SCIENTIFIC LITERATURE.

Nicolái Ivánovich Lobachévsky.—Address pronounced at the commemorative meeting of the Imperial University of Kasán, October 22, 1893, by Professor A. VASILIEV, President of the Physico-Mathematical Society of Kasan.—Translated from the Russian, with a preface, by DR. GEORGE BRUCE HALSTED, President of the Texas Academy of Science.—Volume one of the neomonic series.—Published at The Neomon, 2407 Guadalupe Street, Austin, Texas, U. S. A. 1894. Sm. 8vo, pp. 8+40+17.

Within the last thirty years the name of Lobachevsky has become widely known as that of one of the earliest discoverers in the field of non-Euclidean geometry, a subject which has not only revolutionized geometrical science, but has attracted the attention of physicists, psychologists and philosophers.

Professor Vasiliev's life of Lobachevsky, which we welcome here in an English translation, is without question the best and most authentic source of information on this original mathematical thinker who spent his whole life in a remote Russian town, almost on the confines of civilization, and whose work began to be appreciated by the scientific world only after his death (1856). What lends a peculiar interest to the story of this uneventful life is its intimate association with the growth of the University of Kazàn. Lobachevsky entered this university as a student soon after its foundation, became, immediately after graduation, an instructor, and then a professor in it, was its president for nineteen years during its formative period, and contributed largely to its rise and progress through his administrative ability and untiring energy. This man, who is known abroad as an original investigator in one of the most abstruse branches of mathematics, endeared himself, moreover, to his townsmen in many respects as a progressive and public-spirited citizen, delivering popular lectures on scientific subjects, conducting evening classes in elementary science for workingmen, taking a most active part in the work of the Kazàn Economic and Agricultural Society, and so on.

It is due to these facts that the centennial celebration held by the Physico-Mathematical Society of the University of Kazàn, in 1893, in commemoration of his birth, was participated in not only by professional mathematicians, but also by the whole university and the citizens of Kazàn. It is for this occasion that Professor Vasiliev prepared his biography.

The celebration began with religious services in the University chapel, on Lobachevsky's one hundredth birthday, November 3 (or, according to the old calendar still used in Russia, October 22); at noon the University Senate assembled in solemn session, the foreign delegates were greeted by the president of the university, letters and telegrams of congratulation were read, and several addresses were made commemorating the life and work of the great Russian geometer. On the next day the Physico-Mathematical Society held a public session for the reading of various papers on subjects connected with non-Euclidean geometry. On the 5th of November the Municipal Council of the city of Kazàn dedicated with appropriate ceremonies a memorial tablet, inserted in the front wall of the house in which Lobachevsky had lived. Another meeting of the Physico-Mathematical Society brought the celebration to a close. A sum of several thousand rubles had been collected in the course of the year for the purpose of founding a Lobachevsky medal or prize to be awarded annually, and of erecting a bust of Lobachevsky at Kazàn, in the public square that bears his name.

It is well that this late justice should be done to the memory of a man who during his lifetime never received any public recognition for his scientific work. At the present time no competent mathematician doubts the value of Lobachevsky's investigations in non-Euclidean geometry. For those not familiar with modern mathematical thought it is, however, difficult, if not impossible, to fully appreciate the true value of this subject; they are inclined to attribute undue importance to its possible bearings on non-mathematical questions and to neglect and underrate what is most valuable.

The starting point for Lobachevsky's researches, as for those of all the earlier writers on non-Euclidean geometry (Saccheri, Lambert, the two Bolyais), is given by the theory of parallels in elementary plane geometry which is based by Euclid on his fifth postulate (usually called his "eleventh axiom"). This postulate refers to two lines cut by a transversal, and states that if the sum of the interior angles on one side of the transversal be less than two right angles the lines will meet on this side if sufficiently produced. The numerous attempts that have been made to make a theorem of this proposition, and to *prove* it, have always remained as futile as the attempts to square the circle. They have only shown that it can be replaced by other postulates, such as that only one parallel can be drawn to a given line through a given point, or that the sum of the angles of a triangle is equal to two right angles, etc.

Does it follow that these postulates express an absolute necessary truth? Certainly not. For it can be shown—and this is just what Lobachevsky did—that a perfectly consistent system of geometry can be constructed by rejecting Euclid's postulate and its equivalents, and assuming, say, that more than one parallel can be drawn to a given line through a given point, or that the sum of the angles of a triangle is less than two right angles.

The question of the character of the so-called geometrical axioms thus assumes an aspect very different from the one it had at the beginning of the present century, when they were commonly regarded as necessary logical truths. It is, however, not for the mathematician to decide whether ultimately these axioms express facts of observation unconsciously acquired and made familiar through the constant perception of an actually existing space. For him they represent mere assumptions selected for the purpose of defining his space or his methods of measuring this space.

It would, of course, be very important to know which of the different spaces that the mathematician can thus define corresponds most closely to the facts of observation. But this question is difficult to decide; for while the ordinary Euclidean space appears in this respect to satisfy all demands, the non-Euclidean spaces do the same, at least, approximately within certain limits; and all our observations give only approximate results and are confined within a narrow range of space.

What the mathematician has gained through the generalization of non-Euclidean geometry is a broader horizon and a vastly extended field of research. The multifarious relations by which this new science is connected with the various banches of geometry are admirably set forth by Professor F. Klein, of Göttingen, in his *Vorlesungen über nicht-Euklidische Geometrie* (1889–90). These lectures also trace the historical development of the subject since the times of Gauss. A few more recent investigations were discussed by him in the *Evanston Colloquium* (New York, Macmillan, 1894), in the 6th and 11th lectures.

What Professor Vasiliev tells us about Bartels, who in his earlier years had intimately associated with Gauss, and later, as the first professor of mathematics at the University of Kazàn, became the teacher and protecting friend of Lobachevsky, confirms the supposition that the first impulse to these studies came to him, at least indirectly, from Gauss. To the same source of inspiration must be traced the almost simultaneous, but independent, researches of the Hungarian Wolfgang Bolyai and his son Johann. Gauss himself never published anything on the subject of non-Euclidean geometry; but we know from his letters to Schumacher that he had spent much thought on these questions, which had occupied him from his earliest youth, and had arrived at practically the same results as Lobachevsky and the Bolyais.

In the later development of non-Euclidean geometry and the closely related theory of n-dimensional spaces or manifoldnesses we find among others the names of Grassmann, Riemann, Helmholtz, Cayley, Klein, Lie; and in these the uninitiated may find a sufficient guarantee for the value of the subject.

In conclusion, a few words must be said of the present English translation. The original has been followed so faithfully that anybody possessed of an adequate knowledge of the Russian language will understand the translation very readily. The reading of such unidiomatic English is, however, exceedingly painful. Were it not for the direct statement on the title-page, we should never have ascribed this translation to Professor Halsted, whose vigorous command of the English language is well known. It seems almost incredible that a person whose native language is English should have written, or even passed in the proof, such sentences as these: (p. 3) "So in celebrating this day to Lobachevsky, we must remember with gratitude his teachers." (ib.) "His destiny was to be the teacher and protector not only of Lobachevsky, but of the scientist of our century most influential on the development of mathematics, Gauss. (ib.) "The mathematical ability of the boy-genius awakened the attention of the science-hungry Bartels." (p. 4.) " . . . he received the grade of 'Magister' July 10, 1811, for extraordinary advance in mathematics and physics." (ib.) " . . . the question of the lowering of the grade of a two-termed equation . . ."

The transliteration of Russian names is faulty and inconsistent; thus we find Pouchkin for Pushkin, Demidef for Demidov, Karamzen for Karamzin, Simenov for Simônov, etc. It is inconceivable why the name of the well-known astronomer Littrow should be persistently misspelled Lettrov. On p. 1, for 'November 9, 1807' read 'January 9, 1807.' The statement in the preface, p. vii., that "in 1500 Copernicus was enjoying the friendship of Regiomontanus and fulfilling with distinction the duties of a chair of mathematics" is singularly incorrect. Regiomontanus died in 1476, when Copernicus was three years of age; and, although Rhaeticus, in speaking of the residence at Rome in 1500, refers to Copernicus as 'professor mathematum,' it is now, in the absence of any direct evidence, generally accepted that the author of the *De revolutionibus* was never connected as teacher with any scientific institution.

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Laboratory Exercises in Botany, designed for the use of colleges and other schools in which Botany is taught by laboratory methods, by EDSON S. BASTIN, Am. Professor of Materia Medica and Botany and Director of the Microscopical Laboratory in the Philadelphia College of Pharmacy. Philadelphia. 1895. $2.50.

In a review of this volume it should be considered for whom it was written and from that standpoint an estimate should be made whether the purpose has been really accomplished. Being designed for students who are beginners, it leads them from the simple to the complex, and does it, we think,