.

With a new edition of Lock's Trigonometry, The Macmillan Company enters the field, and with its usual business sagacity have secured its revision by an American. Professor Miller has certainly earned the right to have his name on the cover, indeed, because of additions and improvements far less necessary and fundamental many a man would have called the volume his own. As claimed, the new edition corrects the fundamental weakness of its predecessor by carefully emphasizing the necessity for proofs for all relations, especially for the addition formula, that are rigidly correct for all values of the angles involved. To this end, as is necessary, we find clear demonstrations of such relations as, for example, $\sin(90° + A) = \cos A$ for all values of A. This enables the author in §79, while attempting to generalize the addition formula, to write.

$$\sin[90°+(A'+B)] = \cos(A'+B), \quad 0 < A' < 90°, \; 0 < B < 90°.$$

He then remarks since A' and B are now both less than 90° we may write

$$\cos(A'+B) = \cos A' \cos B - \sin A' \sin B. \quad (§\,76).$$

There is certainly a flaw in the general accuracy of the argument up to this, the pivotal point of trigonometric analysis, for § 76 does not completely justify this expansion, since the demonstration to which reference is made depends upon a figure representing both A' and B, as in the first quadrant, and their sum also as in the first quadrant, whereas, in the case before us, we have no means of knowing whether $A'+B$ is greater or less than ninety degrees. In fact, before analytic demonstrations for particular cases can be accurately defended, geometric demonstrations must be given for $0° < A < 90°$, $0° < B < 90°$ for the four possible cases under this head, namely,

$$(A+B) < 90°, \quad (A+B) > 90°, \quad (A-B) > 0°, \quad (A-B) < 0°.$$

On the other hand, it must be admitted that the author, in paragraph seventy-eight, calls attention to the fact that a similar construction will apply to all possible cases, and even gives a third example, different from either, of those that are necessary. The criticism is, however, that, since up to this point unusual effort has been made to demonstrate the addition theorems, this is certainly not a good place to leave necessary steps to the student.

Like most English text-books, the present volume is very rich in illustrative examples carefully and admirably adapted to meet the requirements of both teacher and student. Indeed, this feature of the book will be its strongest recomendation to many practical teachers. In revising the work Professor Miller has placed Chapter VII., which, in the old edition, followed Chapters VIII. and IX. in its logical position, and by thus developing first the Cartesian system of coordinates has succeeded in making the results of the later chapters general. To this portion of the text other tables might be added to some advantage, for example, one tabulating the values of functions of angles that are multiples of five, and one giving an expression for each function in terms of the others. While of undoubted worth as a means of reference, such tables, it might be argued, are of doubtful value from a pedagogic standpoint. At least they should be required to be established by every student of trigonometry, and it would have been well to have required them among the examples. The book is also unique in respect to the absence of the time-honored figures illustrating the positions and comparative lengths of the functions other than the sine and cosine in the different quadrants of a circle whose radius has been assumed as the unit. Here, again, the omission may be defended on the ground that nine students out of ten, having these figures for the special case in mind, will carry to the grave the impression that the functions are lines instead of ratios. Nevertheless, they are of value in mechanical drawing, and especially as affording a ready means of prompting the memory in the thousand and one simple relations they illustrate, and by constant emphasizing of the ratio definitions on the part of the teacher they can be used without confusion of ideas.

The added chapter on inverse functions, and the one that has been much improved on the solution of trigonometric equations, are valuable and essential parts of the new edition, the importance of which will not be underestimated by the advanced student of mathematics. Finally the analytic portion of the plane trigonometry is completed by the establishment of the so-called tangent formula, $a - b : a + b = \tan \frac{1}{2} (A - B) : \tan \frac{1}{2} A + B)$, by a direct development from the rigidly demonstrated addition theorems. Many of the old treatises prove this formula, probably because of its importance, geometrically, and lose thereby in generality. The geometric proof is of exceeding interest, however, and the present demonstration would be emphasized and the book gain in pedagogic strength were it given in a foot-note with a corresponding valuable reference to its limitations.

In outline the design of the book is to discuss, in the first thirteen chapters, the general theory of the trigonometric functions; then, in chapter fourteen, to give a short review of the theory of logarithms followed by a discussion of the solution of the triangle, and, finally, by two short chapters giving applications to engineering and geometrical problems. This is the time honored arrangement, certainly in the hands of a good teacher sufficiently effective. As our author says, "the discussion of logarithms belongs properly to algebra," and as a rule the student has met with them during the preceding term's work, but has, nevertheless, far from mastered their application and still less their theory. Why not then begin at once with a review of the theory of logarithms and insure a thorough mastery of their application by constant practice from the beginning? While the student is learning about angular measurement and the trigonometric ratios the class can be exercised in the evolution of complicated numerical expressions and in the solution of logarithmic equations, and, as soon as the functions are developed, a large number of the applications to right triangles reserved for the last chapters may be discussed immediately, affording new material for logarithmic work, thus arousing at the start the keen interest of the practical mind.

The last thought has been acted upon by the author. Pages 10, 11, 26 and 27 are filled with practical problems of a most interesting character, all of which, however, are intended for solution without the aid of logarithms. Finally Professor Miller has added two short chapters, in which he develops the theory of the solution of spherical triangles. Here, as in the plane trigonometry, the author is fully alive to the limitations of geometric proof and carefully renders the fundamental formula $\cos a = \cos b \cos c + \sin b \sin c \cos A$ perfectly general, and then by basing all further deductions on this one determines rigorously all necessary relations.

To some teachers it may seem that on this feature of the work too much emphasis is laid, and that too much time, and, possibly, clearness and definiteness are sacrificed to this end. On the other hand, the careful student will maintain that this one feature should recommend the book most highly, for long after the pupil has forgotten what a cosine is he will have retained the habit of mind which distinguishes clearly between the general and the particular, and will be less apt to make that most frequent of all mistakes in logic, that of arguing to the former having proved the latter. Certainly, if the present criticism is at all just, enough has been said to put in evidence the fact that Professor Miller has succeeded, at important points, in improving Lock's trigonometry, and his work will assuredly be found acceptable to many educators.

So far as the publishers are concerned, the typographic results are excellent; different types have been employed with useful discrimination, and a cheerful appearance is given through liberal use of space. The book, however, is, for practical use, large. It contains two hundred pages, is heavier than Chauvenet, while containing only one-fourth as much subject-matter, and three times as heavy as Wells, while, save for some sixty pages devoted to logarithmic tables of questionable value, it does not contain any more. The student of Lock's text book in its present dress will certainly receive the impression that trigonometry is a very large subject indeed, and the probabilities are that he will never entirely recover from this, his first impression.

J. B. CHITTENDEN.

COLUMBIA UNIVERSITY.

SCIENTIFIC JOURNALS.

THE ASTROPHYSICAL JOURNAL, MARCH.

Résumé of Solar Observations Made at the Royal Observatory of the Roman College During the Second Half of 1896: By P. TACCHINI. A general summary of solar observations, giving the distribution in latitude of spots, faculæ and prominences during the period indicated.

Oxygen in the Sun: By ARTHUR SCHUSTER. In a short note Professor Schuster calls attention to the close agreement in wave-length between two of the triplets of the 'compound line spectrum' of the oxygen and two of Young's chromosphere lines. In view of the recent opening of the question of the existence of oxygen in the solar atmosphere, Professor Schuster suggests that an accurate determination of the chromorpheric lines in question be made.

The Yerkes Observatory of the University of Chicago—I. Selection of the Site: By GEORGE E. HALE. The writer gives a review of the considerations that led to the selection of the site of the Yerkes Observatory. A general discussion of the points to be considered in the selection of an observatory site is followed by a discussion of the conditions to be met in the case in hand.

Preliminary Table of Solar Spectrum Wave-lengths: By HENRY A. ROWLAND.

On the Occurrence of Vanadium in Scandinavian Rutile: By B. HASSELBERG. The paper describes the detection of the heretofore unsuspected existence of Vanadium in Norwegian and Swedish Rutile. The research was entirely spectroscopic.

A New Formula for the Wave-lengths of Spectral Lines: By J. J. BALMER. The author discusses a generalization of his formula for the hydrogen spectrum. This formula, which is generally known under the name of 'Balmer's law,' is $\lambda_n = 3645.6 \frac{n^2}{n^2 - 4}$. By introducing a new constant c, we have $\lambda_n = a \frac{(n+c)^2}{(n+c)^2 - b}$, which is found to satisfy the series of lines hitherto investigated by Kayser and Runge by the aid of the formula $\frac{1}{\lambda_n} = A - \frac{B}{n^2} - \frac{C}{n^4}$. Balmer's new law is similar to that due to Rydberg, except that the latter considered the value of $\frac{b}{a}$ to be constant for all elements. Several geometrical constructions based upon the formulæ are given.

Minor Contributions and Notes, Reviews of Recent Astrophysical Literature.

Bibliography of Recent Astrophysical Literature.