

*Die Theorie der Parallellinien von Euklid bis auf Gauss, eine Urkundensammlung zur Vorgeschichte der nichteuklidischen Geometrie, in Gemeinschaft mit Friedrich Engel, herausgegeben von Paul Stäckel. Leipzig, Teubner. 1895. [July].*

This book is a striking example of one of the many beneficent characteristics of our present civilization. Here all the works which show the gradual but sure development of the human mind toward an achievement of modern thought unsurpassed for interest and importance, books so rare that, so far as I know, not one is contained in any public library on the western continent, are put within the reach of the poorest student. Here we have, edited with the most painstaking accuracy, Wallis, Saccheri, Lambert, Schweikart, Taurinus, the forerunners of the non-Euclidean geometry.

The jump made by Bolyai and Lobachevski, the Magyar and the Russian, will no longer seem so bewilderingly long and unanticipated. How they, about the same time, 1829, came to publish each a complete, a full-fledged non-Euclidean geometry was a problem which provoked an unfortunate pseudo-solution, a hypothetical construction, which is still repeated, and even to be found in the pages of SCIENCE. [March 29, 1895, pp. 357-8.]

After a lecture on Saccheri at the World's Fair Science Congress, since published under the title 'The non-Euclidean Geometry Inevitable,' in the *Monist*, July, 1894, pp. 483-493, Professor Felix Klein, of Göttingen, who was present and said that never before had he so much as heard even the name of Saccheri, was asked why in his *Nicht-Euklidische Geometrie*, 1889-90, he says: "Kein Zweifel bestehen kann, dass Lobatscheffsky sowohl wie Bolyai die Fragestellung ihrer Untersuchungen der Gaussischen Anregung verdanken" [p. 175, Zweiter Abdruck, 1893]. He answered, that he believed he would be justified when Schering published the 'Nachlass von Gauss.' Such special personal information from Schering perhaps is referred to on the preceding page, 174, in the sentence: "Dies sind die saemmtlichen Notizen, die man in *allgemeinen Kreisen* ueber die Gaussischen Untersuchungen, betreffend die nichteuklidische Geometrie besitzt."

This very question to Professor Klein, as to how he could justify his ungenerous statement, must have been again put to him by Engel and Stäckel, and he must have given essentially the same answer; for, after stating his opinion, they say of it, p. 243: "Eine Entscheidung ueber die Richtigkeit dieser Vermutungen wird kaum moeglich sein, solange der Nachlass von Gauss der Forschung unzugänglich ist."

But how little we can trust the unchecked judgment of Klein in this matter is strikingly shown by what he says of Gauss's letter to Bolyai of 1799, on this very page 174: "Dies ist das interessanteste hierher gehoerige Dokument, da es noch ganz aus Gauss' Jugendzeit stammt. In diesem letzteren Brief ist besonders gesagt, dass es in der hyperbolischen Geometrie ein Maximum des Dreieckinhaltes gebe."

This letter is given in full in the English translation of the *Science Absolute of Space* by Bolyai János, and again in the *Monist*, (p. 486), and is reproduced by Stäckel (p. 219). What it really says is about as far as could be well imagined from the statement of Professor Klein. If Schering can do no better than that, we need not wait to declare that there is not the slightest particle of evidence that either Bolyai or Lobachevski were even in the remotest degree influenced by Gauss.

A certain 'Gymnasiallehrer,' Richard Beez, mentioned slightly by Professor Klein on p. 277 of Part I. of his *Nicht-Euklidische Geometrie*, as incapable of grasping the subject, yet presumed on p. 15 of a pamphlet published at Plauen to use the expression, 'Gauss, der Lehrer von Bolyai und Lobatschewsky.' This irritating misstatement was reproduced by Dr. Emory McClintock in the *Bulletin of the New York Mathematical Society*, and when written to about it he asked Beez his grounds, and of course found there were none. In his retraction, *Bulletin*, March, 1893, p. 146, he cites, as some justification, the paragraph from Professor Klein already discussed, and says: "In the paper already cited I followed Beez in stating too strongly the probable connection between Gauss and Lobatschewsky. I am indebted for my first knowledge of Beltrami's account of Saccheri to a letter from Professor Beez, in which he admits his mention of Gauss as the

teacher of Lobachewsky to be partly inferential, and not to be taken literally." It is to be taken, we suppose, in some 'Pickwickian' sense.

This letter of Beez incited Dr. McClintock to an examination of Beltrami's article and a paper on it under the title 'On the early history of the non-Euclidean geometry,' where among other mistakes he makes one peculiarly entertaining. He says, p. 145, Bulletin, Vol. II., of Saccheri: "He confessed to a distracting heretical tendency on his part in favor of the 'hypothesis anguli acuti,' a tendency against which, however, he kept up a perpetual struggle (diuturnum proelium). After yielding so far as to work out an accurate theory anticipating Lobatschewsky's doctrine of the parallel-angle, he appears to have conquered the internal enemy abruptly, since, to the surprise of his commentator, Beltrami, he proceeded to announce dogmatically that the specious 'hypothesis anguli acuti' is positively false." Who would suspect that all that is a pure fairy tale evolved by Dr. McClintock from his mistranslation of a passage immediately announced by the two Latin words he fortunately retained in parenthesis!

As some slight acknowledgment of the fine spirit in which the previous criticisms had been received, a transcript was made of a considerable portion of a copy of Saccheri then being translated into English, the only copy then on this continent, and sent to Dr. McClintock. After another examination and comparison of the article by Beltrami, Dr. McClintock wrote a frank acknowledgement of his mistake, but this time published no correction.

Mr. A. Ziwet, noticeable as a converted anti-non-Euclidean, repeats the older error in a review of the translation of Vasiliev's Address on Lobachevski:—"confirms the supposition that the first impulse to these studies came to him, at least indirectly, from Gauss. To the same source of inspiration must be traced the almost simultaneous, but independent, researches of the Hungarian Wolfgang Bolyai and his son Johann." [SCIENCE, March 29, 1895, p. 358.] It is rather a pity if it 'must,' since it never can be. A life of Bolyai from original Magyar sources, which is now in press, puts a totally new aspect upon the whole matter, which need not here be anticipated. These Magyar docu-

ments make it possible to offer to Professor Staeckel a slight correction, which is given as homage to the extraordinary accuracy of his book. On p. 241 the title of the Tentamen includes the words 'Cum appendice triplici.' Then follows the statement, "In dem dritten Anhang, der nur 28 seiten umfasst, hat Johann Bolyai seine neue Geometrie entwickelt."

It was not a third appendix, nor is it referred to at all in the words 'cum appendice triplici.' These words, as explained in a prospectus issued by Bolyai Farkas asking for subscribers, referred to a real triple appendix, which appears, as it should, at the end of the book, Tomus Secundus, pp. 265-322.

The now world renowned Appendix by Bolyai János was an afterthought of the father, who prompted the son not 'to occupy himself with the theory of parallels,' as Staeckel says, but to translate from the Magyar into Latin his treatise discovered in 1823, given in writing to J. W. von Eckwehr in 1825. The father, without waiting for Vol. II., inserted this Latin translation, with separate paging, as an appendix to his Vol. I., where, counting a page for the title and a page 'Explicatio Signorum,' it has 26 numbered pages, followed by two unnumbered pages of Errata. The treatise itself, therefore, contains only 24 pages—the most extraordinary two dozen pages in the whole history of thought!

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*Chinook Texts.* FRANZ BOAS. Washington, 1894. Pp.278.

The linguist who in publishing elementary treatises on the languages of primitive peoples was the first to subjoin national texts and to comment on these texts philologically, certainly found the correct method. But it is a pity that so few of his colleagues and co-workers have followed his example, for ten pages of well-edited texts of aboriginal, oral literature accomplish more for the deeper study of these forms of human speech than one hundred pages of vocabulary or of crude, undigested grammatic information. But recently the publishing of such texts has become quite the fashion. The late James O. Dorsey intended to publish a series of works on the Omaha and Ponka language, and