

The Congress was decidedly cosmopolitan in the authorship of the papers presented and the subjects treated. Of the forty-five papers, fourteen were from America, two from Austria, four from France, twenty from Germany, three from Italy, one from Russia and one from Switzerland. Germany was officially represented at the Congress by an Imperial Commissioner, Prof. Felix Klein, of Göttingen, who brought nearly all the papers contributed by his countrymen. It is a singular circumstance that the British Empire did not furnish a single contribution nor a single representative. Roughly classified, seven of the papers deal with geometry, ten with theory of functions, eight with the theory of groups, seven with the theory of numbers, two each with differential equations, invariants and mechanics, and seven with miscellaneous subjects.

In his opening address before the combined Congresses on Mathematics and Astronomy, Prof. Klein drew attention to a matter of great interest to all scientists, viz., a present marked revulsion from the tendency of mathematics to run into isolated specialities, which has been so pronounced for a century. At present the movement is decidedly toward unification and breadth, not only in mathematics itself, but in its relation to other sciences. The general conceptions of 'function' and of 'group' are powerful coordinating elements. Two of the Chicago papers are especially mentioned by Klein as representing the new tendency. One of these, by Schönflies, deals with the connection between the theory of groups and crystallography. The other, by Burkhardt, discusses the relations between astronomical problems and the theory of linear differential equations. A third paper, by Fricke, on the automorphic functions and arithmetic (*i. e.*, theory of numbers), illustrates the work of Klein's own school in the unification of strictly mathematical branches.

Of the other papers on the theory of functions, one is by Bolza, on Weierstrass' system of hyper-elliptic integrals; one by Hermite, on certain fundamental propositions in the theory of elliptic functions; one by Krause, on the transformation of the fifth degree of the hyperelliptic functions of the first order; two by Macfarlane,

on the definition of the trigonometric functions, and on the principles of elliptic and hyperbolic analysis; one by Pincherle, in summary of certain results relative to the theory of recurrent systems of functions; two by Pringsheim, on the expansion of functions in series and convergency and divergency; and one by Stringham, on a formulary introductory to elliptic functions. Maschke, Moore, Meyer, de Perott, Taber and Cole have articles on special topics in the theory of groups. Halsted gives an account of some salient points in the history of non-Euclidean and hyper-geometries. It is, of course, impossible here to give any detailed account of the contents of these or the other valuable papers in the list. The moral of the present publication, as representing the International Congress, lies not so much in the specific contents of the book, as in the fact that America's workers in mathematics are constantly coming into closer affiliation with those of other lands and strengthening their scientific position by this connection. The other sciences have long had this advantage, but mathematics has received it only within a few years. Our position in this country has been so isolated, and our science so backward here in development, that it will probably be news to most of our collaborators in other fields that the development of mathematics in the present century has probably been as great in actual permanent substance as that of any other science now existing. And it is the function of such meetings as that at Chicago to stimulate the growth of the science and to secure it the recognition to which it is entitled.

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RECENT TEXT-BOOKS OF GEOMETRY.

Elements of Geometry. By GEORGE C. EDWARDS, Associate Professor of Mathematics in the University of California. New York, The Macmillan Company. 1895. 8°. Pp. xvi+293.

Plane and Solid Geometry. Suggestive Method. By C. A. VAN VELZER, Professor of Mathematics in the University of Wisconsin, and GEO. C. SHUTTS, Professor of Mathematics in the State Normal School, Whitewater, Wis.

Madison, Wis., Tracy, Gibbs & Co. 1894. 8°. Pp. viii+395.

Plane and Solid Geometry. By WOOSTER WOODRUFF BEMAN, Professor of Mathematics in the University of Michigan, and DAVID EUGENE SMITH, Professor of Mathematics in the Michigan State Normal School. Boston, Ginn & Co. 1895. 8°. Pp. ix+320.

These three text-books of geometry all show points of interest and excellence, and bear testimony scientifically and pedagogically to a spirit of progress. It is natural to turn at once to the first few pages of each book, and examine the manner in which the difficult problem of giving the beginner a good start is treated. It is not easy to communicate to the student the fundamental principles upon which the geometric structure is to be based. A rigorous scientific analysis or discussion of these principles is entirely beyond his comprehension. It is generally necessary for him to commit to memory the first dozen pages, to exercise his powers of reason by tracing the subsequent development, and, having become somewhat familiar with the methods of logical argument, to return to the beginning of the text-book for a more complete appreciation of the fundamental ideas.

In order that the student may have some acquaintance with geometric notions before taking up the study of demonstrative geometry, it is desirable that such study should be preceded by simple mechanical drawing and a course in inventional geometry so-called. But the training that he will gain only from the study of demonstrative geometry is necessary before he can appreciate to any extent the character and content of the definitions, axioms and postulates of geometry. It is, of course, very satisfactory to have the first few pages of the text-book constructed so that they meet all the needs of the mature student who turns back to them for a rigorous scientific discussion; but it is of prime importance that these pages should convey to the beginner intelligibly and helpfully the information which is necessary for his first steps in the demonstration of geometrical truth. That the definition of a straight line should be based upon the notion of direction, as it is done in the text-books of Wentworth and Wells, in Davies's *Legendre*, in Byerly's edition of

Chauvenet and in other popular works, or that it should be based on the notion of distance, as is done in the original edition Chauvenet, are points which may seem appropriate for criticism to the mature student who seeks to subject every definition to a scientific analysis; but the beginner will accept at once certain notions of direction and distance, of straight lines and curved lines, and it will be fatal to his progress to stop him at the threshold of the subject for a complete discussion of these ideas.

In the introductory portions of the three books before us intuitive methods are used most largely by Edwards; scientific accuracy of treatment is maintained most fully by Beman and Smith. For example, Edwards uses direction in defining a straight line, and assumes that a straight line is the shortest distance between two points. Van Velzer and Shutts avoid the use of direction as a fundamental notion, but adopt as an axiom that the straight line is the shortest distance between two points. Beman and Smith, however, wishing to make no unnecessary assumptions, demonstrate at length, like Euclid, that one side of a triangle is less than the sum of the two other sides. The last mentioned work will probably be most satisfactory to the advanced student, but it is quite likely that the beginner may prefer one of the others.

The *Elements of Geometry* by Edwards consists of fourteen chapters, of which the first eight relate to elementary plane geometry, the following five to solid geometry, and the last one to the conic sections. The propositions are not numbered in the traditional manner, but the work is divided into articles which are consecutively numbered.

In many of the propositions the demonstration is preceded by a discussion, or 'analysis,' in which are obtained the materials for the construction of the formal proof. In many others the demonstration is of an informal character, a mere outline being given, which, however, the student will have no difficulty in completing. Unusual and ingenious methods of proof are frequent. The surfaces of the cylinder and cone are measured by unwrapping them upon a plane. In applying the method of successive approximation to calculating the area of the

circle of unit radius, the formulæ are put in the very convenient form

$$\frac{1}{a_{2n}} = \sqrt{\frac{1}{a_n} \cdot \frac{1}{A_n}}, \quad \frac{1}{A_{2n}} = \frac{1}{2} \left(\frac{1}{A_n} + \frac{1}{a_{2n}} \right),$$

where small letters denote the areas of inscribed, large letters the areas of circumscribed regular polygons, the subscript in each case indicating the number of sides. Several of the proofs given deserve censure. For example, that in which the author claims to prove that the sum of the exterior angles of a polygon is four right angles is worthless, and that which establishes the area of a rectangle is very incomplete.

The Plane and Solid Geometry of Van Velzer and Shutts consists of eight chapters, five relating to plane geometry and three to geometry of three dimensions. The so-called suggestive method is employed. Each theorem is illustrated by a figure, and a series of suggestions follows, so arranged as to indicate the successive steps of the demonstration. Frequently a 'model' is given showing how the suggestions are to be combined so as to produce a complete formal proof. Numerous original exercises are scattered through the text, the whole number being almost four hundred. Each chapter closes with a syllabus of the preceding propositions so as to facilitate the student's reference to them.

Although the Plane and Solid Geometry of Beman and Smith is more extensive than either of the other works under consideration, its publishers by giving it a very compact form have been able to make it even less bulky than the others. It consists of eight books, of which five are devoted to plane geometry and three to solid geometry. An appendix to book III. discusses the various methods of attacking geometrical propositions, while an appendix to book V. treats of the special topics, maxima and minima, concurrence and collinearity. The work contains almost eight hundred original exercises, and at the end are appended a table of formulæ and numerical constants, a biographical table, a table of etymologies and an index. A slight but sufficient treatment is given of symmetry, the notion of positive and negative magnitudes, the principle of continuity, duality,

the method of loci and the parallelism between certain propositions of geometry and algebra. The work thereby gains interest and modernness. Clearness and conciseness are given to the proofs by breaking them up into successive steps numbered, in order that they may be the better referred to, somewhat after the manner of the equations in many text-books of algebra. From a scientific point of view, this work is one of the best yet published.

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American Fonostenography.—A modern system of rapid shorthand * * * formulating and applying an entirely original principle of legibility and brevity—the fonostenographic root. By WILLIAM McDEVITT, LL.M. Judd & Detweiler, Printers, Washington, D. C.

The author of this work has sought to raise shorthand to the rank of an art of expression coordinate with speech and writing, and he has wrought with masterly hand his materials into a definite system based on practical scientific principles. The requirements of a rapid and legible system of graphic expression are clearly set forth and the means for supplying them are provided; and the structural laws on which the system is based are fundamental principles in the genesis and development of human expression, which the author seems to have attentively studied. He grasps firmly a cardinal principle of linguistic growth, *i. e.*, that meaning and sense inhere in context and not in the isolated syllable or word, and teaches that he who sets himself the task of mastering the composition of phrases, clauses and sentences acquires thereby the key to successful fonostenographic practice. A fundamental distinction between the principles of contraction employed in this system of shorthand and the rules of abbreviation applied in older stenographies is that in the latter vowels are not specifically symbolized because they *are* vowels, consonants because they *are* consonants, and final initial syllables because they are such—for in McDevitt's system of vocalized fonostenography, contraction is attained by the omission of factors or elements which are not 'strong and significant,' without regard to the adventitious

circumstance that the discarded element may be a vowel, a consonant, or even one or more syllables, *i. e.*, a sound or combination of sounds. This method of abridgement admits of a comprehensive application of the principle of the stenographic root to which the proper affixes may be joined.

The recognition and use of the fonostenographic root introduces for the first time in shorthand systems the rational application of the natural but unconscious linguistic process of adaptation to the purposive abbreviation of words. This root (also called 'sound root' or 'phonetic radical') is not of course the etymologic root of the word or words to be written; it is defined as the 'strong and significant' element of the word; in its use the author grasps and utilizes a well known law of verbal abridgement which supplies an easy and natural rule for the simple and effective contraction and abbreviation of vocables. It would seem that in his new system the author has found the golden mean between the highest rapidity and readiest legibility—between those phonographic systems which, owing to complex structure and consequent illegibility in practice, can be acquired only by the highly gifted or the tirelessly industrious, and those other systems which have been rendered simple in structure by 'such limitations in principles and development' as to deprive them of the requisite adaptability to the exacting needs of the reporter. In the terse language of the author, "the former class appeals to a high standard of culture; the latter addresses itself to a lower plane of mental capability and development; the former taxes too heavily the head, and the latter demands an impossible dexterity of the hand." Hitherto shorthand has commonly been pursued empirically, with little or no regard to linguistic principles, and it is a gratification to note an attempt to bring it within the domain of linguistic science.

J. N. B. HEWITT.

A Concise Handbook of British Birds. By H. KIRKE SWANN. London, John Wheldon & Co. 1896. 16°. pp. 210.

The author of this recent addition to the apparently endless series of books treating of British birds, claims for his work a unique place

among its fellows on the ground of its small size and conciseness.

No space is given to remarks on the faunal position of the region under consideration, analyses of the birds which occur in it, synopses or keys to higher groups, or other preliminary or explanatory matter, it evidently being assumed that the reader is already more or less an ornithologist who will use the book as a pocket manual for ready reference.

Consequently, immediately after a 'list of genera,' we begin with 'Order Passeres,' 'Family Turdidæ,' 'Subfamily Turdinæ'—names merely—'Genus Turdus,' which is briefly defined. Then follows the species with a short statement of its 'habitat'—a term which is arbitrarily used as "meaning the region inhabited during the breeding season"—plumages, manner of occurrence, haunts, notes, nest, eggs and food, all very much condensed and with no striving for literary effect, but making useful, if not very readable, summarized biographies.

The most interesting point in Mr. Swann's book for American naturalists is his pronouncedly un-British stand in favor of trinomials. He seems to fully recognize—indeed, to glory in—the depth of his depravity, and remarks that he cannot expect to "escape censure for adopting the despised system," adding the fair challenge that, "until some of our ornithologists can suggest some other way of allowing a name to a recognised race without giving it the name of a species, I will adhere to trinomials." After this bold declaration can any one doubt that Mr. Swann's excommunication will speedily follow?

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SCIENTIFIC JOURNALS.

PSYCHE, AUGUST.

THE genus *Orphula* with its three New England species forms the subject of the continuation of A. P. Morse's paper on *N. E. Tryxalinæ*. A. R. Grote writes on the condition of the nomenclature of the species of *Apatela*. H. G. Dyar describes the early stages of *Cosmosoma auge*, and also the apparatus by which he conceives the noise made by *Dionychopus niveus* to be produced. Notice of a few recent publications completes the number.