

by Meissel. The next thirty roots of the equation $J_0(x)=0$, and the values of $J_1(x)$ corresponding to the first forty roots, have just been computed by Prof. B. O. Peirce and Mr. R. W. Willson by means of Vega's ten place table of logarithms, except in the few cases where a greater number of places was necessary, and then recourse was had to Thoman's tables. The computation has been done twice.

The total number of papers read was greater by two than the number read at last year's summer meeting. The attendance was the same as last year. The Council announced that the regular October meeting of the Society would be replaced by a special meeting to be held at Princeton, on October 17th, in honor of Profs. Felix Klein and J. J. Thomson, who would be in Princeton at that time as delegates to sesquicentennial celebration of Princeton University.

THOMAS S. FISKE.

COLUMBIA UNIVERSITY.

A CURVE-TRACING TOP.

SOME time ago I constructed a top (since called the gyrograph) for directly mapping out the curves corresponding to the precessional and progressive motion of a spinning body. I have since found the instrument of service in teaching this rather troublesome subject, and I will, therefore, venture to give an account of some of its performances.

The instrument is exceedingly simple, and consists merely of a form of stably spinning top, not too heavy, having a socket at the bottom of the stem for the axial insertion of the pencil on which the top is to spin. Particular care must be taken, however, to have the top well balanced and the pencil centered, and I have, therefore, sketched in the annexed figure the form with which I obtained my best results. Here *a* is a thin disc or web of tin plate carrying a circular ring (*b*) of $\frac{1}{8}$ -inch

copper wire; *c* is a thin conical brace to sustain the brass tube (*d*), which holds the pencil (*e*) normally to the web. The whole

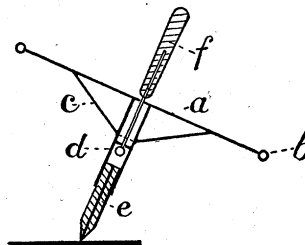


FIG. 1.—Sectional elevation of the top in position for curve tracing (reduced to one-fourth).

is revolvable around the handle (*f*), the round stem of which passes nicely through central perforations in the web and a diaphragm fixed within the brass tube. The string for spinning is wound around *d*. The figure is drawn to scale the diameter of the web, being about 6 inches. The weight should not exceed about 5 ounces. It is not desirable to hasten the precessional motion as the curve tracing is best shown with a leisurely swinging top. Furthermore, a tablet at least a foot square, preferably of plate glass, framed and provided with leveling screws, is needed for the accurate delineation of the curves. On this is placed a smooth sheet of white paper. The top, after being spun on the handle, is placed down with its *axis oblique* to the tablet, so that precessional motion may be initiated at the outset.

If the tablet is quite level the curves obtained are spiral with but very slight, if any, lateralness. Two cases are to be distinguished: If the pencil is hard and blunt (preferably cut square off at the lead with a sharp circular edge) precession is markedly accelerated; the top begins with a wide sweep and gradually rising from the oblique to the vertical position, describes a series of spires which converge rapidly from a wide circumference towards a center. If the point is hard and sharp, the top does not rise so

fast, and the tendency is to retain small contracted figures. In both cases, however, the occurrence of spirals is chiefly the result decreasing angular velocity due to friction of the pencil on the tablet, as will be presently shown.

The most interesting results and by far the most beautiful curves are obtained when the tablet is not quite horizontal. In proportion as larger angles of dip are chosen, the spirals increase in lateralness until in an extreme case they merge more and more fully into contracting prolate cycloids. The figures are in the main cornucopia-like and the tracery at proper angles of dip is

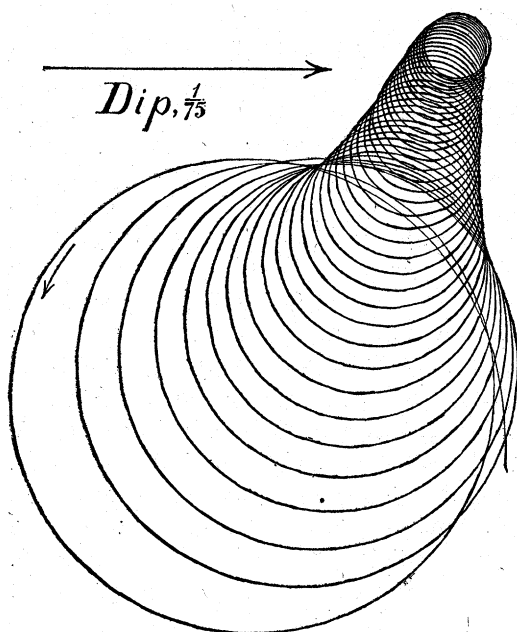


FIG. 2.

exceedingly delicate. Figure 2 is a type of these curves *, to which however an infinite variety may be given, a dip of $\frac{1}{4}$ to 1 inch in two feet being favorable.

One would at first thought suppose that a top on an inclined nearly smooth tablet

* Unfortunately the photographic reproduction of these curves was not satisfactory. The above figure is made from a hand tracing of a coarsely drawn curve and does not convey the finish of the originals.

would tend to describe elongated figures by sliding down the plane. Such, however, as is otherwise known, is only in small part the case. The top moves across the dip, tending to remain, if not quite, at least very nearly, on an average level. Moreover there is a necessary relation between the dip, the direction of rotation and the march of the top across the dip. If the tablet slopes downward from left to right parallel to the observer, the top moves away from him if spun counter-clockwise, and towards him if spun clockwise. If the dip be downward from right to left, the opposite relations of rotations and progression will hold. In other words, if the pivot or stylus of the top were to point in the direction of the dip, the rim or web would roll in the direction in which the top actually moves across the dip.

The reason for this curious behavior might perhaps most simply be looked for in the fact that precession* is relatively less accelerated when the end of the pencil moves up hill and relatively more accelerated when the end of the pencil rolls down hill. Hence if the dip be from left to right and the rotation counter-clockwise as seen from above, the pencil sweeps further out from the center, *i. e.*, away from the observer, because the obliquity of the top axis is being relatively increased. In rolling down hill the pencil sweeps nearer towards the center of motion (*i. e.*, also away from the observer), because the obliquity of the top axis is being relatively decreased. An inspection of the experiment and of the curves drawn by the top does not bear this out. It appears rather that the angular velocity of the top is continually decreased when the pencil rolls up hill and is continually increased again when the pencil rolls down hill. The tops of the spires therefore correspond to a rela-

* Following Lord Kelvin's well known explanation.

tively *later*, and the bottoms of the spires to an *earlier* stage than the corresponding mean time of spinning on a plane tablet.

Now if the pencil is hard, so that its circular edge remains nearly constant in radius, the envelope of the cycloido-spirals will consist of two straight lines converging at the point where the top would cease to rotate if the other conditions of motion remained similar. In other words, supposing the period of precession to be nearly constant, the angular velocity of the top would vanish at the point of intersection in question. The cause of the gradual cessation of motion here, as in case of the horizontal tablet, is friction; but in case of the oblique tablet, if the period of precession remains nearly constant, the crests of the spires correspond to smaller angular velocities, and will therefore have smaller radii of curvature than the troughs of the spires where angular velocity passes through a maximum. In other words, the loops will be less obtuse at the top and more obtuse at the bottom of the dip. Part of the energy of rotation is periodically potentialized. To draw such curves the top must necessarily move across the dip.

If the end of the pencil is convex, so that the rolling on the pivot is relatively decreased as the top rises, the envelope of the spires will no longer be straight, but consist of two converging curved lines as shown in the figure.

In the preceding instances the direction of the progressive motion of the top, *i. e.*, the trend of the 'cornucopias,' is nearly a straight line at right angles to the dip. Suppose, however, that for the plane tablet a flat conical one be substituted, which may be either raised or depressed in the center. The dip is now everywhere radial. In this case the progressive motion of the top becomes orbital around the axis of the cone, if the dip be suitably chosen. We have then a very simple arrangement for simula-

ting (except, of course, as to cause, and quantity) the orbital and precessional motion of the earth. Indeed, beautiful fluted curves corresponding to nutational movement may also easily be obtained by slightly destroying the balance of the top, though this interferes somewhat with the smoothness of motion.

C. BARUS.

BROWN UNIVERSITY,
PROVIDENCE.

MEETING OF THE MAZAMAS AT CRATER LAKE, OREGON.

THE annual field meeting of the 'Mazamas,' a club of mountain climbers with headquarters at Portland, Oregon, was held at Crater Lake, in the Cascade Mountains, during the latter part of August. This meeting was one of the most important and successful ever held and was a memorable one in many ways. Through the cooperation of local Crater Lake clubs, of Ashland, Medford and Klamath Falls, about 500 persons were present. There were also present members of four of the scientific bureaus of the government. The various parties arrived from the 14th till the 19th, and the camp began to break up about the 25th.

The Mazamas pitched their tents on an eminence overlooking the wonderful crater, and meetings were held evenings around a huge bonfire in front of the tents. On the evening of August 21st the ancient volcano, whose summit is occupied by Crater Lake, was christened Mt. Mazama. An appropriate address was read by the President, Mr. C. H. Sholes, of Portland. This was followed by a dedicatory poem by the Vice-President, Miss Fay Fuller, of Tacoma, Washington. Then the energetic Secretary, Rev. Earl M. Wilbur, of Portland, acting as 'toastmaster,' introduced the following toasts: To Mt. Mazama, responded to by Mr. J. S. Diller, of the U. S. Geological Survey; to the Poetry of Crater Lake, by Capt. Oliver Applegate, of Klamath