William L. Root; and in Physics, George K. Burgess, William D. Coolidge and Ralph R. Lawrence.

DR. E. LESSER has been appointed associate professor of dermatology at Berlin and Dr. Chermak to the chair of comparative anatomy and embryology at Dorpat. Dr. Winkler, professor of chemistry, has been appointed director of the School of Mines at Freiberg i. S., and Dr. Godschmidt has been promoted to an assistant professorship of chemistry in the University of Heidelberg.

DISCUSSION AND CORRESPONDENCE.

THE STRAIGHT LINE AS A MINIMUM LENGTH.

TO THE EDITOR OF SCIENCE: In looking over the beautiful new text-book of geometry by Profs. Phillips and Fisher one meets with the following proposition of spherical geometry:

The shortest line that can be drawn on the surface of a sphere between two points is the arc of a great circle, not greater than a semi-circumference, joining these points.

The demonstration given is one which has been given before. It appears, for example, in the treatise of Chauvenet (1869) and also in that of George Bruce Halsted (1885). In connection with this demonstration, the reader can hardly escape noticing that every step of it applies equally well to plane geometry. In fact, it is perfectly easy for any student of Euclid's Elements to construct, step by step, a precisely similar proof of the corresponding proposition of plane geometry:

The shortest line that can be drawn between any two points is the straight line which joins them.

The definition of a straight line given by Profs. Phillips and Fisher, therefore, embodies a statement capable of deduction from the geometrical axioms by a chain of logical reasoning, and as a definition, is on strictly scientific grounds, quite indefensible.

Upon examining Prof. Halsted's book, the definitions of which more closely conform to the Euclidean models, one naturally wonders why this demonstration, even more simple in plane than in spherical geometry, has been introduced only in connection with spherical geometry; and one is led to inquire at how early a point the proposition of plane geometry could properly be introduced.

In attempting to establish between any two lines a relation of equality or inequality, we find ourselves compelled to start from the following principles: *The whole is greater than any of its parts; The whole is equal to the sum of all its its parts; Lines which may be placed so as to coincide are equal.* Using these principles alone, it is evident that we cannot compare every two arbitrary lines in magnitude. In any such comparison we must be able to place one of the lines, or portions of it, in complete or partial coincidence with the other. No direct comparison can be instituted, for example, between a straight line and a line no part of which is straight. For the purposes of the proposition in question, therefore, it is necessary to make the distinct assumption, that *the magnitude of every line is comparable with the magnitude of every other line, and between these magnitudes there exists a relation either of equality or of inequality;* or else, what is better, to await the method of limits and the development, by means of it, of metrical ideas, not only for straight lines, but also for curves. Prof. Halsted, accordingly, in spite of his apparent lateness in introducing the proposition, is guilty of an error in theory. He has attempted to give a complete discussion of a proposition, and appears to believe that he has done so, when in reality assumptions additional to those previously made must be introduced before such a discussion can be undertaken.

It seems worth while to make these criticisms, because the two books above referred to are at other points remarkable for their scientific accuracy, and are of so high an order of excellence generally that the student may not readily appreciate the existence of such errors as occur.

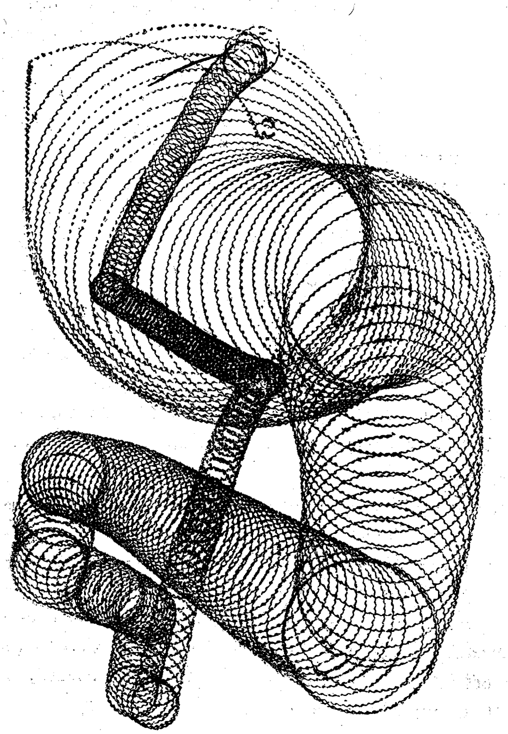
THOMAS S. FISKE.

SEPTEMBER 30, 1896.

'A CURVE-TRACING TOP,' AND A CURIOUS OPTICAL ILLUSION.

EDITOR OF SCIENCE: If Prof. Barus will use a smoked glass for his curve-tracing top to spin on, he will get more beautiful tracings than with any lead pencil arrangement. Then let him flow it over with *thin* demar varnish, and dry; the tracings will be permanent and can be photographed or printed directly by transmitted light. Some ten years ago I had a top made from an excellent gyroscope by removing the supporting ring and fitting a socket on one arm of the axle, in one end of which (the socket) was a female screw. I also had several 'points' made of hardened polished steel, one ending in a very fine point, one in a truncated cone $\frac{1}{20}$ of an inch across the smaller base, others smaller and one in a hemisphere say $\frac{1}{4}$ of an inch in diameter.

These were made to screw into the socket, and the whole most carefully centered by the very best mechanical skill to be had. It was set in motion as humming tops usually are, with a string and wooden handle.



I send you a few of my tracings, with the sharp point.* The abrupt changes in direction are due to my tilting the glass, and are always approximately perpendicular to the inclination, but never exactly so. In the tracings which I send you they begin at the larger curve and grow smaller as they progress. In a few cases, very few out of hundreds, this is reversed. The very small undulations, which are so marked a feature in most of the spirals, are due to minute nutation and precession resulting in the larger effects, as the minute movements of the earth result in the grand precession of the equinoxes.

Sometimes the smaller movements are so very small that they leave no visible traces. All that is seen is what I may call the secondary curve. Sometimes even that so nearly disappears that the path becomes to the eye a straight line.

If the glass plate is 'level,' *i. e.*, approximately so, interesting figures are traced, oblique spirals I may call them, *i. e.*, spirals traced about a point which is not quite stationary.

At first glance they appear merely like flat spirals out of center. Looking at one of them steadily, with one eye or both, you look into a deep basket resting on its smaller end. Look a little longer, and without knowing how it happens, the basket is reversed, it rests on the larger end, and you see only the small bottom and the outside.

Look longer, and without seeing any change you are looking again into the basket.

Now look at the figure with both eyes, but as if focussed for a distant object. You will see two baskets, and probably both in the same position, *i. e.*, both with the small end, or both with the large end, toward you. Keep looking; move the paper quickly a little in any direction, both will reverse; if you have good luck, with a little practice, you will soon get it. Look a little longer. One basket will stand on the small end, and the other on the large one. Focus the eyes on them or near by, and there will be only one basket. This is all well shown in the short spiral I send you.

Double images are common enough, but the new and singular thing is that they appear to each eye so different, and that all these changes take place without effort. You do not see them change, but only that they are changed.

C. B. WARRING.

SEPTEMBER 26, 1896.

* These were made so long ago that I cannot be certain whether the sharpest point was used, or one that measured $\frac{1}{50}$ of an inch across its face.