thoroughly studied it will be found to belong to one of these recently differentiated genera. F. H. KNOWLTON.

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Mean Values.

MISS PORTER'S kindly criticism (*Science*, June 2) of one point in the article, "Sun-Heat and Orbital Eccentricity" (*Science*, Apr. 28), gives occasion to say a work in regard to mean values. Since the mean value of n quantities is the arithmetic mean of their sum, it would appear at first glance as if the term were a perfectly definite one; but if the quantities to be averaged are successive values of a function of some variable, then clearly their magnitudes depend not only on the nature of the function, but also on the law of variation of the fundamental. Thus, suppose we have the isotherm, $pv = c$, and wish to know the average pressure between the volumes $v = v_1$ and $v = v_2$. It is necessary to make some assumption in regard to the variation of v. If its increments are supposed equal, we understand by the "mean value" of the pressure the average of the pressures corresponding to the values of v. If the volume is assumed to depend in turn on some other variable in such a manner that the abscissa-increments are not equal, the mean value will now be the average of the new series of pressure-ordinates corresponding to the series of values of v arising under the second assumption. Evidently these two means will in general be unequal, but one is just as properly the "real average" as the other. The formula for mean value may be derived by a method even simpler than the usual analytical one as given by Williamson and Todhunter. Let it be required to find the mean value of y where $y = f(x)$ and x is an equicrescent variable. If $y = f(x)$ be treated as a curve referred to rectangular axes, $\int_a^b f(x)\,dx$ is the expression for the area, A, bounded by the X-axis, two ordinates, and the portion of the curve intercepted between the bounding ordinates. Let $A = A'$, where A' is a rectangle whose base equals the base of A. Then the altitude of A' is the average of the ordinates in A. For let

$$\frac{y_1 + y_2 + \ldots\ldots y_n}{n} = y_0,$$

the average of the series of ordinates.

Then $y_1 + y_2 + \ldots\ldots = y_0 + y_0 + \ldots\ldots$ on to n terms.

Multiplying by $\triangle x$ and summing,

$$\Sigma\,(y_1 + y_2 + \ldots.)\,\triangle x = \Sigma\,(y_0 + y_0 + \ldots.)\,\triangle x;$$

or, making n indefinitely large,

$$\int_a^b y\,dx = y_0 \int_a^b dx = y_0\,(b - a).$$

But $$\int_a^b y\,dx = A, \text{ hence } y_0\,(b - a) = A',$$

and, since $b - a$ is the base of the rectangle, A', y_0 is it saltitude.

For example, let it be required to find the mean pressure between the volumes v_1 and v_2. If the isotherm is $pv = c$, the area, A, in this case becomes

$$\int_{v_1}^{v_2} \frac{c}{v}\,dv = c \log\left(\frac{v_2}{v_1}\right);$$

its base is $v_2 - v_1$, hence the mean pressure is

$$\frac{c}{v_2 - v_1} \log\left(\frac{v_2}{v_1}\right).$$

This conception of mean values may be readily [illegible] when a curve is expressed in polar coördinates. If $r = f(\theta)$, let x be written for θ and y for r. The Cartesian equation thus arising furnishes a curve which sustains peculiar relations to the original polar curve. The radii-vectores are taken out of their fan-shaped arrangement and placed equi-distant and parallel, with their extremities on the comon line, the X-axis. The pole may be viewed as having developed into this axis, whilst a circle of unit radius with pole as centre has developed into a straight line parallel to the axis, the radii-vectores keeping their normal position with respect to the circle. In finding the mean value of the radius-vector of an ellipse, $d\theta$ being constant, the figure A has three rectilinear sides: $x = 0$, $x = \pi$, and the X-axis. Its fourth side is the curve,

$$y = \frac{a\,(1 - e^2)}{1 + e \cos x}.$$

The base of the figure is π; hence the mean value is

$$\frac{1}{\pi} \int_0^\pi \frac{a\,(1 - e^2)}{1 + e \cos x}\,dx = a\,\sqrt{1 - e^2}.$$

It will be seen that the area-method serves only when the ordinates are equally distributed throughout the area A. In the dynamical problem of the earth's mean distance from the sun it is not θ (or x) which is the equicrescent variable, but t, the time. A must therefore be taken equal to

$$\int_{t_1}^{t_2} r\,dt,$$

for which $r = f(t)$ must be given; but, as is well known, the equation expressing the relation between r and t is transcendental and cannot be written in the form $r = f(t)$. Recourse must therefore be had to other devices for finding the mean distance when the problem is rendered kinematical by taking Kepler's second law into account. ELLEN HAYES.

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Iron and Aluminium in Bone Black.

WILL you kindly, in your next issue, print the following corrections to my article on "Iron and Aluminium in Bone Black," which has just reached me.

Page 300, first column. In twentieth line (from the bottom of page), after the word "permanent," insert, *and boil*. In nineteenth line (from bottom of page) remove the first two words: "and boil."

In twelfth line (from bottom of page) insert a decimal point between 5 and 0 at end of this line, for the figure must read 5.0 and not 50 grammes.

Page 301, first column. In twentieth line (from bottom of page) transpose after "iron." Instead of "aluminium, or the phosphate" then should stand: or the aluminium phosphate predominates.

J. G. WIECHMANN.

New York, June 7.

Estimated Distance of Phantoms.

IN *Science* of May 19, p. 269, Mr. Bostwick mentions the familiar experiment of binocular combination of regular patterns, such as a tessellated pavement or figured wall-paper, by means of ocular convergence, and states that in his case, although the figures of the phantom thus formed appear smaller, yet contrary to the statements of all other writers they do not appear nearer but farther off than the real object. This seems to me inexplicable if the phantom is really distinct.

As I have very unusual facility in making such binocular combinations, I will very briefly describe an experiment of this kind. I stand now looking down on the tesselated oil-cloth covering the floor of the library. By ocular convergence I slide the two images of the floor over one another in such wise as to combine contiguous figures. After perhaps a brief interval of indistinctness, the pattern appears with perfect clearness at half the distance of the floor and the figures of the pattern of half the real size. The sense of reality is just as perfect as in the case of a real floor at that distance. It seems to me as if I could rap it with my knuckle. Taking now this phantom as a real object, by greater convergence the plane can be brought up higher and higher, until by extreme convergence it is brought within three inches of the root of the nose and seen there with the greatest distinctness in exquisite miniature, the figures being only one-quarter inch in diameter. By relaxing the convergence a little, the phantom-plane may be dropped and caught on lower and