Nowhere else in the country are men as free to delve into the unsolved mysteries and work out the practical application of discoveries as here. If, as President Jordan asserts,* and as, I think, no one will deny, "The National University should not be an institution of general education, with its rules and regulations, college classes, good-fellowship, and football team; it should be the place for the training of investigators and men of action," can a more favorable plan be formulated, for at once realizing the popular idea of a truly National University and meeting the need for a reorganization and centralization of the National Scientific Departments, than to reorganize the latter as the former, charged with the twofold duty of prosecuting all needful investigation and of training all competent students desirous of devoting their lives to a like purpose? To this scientific foundation, history, literature and the arts would be readily added, without wasteful duplication.

WILLIAM TRELEASE.

SCIENTIFIC LITERATURE.

Die Spiele der Thiere. By KARL GROOS, Professor of Philosophy in the University of Giessen. Jena, Gustav Fischer. 1896. Pp. xvi +359.

In this volume Professor Groos makes a contribution to three distinct but cognate departments of enquiry: philosophical biology, animal psychology, and the genetic study of art. Those who have followed the beginnings of enquiry into the nature and functions of play in the animal world and in children will see at once how much light is to be expected from a thorough-going examination of all the facts and observations recorded in the literature of animal life. This sort of examination Professor Groos makes with great care and thoroughness, and the result is a book which, in my opinion, is destined to have wide influence in all these departments of enquiry.

I cannot take space for a detailed report of Professor Groos' positions. It may be well, therefore, before speaking of certain conclusions which are to me of especial interest, to give a résumé of the contents of the book by *l. c. 603.

chapters. Chapter I. is an examination of Mr. Spencer's 'surplus energy' theory of Play; the result of which is, it seems, to put this theory permanently out of court. The author's main contention is that play, so far from being 'byplay,' if I may so speak, is a matter of serious business to the creature. Play is a veritable instinct, true to the canons of instinctive action. This view is expanded in Chapter II., where we find a fine treatment in detail of such interesting topics as imitation in its relation to play, the inheritance of acquired characters apropos of the rise of instincts, the place and function of intelligence in the origin of these primary animal activities. This chapter, dealing with the biological theory of play, is correlated with Chapter V., later on in the book, in which the 'Psychology of Animal Play' is treated. gether they furnish the philosophical and theoretical basis of the book, as the chapters in between furnish the detailed data of fact. return to the biological matter below. Chapters III. and IV. go into the actual 'Plays of Animals' with a wealth of detail, richness of literary information and soundness of critical interpretation, which are most heartily to be commended. Indeed, the fact that the first book on this subject is, at the same time, one of such unusual value, both as science and as theory. should be a matter of congratulation to workers in biology and in psychology. The collected cases, the classification of animal plays, as well as the setting of interpretation in which Professor Groos has placed them—all are likely to remain, I think, as a piece of pioneer work of excellent quality in a new but most important field of enquiry.

As to the plays which animals indulge in, Professor Groos classifies them as follows: 'Experimenting,' 'Plays of Movement,' 'Play-Hunting' ('with real living booty,' 'with play living booty,' 'with inanimate play booty'), 'Play-fighting' ('teasing, scuffling among young animals,' 'play-fighting among adult animals'), so-called 'Building Art,' 'Nursing' plays, Imitation' plays, 'Curiosity,' 'Pairing' plays, 'Courting by Means of Play of Movements,' 'Courting by the Exhibition of Colors and Forms,' 'Courting by Noises and Tones,' 'Coquetry on the part of the Female.'

With this general and inadequate notice of the divisions and scope of the book, I may throw together in a few sentences the main theoretical positions to which the author's study brings him. He holds play to be an instinct developed by natural selection (for he does not accept the inheritance of acquired characters), and to be on a level exactly with the other instincts which are developed for their utility. It is very near, in its origin and function, to the instinct of imitation, but yet they are distinct (a word more below on the relation between play and imitation). Its utility is, in the main, two-fold: First, it enables the young animal to exercise himself beforehand in the strenuous and necessary functions of its life and so to be ready for their onset; and second, it enables the animal by a general instinct to do many things in a playful way, and so to learn for itself much that would otherwise have to be inherited in the form of special instincts; this puts a premium on intelligence, which thus comes to replace instinct (65f.). Either of these utilities, Professor Groos thinks, would insure and justify the play instinct; so important are they that he suggests that the real meaning of infancy is that there may be time for play.*

It is especially in connection with this latter function of play that the instinct to imitate comes in to aid it. Imitation is a real instinct, but it is not always playful; play is a real instinct, but it is not always imitative. Professor Groos does not suggest, I think, closer relations between these two instincts. There is likely, however, to be a great deal of imitation in play, since the occasion on which a particular play instinct develops is often that which also develops the imitative tendency as well, i. e., the actual sight or hearing of the acts and sounds of other animals. Moreover, the acquisition of a muscular or vocal action through imitation makes it possible to repeat the same action afterwards in play.

It is only a step, therefore, to find that imitation, as an instinct, has to have ascribed to it,

*"Die Thiere spielen nicht weil sie jung sind, sondern sie haben eine Jugend, weil sie spielen müssen" (68). Other capital utilities which might be added are (1) the exercise of the intelligence itself and (2) direct social utility as such.

in a measure, the same race utility as play—that of going before the intelligence and preparing the way for it, by rendering a great number of specialized instincts unnecessary. It is interesting to contrast this view with that which the present writer has recently developed in these pages (Science, March 20, 1896), i. e., the view that imitation supplements inadequate congenital variations in the direction of an instinct and so, by keeping the creature alive, sets the trend of further variations in the same direction until the instinct is fully organized and congenital. If both these two views be true, as there seems reason to believe, then imitation holds a remarkable position in relation to intelligence and instinct. It stands midway between them and aids them both. some functions it keeps the performance going, and so allows of its perfection as an instinct; in others it puts a stress on intelligence, and so allows the instinct to fall away if it have no independent utility in addition to that served by intelligence.* In other words, it is through imitation that instincts both arise and decaythat is, some instincts are furthered and some suppressed, by imitation. And all this is accomplished with no appeal to the inheritance of acquired characters, Professor Groos agreeing with Weismann that the operation of natural selection as generally recognized is sufficient.

The difficulty which I see to this conception of play as a pure instinct is that which is sometimes urged also against considering imitation an instinct, *i. e.*, that it has no definite motor coordinations, but has all the variety which the different play forms show. If the definite congenital plays are considered each for itself, then we have a great many instincts, instead of a general play instinct. But that will not do, for it is one of Professor Groos' main contentions, in the chapter on the psychology of animal plays, that they have a common general char-

* In a private communication Professor Groos suggests to me that the two views might be held to supplement each other. The case is very much like that of early intelligence, in the form of association; where it fully accomplishes the utility also subserved by an instinct, it tends to supersede the instinct; otherwise, it tends to the development of the instinct (Groos, p. 64).

acter which distinguishes them from other specialized instinctive actions. They are distinguished as play actions, not simply as actions. This difficulty really touches the kernel of the matter, and serves to raise the question of the relation of imitation to play; for imitation presents exactly the same conditions—a general instinct to imitate, which is not exhausted in the particular actions which are performed by the imitation. I shall remark on the solution of it below, in speaking of Professor Groos' psychology of play. It will be interesting to see how he treats this problem in his promised work on the Spiele der Menschen; for the imitative element is very marked in children's plays.

Other points of great interest in this biological part are the great emphasis which Groos finds it necessary to put on 'tradition,' instruction. imitation, etc., in young animals, even in enabling them to come into possession of their natural instincts; in this the book tends in the same direction as the new volume of Professor C. Lloyd Morgan. Again, there is a remarkably acute discussion of Darwin's Sexual Selection. which the author finally accepts in a modified form by saying that the female's selection is not necessarily conscious, but that she has an inherited susceptibility to certain stimulating colors, movements, etc., in the male. It is not so much intelligence on her part as increased irritability in the presence of certain visual and other stimulations. *Over against the charms of the male he sets the reserve or reluctance (Sprödigkeit) of the female, which has to be overcome and which is an important check and regulator at the mating time. Again, the imperfect character of most instincts is emphasized, and the interaction with imitation and intelligence. He finds a basis for the inverse ratio between intelligence and instinct is an animal's equipment on natural selection principles, i. e., the more intelligence develops the less does natural selection bear on special instincts, and so they become broken up.

*'Sexual' is thus referred back to 'natural' selection (p. 274), although the direct results of such preferential mating would still seem to give very 'determinate' variations for natural selection to work upon (Cf. SCIENCE, Nov. 23, 1896, p. 726).

Finally, I should like to suggest that a possible category of 'Social Plays' might be added to Groos' classification-plays in which the utility of the play instinct seems to have reference to social life as such. Possibly in such a category it might be possible to place certain of the animals' performances, which seem a little strained under the other heads—for example. those performances in which the social function of communication is exercised early in life. A good deal might be said also in question of the author's treatment of 'Curiosity' (Neugier). He makes curiosity a function of the attention, and finds the restless activity of the attention a play function, which brings the animal into possession of the details of knowledge before they are pressed in upon him by harsh experi-My criticism would be that attention does not fulfil the requirements of the author's psychological theory of play, as indicated below.

Turning now to the interesting question of the psychological theory, we find it developed, as it would have to be, in a much more theoretical way. The play consciousness is fundamentally a form of 'conscious self-illusion' (311 ff)—bewusste Selbsttäuschung. It is just the difference between play activity and strenuous activity that the animal knows, in the former case, that the situation is not real, and still allows it to pass, submitting to a pleasant sense of illusion. It is only fair to say, however, that Herr Groos admits that in certain definite instinctive forms of play this criterion does not hold; it would be difficult to assume any consciousness of self-illusion in the fixed courting and pairing plays of birds, for example. The same is seen in the very intense reality which a child's game takes on sometimes for an hour at a time. Indeed, the author distinguishes four stages in the transition from instincts in which the conscious illusion is absent, to the forms of play to which we can apply the phrase 'Play activity' in its true sense, i. e., that of Scheinthätigkeit (298 f). The only way to reconcile these positions that I see is to hold that there are two different kinds of play: that which is not psychological at all, i. e., does not show the psychological criterion at all, and that which is psychological as Scheinthätigkeit. Herr Groos does

distinguish between 'objective' and 'subjective' Scheinthätigkeit (312). The biological criterion of definite instinctive character might be invoked in the former class, and the psychological criterion in the other. And we would then have a situation which is exemplified in many other functions of animal and human life—functions which are both biological and instinctive, and also psychological and intelligent, as sympathy, fear, bashfulness. Then, of course, the further question comes up as to which of these forms is primary, again the old question as to whether intelligence arose out of reflexes or the reverse.

I think some light falls on this time-honored question from the statement of it in connection with this new question of play, and especially when we remember Herr Groos' theory of the function of imitation and the extension of his view suggested above. If imitation stands midway between instinct and intelligence, both furthering the growth of instinct, and also leading to its decay in the presence of intelligence, then we might hold something like this: In proportion as an action loses its consciously imitative and volitional character, to that degree it loses its Schein character, and becomes real in consciousness and instinctive in performance (and this applies to the cases in which imitation has itself become habitual and instinctive); and on the contrary, in proportion as an instinctive action is modified and adapted through imitation and intelligence, to that degree it becomes capable of assuming the Schein character and is indulged in as conscious play. I cannot enlarge upon this here, but it seems to square with a good many of the facts, both those which Groos cites as showing that imitation opens the way for the decay of instinct with the growth of intelligence, and those which Morgan and I have cited as showing that imitation keeps congenital variations alive and so allows them to accumulate into instincts. And I think it so far confirms the view that imitation is a sort of meeting point of race habit, represented by instinct, and race accommodation, represented by intelligence-just the double function which imitation serves also in the development of the individual (Cf. My volume on Mental Development, in loc.).

Going into the analysis of the play psychosis, Herr Groos finds several sources of pleasure to the animal in it (203 ff): pleasure of satisfying an instinct, pleasure of movement and energetic action, but, most of all, 'pleasure in being a cause.' This last, together with the 'pleasure in experimenting,' which characterizes many play activities, is urged with great insistence. Even the imitative function is said to produce the joy of 'victory over obstacles.' Yet, here again, the author is compelled to draw the distinction between the play which is psychological enough to have a represented object, and the instinctive sort in which the pleasure is only that of the instinct's own performance. The pleasure of overcoming friction of movement, also, is very doubtful, since in any but the instinctive games which are cited (Chapter I.) to prove that the animal is not using up surplus energy (seeing that he plays after he is tired)—in other games we stop playing when the friction and inertia of the muscles become conscious as fatigue. Much more, however, is to be said for the pleasure of rivalry, or of overcoming an opponent, in the higher types of play; but Herr Groos scarcely does this justice.

Returning to the element of illusion in play, we find two ingredients in it (313 ff): a division of consciousness (Spaltung des Bewusstseins), i. e., a division between the activity treated as real and the sense that it is unreal. There is considerable oscillation between these two poles. This ability to treat representations as realities is, according to Herr Groos, the essential of all imagination. In play it is akin to the division of consciousness found in certain pathological cases of double personality. It is a sort of hypnotization by the stream of representations, but with the sense that it is all an illusion and may be pierced through by a return to reality at any moment. This seems to me a true and valuable characterization of the play consciousness (it is taken from K. Lange), but Professor Groos' extension of it to all imagination does not seem to hold. In his criticisms of others (as the present writer) he fails to honor the current distinction between 'fancy' and 'constructive imagination.' In fancy we do yield ourselves up to a play of images, but in the imagination of scientific thinking or of artistic creation are not both the goal and the process strenuous enough? This, indeed, leads Professor Groos to a view of art which allies it closely with the play function, but to that I return below.

The second element in the play or 'Schein' consciousness is the feeling of freedom (Freiheitsgefühl) (331f). In play there is a sense, so to speak, of 'don't-have-to,' which is contrasted both with the necessity of sense and with the imperative of thought and conscience. This idea seems to be part of Schiller's theory of play. So Groos thinks the general feeling of freedom holds in consciousness only while there is a play of motives to which the agent may put an end at any moment-a sense of 'don't-have-to' in the life of choice. This sense of freedom keeps the Schein consciousness pure and prevents our confusing the play content with the possible real contents of life. This is very interesting and suggestive. The sense of freedom is certainly prominent in play. Whether it should be identified with the sense of control which has been used by some writers as a criterion (both in a negative and in a positive sense) of the belief in realities already experienced, or again with the freedom with which choice is pregnant, is more questionable. Without caring to make a criticism of Professor Groos' position, I may yet point out the distinction already made above between the two sorts of imagination, one of which has the 'don't-have-to' feeling and the other of which does not. So also in our choices there are those which are free with a 'don't-have-to' freedom, but there are choices—and these are the momentous ones, the ones to which freedom that men value attaches—which are strenuous and real in the extreme. Indeed, it seems paradoxical to liken the moral life, with its sense of freedom, to a 'game of play,' and to allow the hard-pressed sailor on the ethical sea to rest on his oars behind a screen of Schein and plead, 'I shan't play.' Seriously, this is something like the result, and it comes out again in the author's extremely interesting sections on art, of which I may speak in conclusion.

Those who have read Professor Groos' former

stimulating book, Einleitung in die Æsthetik, will anticipate the connection which he finds between play and art. The art consciousness is a consciousness of Schein; it is also a play consciousness, inasmuch as it is the work of imagination—both the creative and the appreciative art consciousness-and the meaning of imagination is just that it takes Schein for The 'self-conscious' illusion of the reality. play consciousness is felt in extreme form in the theatre, and the pleasure of it is felt even when we play with painful situations, as in tragedy. In art the desire to make an impression on others shows the 'pleasure of being cause.' This intent to work on others is a necessary ingredient in the art impulse (312f). Groos differs from K. Lange, who holds a similar view of the necessary division of consciousness between reality and Schein in the æsthetic psychosis, in that Lange thinks there must be a continual oscillation between the two poles of the divided consciousness, while Groos thinks there is rather a settling down in the state of illusion (as in an artist's preoccupation with his creations, a novelist with his characters, and a child with her doll (323). In art the other great motive of play, 'experimenting,' is also prominent, and is even more fundamental from a genetic point of view; of that a word below.

Here, again, the question left in my mind is this: whether the play motive is really the same as the art motive. Do we not really distinguish between the drama (to take the case most favorable to the theory) as amusement and the drama as art. And does the dramatist who is really an artist write to bring on selfillusion in the spectator by presenting to him a Schein scene. Possibly, art theorists would divide here; the realists taking more stock in Schein, since realistic art is more nearly exhausted by imitation. This sort of illusion undoubtedly gives pleasure, and it is undoubtedly part of art pleasure. Yet there does seem to be, in a work of fine art, a strenuous outreach toward truth, which is additional to the instrument of appearance used by the artistboth in the production and also in the enjoyment. It may be that we should distinguish between truth which comes to us didactically and truth which comes artistically, and make

the method of the latter, and that alone, the source of æsthetic impression. In any case the theory of Groos, which has its roots in the views of Lange and v. Hartmann, is extremely interesting and valuable, especially as contrasted with the recent psychological theory of Mr. H. R. Marshall. In the present theory, the 'self-exhibition' of which Mr. Marshall makes so much, enters as the need of impressing others with the play illusion. As to the hedonic element and its ground, however, the two theories are in sharp contrast, and that of Groos seems to me, on the whole, more adequate. In the wealth of literary reference in his book Mr. Marshall pays singularly little attention to the authors from whom Groos draws, and none to the earlier work of Professor Groos himself, but treats the play theory only in the form of Mr. Spencer's surplus energy construction. As to Groos' theory musical art would present difficulties and so would lower sensuous æsthetic effects generally.

Genetically art rests upon play, according to Herr Groos, in that the three great motives of art production, 'Self-exhibition' (Selbstdarstellung), 'Imitation,' and 'Decoration' (Ausschmückung), are found in the three great classes of animal plays, respectively, 'Courting,' 'Imitation,' and Building Art' (Baukünste, seen in birds' nest-building, etc.). On the strength of this, Groos finds both æsthetic appreciation and impulse in the animal, and all rests upon the original 'experimenting' impulse. Of this, however, Professor Groos does not give a satisfactory account. Experimenting is a necessary part of effective learning by 'imitation,' I think, and the use made of it in the selection of movements may be its original use.

On the whole, Professor Groos' book is both a pioneer work and one of great permanent value; it should be translated into English. It contains a good index and a full list of the literary sources.

J. MARK BALDWIN.

PRINCETON.

A Primer of the History of Mathematics. By W.
W. ROUSE BALL. London, The Macmillan Co. 1895. Pp. 148, 16mo. Price, 65 cents.
A History of Elementary Mathematics, with hints on

methods of teaching. By Florian Cajori. New York, The Macmillan Co. 1896. Pp. viii+304, 12mo. Price, \$1.50.

The object of the 'Primer,' as well set forth in its introduction, is "to give a popular account of the history of mathematics, including therein some notice of the lives and surroundings of those to whom its development is mainly due, as well as their discoveries. Such a sketch. written in non-technical language and confined to less than 140 pages, can contain nothing beyond a bare outline of the subject, and, of course, is not intended for those to whom it is familiar." It consists of the author's larger work* reduced in size by the omission of all detailed and highly technical matter. In a few places the pruning process has been carried too For example, on p. 13 we are told that "after the execution of Socrates, in 399 B. C., Plato spent some years in travel * * * " but we are given no clue to the relationship of Socrates to Plato. However, the few instances of this kind which occur do not appreciably detract from the clear, well ordered and interesting style which the 'Primer' enjoys in common with its source.

The book affords to students in our high schools and colleges a means of gaining, with a small expenditure of time, a sufficiently complete history of the mathematical subjects they are studying, to give them a much greater appreciation of and interest for such subjects.

As its title indicates, Professor Cajori's book does not cover the entire field of mathematics; he restricts it to arithmetic, algebra, geometry and trigonometry, as presented in undergraduate instruction, with a short account of the history of non-Euclidean geometry. The arrangement of the material is first under the headings: 'Antiquity,' 'Middle Ages,' 'Modern Times;' under each of these are the subdivisions: 'arithmetic,' 'algebra,' 'geometry' and 'trigonometry.' For a work of its size it contains a great deal of information, and nearly every statement is supported by a reference either to original sources or to other treatises upon mathematical history. The chapters upon arithmetic are par-

*A short account of the History of Mathematics. London, the Macmillan Co. 2d edition. 1893. Pp. xxiv+520, 16mo.