

The author in other publications has claimed that this must have been the law, and explained the phenomena as parallel with that which takes place at the beginning of every series arising in the Paleozoic and Mesozoic, and also according to Minot's law of growth and other phenomena of the earlier stages in the ontogeny of every animal.

All inferences with reference to the length of time that life has existed upon the earth are consequently defective, since, as far as known to the author, they do not take into consideration the differing rates of evolution at different times in the history of organisms.

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THE BLACKBOARD TREATMENT OF PHYSICAL VECTORS.

THE tedious part of geometrical reading is the need of searching for the letters which designate the lines. Frequently this is the chief difficulty in the demonstration. In a measure, the same is also true when a geometrical proof is to be written down, particularly where special vector symbols (*e. g.*, the $[AB]$ of Möbius) are employed. There is, perhaps, no remedy for this in printed work; but in the classroom, with a blackboard available, coplanar vectors may be drawn in great variety at pleasure. I will therefore describe the following method of elementary treatment which, though it contains no essential novelty, is new, I think, from a pedagogic point of view, and for this reason not without value.

Of the four specifications which characterize a vector—position, quantity, direction, sign—the first three usually come within the range of indulgence of the average student; but with the sign he will have nothing to do. Thus it becomes necessary to the author to be simply a mode of expressing a general fact, or series of facts, that occur everywhere, and in all series more or less through the action of the law of tachygenesis.

to especially emphasize the latter, and this is done by putting an arrowhead on the proper end of it. A physical vector is thus fully given by an arrow of definite length, originating in a definite point and pointing in a definite direction. With this laid down insistently, the principle of vector summation is next developed* in the usual way. Here, again, the sign quality needs to be accentuated. The origin of the first arrow is the given point of application. The origin of every other arrow is the point of the preceding, beginning with the first arrow already placed. If two vector systems are equivalent, this implies that if the free tail of each begins at a common point, then the free tip of each system must terminate in the same final point.

It is simpler to begin with the first kinematic vector, velocity, rather than with displacement. The inherent importance of the space relations is easily pointed out in the course of the development.

With these customary introductions it is my plan to write down vector equations on the blackboard just like algebraic equations, using for my terms definitely specified arrows. Thus I obtain consecutively:

Sum: The equation reads, for instance,

$$(1) \quad \uparrow + \rightarrow = \nearrow$$

To change the direction of an arrow is to change the sign of the term. Hence (1) is identical with (2).

Difference:

$$(2) \quad \uparrow - \leftarrow = \nearrow$$

or by transposing,

$$(3) \quad \uparrow = \leftarrow + \nearrow$$

which may be tested by construction. Again from (3)

$$-\downarrow = \leftarrow + \nearrow$$

* By supposing one of the vectors to be forming on a blackboard moving as specified by the other vector.

and by transposing

$$(4) \quad 0 = \downarrow + \leftarrow \nearrow$$

which is the *triangle of rest*.

Change of velocity: If in the following equation (5) the second term of the first member is given as having changed into the first, then the change of velocity is

$$(5) \quad \downarrow - \rightarrow = \downarrow + \leftarrow \swarrow$$

Polygon of velocity: If any number of velocities are given to be added,

$$(6) \quad \downarrow + \rightarrow + \uparrow + \leftarrow = \searrow,$$

which is the *polygon of velocities*, and all possible constructions are equivalent to a mere change in the order of the terms. If we change the sign and direction of the arrow in the second member of (6) and then transpose the term to the first member

$$(7) \quad \downarrow + \rightarrow + \uparrow + \leftarrow + \searrow = 0,$$

which is the *polygon of rest*.

Acceleration: That accelerations may be compounded like velocities students assert readily enough, but few really understand the assertion. Defining acceleration in the usual way, the product of a time factor and a vector is here encountered. But the time factor is scalar and can be fully given by an ordinary number. Let t be a sufficiently small interval of time. Then, for the case of linear acceleration, the equation reads

$$\frac{1}{t}(\downarrow - \rightarrow) = \frac{1}{t}(\downarrow + \leftarrow) = \frac{1}{t}(\swarrow),$$

where the quantity in parenthesis is the observed change of velocity in the time t . The result merely calls for an increase in the length of the reduced vector, $1/t$ times. The more general case corresponding to (5) may be taken at once, whence,

$$(8) \quad \frac{1}{t}(\downarrow - \rightarrow) = \frac{1}{t}(\downarrow + \leftarrow) = \frac{1}{t}(\swarrow)$$

Two accelerations of the general kind

may be compounded (using a common time t for brevity), as follows:

$$(9) \quad \frac{1}{t}(\downarrow - \leftarrow) + \frac{1}{t}(\uparrow - \rightarrow) = \frac{1}{t}(\downarrow + \rightarrow) + \frac{1}{t}(\uparrow + \leftarrow) = \frac{1}{t}(\searrow).$$

The quantities really compounded are thus the velocities (ultimately displacements) and the effect of the scalar factor is a mere change of the length of the arrow produced.

The case of a finite acceleration and vanishing t is particularly remarkable.

Momentum: If m denote mass, we again have the product of a scalar and a vector, in which, therefore, m is fully given by a number. To compound

$$(10) \quad m(\uparrow) + m(\rightarrow) = m(\swarrow)$$

we virtually reproduce (1). If the momenta are referred to different masses, as in

$$(11) \quad m(\uparrow) + m'(\rightarrow),$$

it will be necessary to change the length of each arrow before compounding. The proposition may be extended to the polygon of moments, etc., as already shown.

Force: If the interval t is sufficiently small, force is defined, in a general way, by

$$(12) \quad \frac{m}{t}(\downarrow - \leftarrow) = \frac{m}{t}(\downarrow + \rightarrow) = \frac{m}{t}(\swarrow),$$

where in the first member the second term (vector) is changed to the first term in the time t for each particle of the mass m . The quantity compounded is again the product of a vector (velocity) and a scalar m/t . To compound forces we thus virtually compound velocities and increase the length of the arrow resulting m/t times. If two forces actuate m we have in the most general case

$$(13) \quad \frac{m}{t}(\downarrow - \leftarrow) + \frac{m}{t}(\uparrow - \rightarrow) = \frac{m}{t}(\searrow).$$

These forces might have been rated in terms of different masses, m and m' , and times, t and t' . In such cases the first resultant would be multiplied m/t times and the

second m' / t' times and the new vectors then compounded.

Center of Mass: To complete the subject of translational motion for an extended body the customary reference is made to the center of mass.

ROTATION.

The case of rotation is treated throughout in complete analogy with the foregoing. What was linear velocity constant throughout the body in the above is now angular velocity also constant throughout the body; what was mass m has become moment of inertia n , and what was force F has become torque T —formally speaking, of course. The results are reached in the usual elementary way.

The first proposition to be laid down is Lagrange's well-known elementary proof for the composition and resolution of angular velocities. This must be most carefully done; for if students growl at the sign of a translational velocity they break out in open mutiny at the sign of an angular velocity. Obviously the arrow is again necessary for the complete specification, and I am in the habit of using the sign of Mars ($\bowtie$) for angular velocities, measuring the arrow from the center of the circle. As a rule, only one of a group of velocities need be so marked. If right-handed relations be postulated (the reverse is the rule in dynamics) then an eye looking in the direction of the arrow sees clockwise rotation around it as an axis, with a speed given by the length of the arrow.

Thus one obtains in succession:

Angular velocity:

$$(7) \quad \uparrow + \rightarrow = \nearrow$$

reproducing all the propositions (1) to (6) above. Stress must be laid on proposition (5).

Angular acceleration: Essentially like (8) and (9) above.

Angular momentum, moment of momentum: If n and n' be the moments of inertia the quantities to be compounded are, for instance,

$$(10) \quad n(\uparrow) + n'(\leftarrow),$$

reproducing (10) and (11') above.

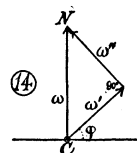
Torque, couple, moment of rotation: If t be sufficiently small, torque is defined in in the most general way by

$$(12) \quad \frac{n}{t}(\uparrow \leftarrow) = \frac{n}{t}(\uparrow \rightarrow) = \frac{n}{t}(\nearrow),$$

where in the first member of the equation the second vectorial term (angular velocity) changes into the first term in the time t for every particle of the mass implied in n . Thus the propositions (12) and (13) are formally reproduced for rotations. In other words, torques, couples, moments are compounded just like forces, and the convention involved is the convention made in representing angular velocities.

I will conclude by giving a few examples, the first of which, *Foucault's Pendulum*, is cited merely as a concrete case of (1').

Let ω be the earth's angular velocity. Let φ be the latitude of the place of observation. Resolve ω as shown in figure.



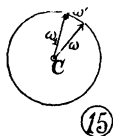
Then ω'' rotates the plane of the pendulum around a line in this plane, horizontal for the place. Hence ω'' produces no deviation. Obviously ω' , the deviating component is $\omega \sin \varphi$.

In physical *meteorology* the same result enters fundamentally into the theory of cyclones. For if $2mV\omega$ be the deviating component of the earth's rotation for a circumpolar body of mass m and velocity V , then the corresponding component for any

latitude φ is $2mV\omega \sin \varphi$, quite independent of the azimuth of V .

Again, if in figure (15) ω and ω_1 be replaced by linear velocities, one easily obtains by (8) the expression for acceleration towards a center, etc.

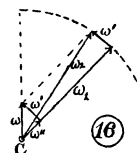
Precession: In instruments like tops, gyroscopes, etc., the mechanism (supposed frictionless) is such as to *exclude all interference* from without, with the magnitude of the angular velocity ω of the top around its axis. This constructive condition is essential. Hence, if the axis changes position, and



if for brevity we suppose the tail of the arrow ω to remain frictionlessly at C , then the locus of the point of the arrow must be the surface of a sphere of radius ω . Let ω change in position to ω_1 , let the axis to which the change of angular velocity ω' (in figure 15) corresponds, pass through C and necessarily rotate around it in a horizontal plane. This is clearly the case with the axis of gravitational torque in the precessional motion of a top or gyroscope. Then must ω' also lie in a horizontal plane, and the locus of ω is the surface of a circular cone with its axis vertical and its vertex at C . If ω' is imparted in unit of time ω' is the mean angular acceleration due to the gravitational torque and therefore equal to T/n by (12'). But the inclination of ω to the horizontal has just been shown to be constant (cone), wherefore gravitational torque is constant and ω' is constant. Hence the precessional motion is uniform rotation around the vertical axis of the fixed cone; for from one point of view ω' is the total change of angular velocity due to gravitational torque, and from another point of view, ω'/ω , constant for the reasons specified, is proportional to the uniform angular velocity of

precession (see figure). If gravitational torque is withdrawn, as in a balanced gyroscope, $\omega'/\omega=0$ and precession ceases. If ω gradually decreases (friction), ω' will subtend a relatively greater angle, or precessional motion will be accelerated, even when the axis of ω is not lowered. In the latter case the result is accentuated, for gravitational torque is increased.

Again, suppose gimbals of a gyroscope *forcibly* rotated around a vertical axis. In Figure 16 let the angular velocity ω be thus



imparted in unit of time. Let ω_1 and ω_2 be the positions of the top axis and its angular velocities before and after the interference. Resolve ω into components ω' and ω'' respectively at right angles and parallel to ω_1 . Then ω'' would rotate the top axis if it were not frictionlessly mounted. It actually rotates the gimbals only. Therefore $\omega_1 = \omega_2$ in length, as is otherwise evident. Thus ω' is the total effective change of angular velocity, and in virtue of this ω_1 passes to ω_2 and the extremity of the top axis rises, describing a circle in a vertical plane. If ω is imparted in a contrary direction the motion of ω_1 will be reversed. The top rolling on a blunt point belongs here.

Finally, if the top axis is forcibly rotated back and forth over a small angle around the *horizontal* axis of gravitational torque, similar considerations will lead to a better explanation of the curves drawn by a top on an inclined plane than I gave in a preceding article. The periodic changes of torque correspond to the rolling of the top up and down the inclined plane.

I have been tempted to enter somewhat at length into this most important subject,

because I failed to find an adequate account in such standard elementary text-books as came to my hands. Thus the explanation given in Daniell's physics is empiric and about within the limits of Perry's little book on tops. Ganot and Deschanel, Barker and Carhart, avoid the matter altogether. Kelvin and Tait's 'elementary' treatise has a single paragraph, intelligible at once, no doubt, to the authors. Peddie puts a slight expansion of this paragraph into his book. Even Violle's large new work says nothing about tops. In the German books, like Müller-Pouillet, Wüllner and the excellent treatise of Mousson, the phenomena are interpreted by aid of a suggestion of Poggendorff's, the very object of which is to dodge the principles of rotation involved under cover of a reference ('nur durch höhere Rechnung') to Euler. Yet gyrostats of diverse forms usually abound in physical cabinets. Supposing an instructor is not on the outlook for special entertainment for his children, of what use is such apparatus, I ask, if it be not to furnish the most striking tests imaginable of the truth of the above fundamental doctrines of rotation.

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ZOOLOGICAL NOTES.

NANSEN'S DISCOVERY OF THE BREEDING GROUNDS OF THE ROSY GULL.

OF the result of Nansen's Expedition thus far announced one of the most interesting, at least to ornithologists, is the reported discovery of the breeding grounds of Ross' Gull, also known as the Wedge-tailed or Rosy Gull (*Rhodostethia rosea*). In a letter published in the *London Daily Chronicle* last November, Dr. Nansen stated that he found flocks of Rosy Gulls on August 6th, in latitude $81^{\circ} 38'$, east longitude 63° . The birds were seen near four small islands called

'Hirtenland' by Nansen, a little northeast of Franz Josef Land. While Nansen did not actually find nests, he found the birds abundant, and concluded that their nests were probably near by. Every item of information regarding this rare bird is of interest, and in the December number of the *Ornithologische Monatsberichte* (pp. 193-196), Dr. Herman Schalow calls attention to the importance of Nansen's announcement and takes occasion to review briefly the history of the species.

There seems to be no reason to question the correctness of Nansen's determination of the birds or his surmise that they were breeding not far away. The wedge-shaped tail and the rosy tinge of the plumage (both noted by Nansen) are unmistakable characters of the species, and the presence of the gulls in such numbers in that high latitude renders it very probable that they were breeding. The Rosy Gull has long remained one of the rarest gulls. It was described from a specimen collected by Sir James Clark Ross in 1823, on Melville Peninsula, but in the next half century only a few individuals were taken and these in widely separated localities. In the autumn of 1881 Murdoch observed large numbers at Point Barrow, Alaska, apparently migrating from the west to the northeast. Although he secured a good series of specimens, he could add little to the life history of the species, and no other naturalist in Alaska has had the good fortune to meet with it in such numbers. This gull has also been taken in North America at St. Michael's, Alaska, and Disco Bay, Greenland, but it was not seen by the Lady Franklin Bay expedition. It was met with off the Siberian coast by the Jeannette Expedition, and was recorded by Payer between Nova Zembla and Franz Josef Land, only a few degrees to the south of the islands where Nansen found it.

The Rosy Gull is a typical arctic circum-