SCIENCE

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THE GROWTH OF CHILDREN.—II.

IN No. 483 of *Science* I have tried to show that measurements of children of a given age are, as a rule, not distributed symmetrically around the average, but that they are distributed asymmetrically, the curve being expressed by the formula—

$$\frac{(1 + c\,u)}{M\sqrt{2\pi}}e^{-\frac{u^2}{2M^2}}.$$

In this expression c is a small constant, M the mean variation, and u the deviation from that measurement which belongs to the individual which will finally be an average individual in regard to the measurement under consideration and whose development corresponds exactly to that of its age. In this sense the measurement may be called that of the average individual, although it is not the average of all the measurements.

Supposing an extensive series of observations on children of a certain age to be given, the question arises, how to find that value which belongs to the average individual and how to find the mean variation. The number of observations between the limits a and b will be

$$\int_a^b \frac{(1 + c\,u)}{M\sqrt{2\pi}}e^{-\frac{u^2}{2M^2}}du = \int_{\frac{a}{M\sqrt{2}}}^{\frac{b}{M\sqrt{2}}} \frac{1}{\sqrt{\pi}}e^{-t^2}dt - \frac{c\,M}{\sqrt{2\pi}}\left(e^{-\frac{b^2}{2M^2}} - e^{-\frac{a^2}{2M^2}}\right)$$

Whenever a and b remain the same multiples of M, the value of this integral depends solely on $\frac{c\,M}{\sqrt{2\pi}}$ and a table of the values of the integral may be computed. It is convenient to assume $a = -\infty$ and to compute the integral. Following is a brief table of the integral:—

$$\int_{-\infty}^{\frac{b}{M\sqrt{2}}} \frac{1}{\sqrt{\pi}}e^{-t^2}\,dt - \frac{c\,M}{\sqrt{2\pi}}e^{-\frac{b^2}{2M^2}}$$

$$\frac{c\,u}{\sqrt{2\pi}}$$

b	−0.10	−0.08	−0.06	−0.04	−0.02	0.00	+0.02	+0.04	+0.06	+0.08	+0.10
− 3.0 M	0.0025	0.0023	0.0021	0.0018	0.0016	0.0014	0.0012	0.0010	0.0007	0.0005	0.0003
− 2.5 M	0.0106	0.0097	0.0088	0.0080	0.0071	0.0062	0.0053	0.0044	0.0036	0.0027	0.0018
− 2.0 M	0.0362	0.0335	0.0308	0.0281	0.0254	0.0227	0.0200	0.0173	0.0146	0.0119	0.0091
− 1.5 M	0.0990	0.0925	0.0860	0.0795	0.0731	0.0666	0.0601	0.0537	0.0472	0.0407	0.0342
− 1.0 M	0.2212	0.2091	0.1970	0.1849	0.1727	0.1606	0.1485	0.1363	0.1242	0.1121	0.1000
− 0.5 M	0.3967	0.3791	0.3614	0.3438	0.3261	0.3085	0.2909	0.2732	0.2556	0.2379	0.2203
0.0 M	0.6000	0.5800	0.5600	0.5400	0.5200	0.5000	0.4800	0.4600	0.4400	0.4200	0.4000
+ 0.5 M	0.7797	0.7621	0.7444	0.7268	0.7091	0.6915	0.6739	0.6562	0.6386	0.6209	0.6033
+ 1.0 M	0.9000	0.8879	0.8758	0.8637	0.8515	0.8394	0.8273	0.8151	0.8030	0.7909	0.7788
+ 1.5 M	0.9658	0.9593	0.9528	0.9463	0.9399	0.9334	0.9269	0.9205	0.9140	0.9075	0.9010
+ 2.0 M	0.9908	0.9881	0.9854	0.9827	0.9800	0.9773	0.9746	0.9719	0.9692	0.9665	0.9637
+ 2.5 M	0.9982	0.9973	0.9964	0.9956	0.9947	0.9938	0.9929	0.9920	0.9912	0.9903	0.9894
+ 3.0 M	0.9997	0.9995	0.9993	0.9990	0.9988	0.9986	0.9984	0.9982	0.9979	0.9977	0.9975

The series of actual observations must correspond to one of these theoretical curves. We must find those values of c and M which agree most nearly with the curve of the observations. c and M may be determined from any two values of the integral. The most probable values will be those which are found by taking into consideration all the given values. This may be done in the following way: We will call the value for which $u = 0$, U; then, any observed value

$$Y = U + u.$$

The average of all observed values

$$A_1 = \int_{-\infty}^{+\infty} \frac{(U + u)\,(1 + c\,u)}{M\sqrt{2\pi}}e^{-\frac{u^2}{2M^2}}du$$

(1) $$A_1 = U + c\,M^2$$

The average of the squares of all observed values

$$A_2 = \int_{-\infty}^{+\infty} \frac{(U + u)^2\,(1 + c\,u)}{M\sqrt{2\pi}}e^{-\frac{u^2}{2M^2}}du$$

$$= U^2 + 2\,U c\,M^2 + M^2$$

$$A_2 = U^2 + 2\,U(A_1 - U) + M^2$$

(2) $$A_2 = -\,U^2 + 2\,U A_1 + M^2; \text{ and}$$

(3) $$U = A_1 \pm \sqrt{M^2 - (A_2 - A_1^2)}.$$

By substituting this value in (1) we find

(4) $$\frac{c\,M}{\sqrt{2\pi}} = \frac{\mp\sqrt{M^2 - (A_2 - A_1^2)}}{M\sqrt{2\pi}}$$

By computing the average of the observations and of their squares, we can, therefore, find easily a series of the three values U_1, M, c, and we have to select the one which gives the most satisfactory agreement between the theoretical curve and the actual curve, i.e., the one in which the sum of the squares of the differences between the two curves are a minimum. The actual computation becomes a little simpler by substituting

$Y = C + y$ where C is equal or nearly equal A_1.

The average of all y $\quad a_1 = U - C + c\,M^2 = 0$

The average of all y^2 $\quad a_2 = (U - C)^2 + 2\,(U - C)\,(A_1 - U) + M$

$$= (C - A_1)^2 - (U - A_1)^2 + M^2$$

$$U = A_1 \pm \sqrt{M^2 - a_2 + (C - A_1)^2}.$$

I will show the application of this method by computing the stature of 12-year-old girls, measured in Worcester, Mass., 1891. 112 observations are available.

$$A_1 = 1446.6;\ \ C = 1447;\ \ a_2 = 5365.$$

$$U = 1446.6 \pm \sqrt{M^2 - 5364.84}.$$

We assume various values for M, and find the corresponding values for U and $\frac{c\,M}{\sqrt{2\pi}}$.

M	U	$\frac{c\,M}{\sqrt{2\pi}}$
73.8	1455.6	− 0.049
74.0	1457.1	− 0.057
74.2	1458.5	− 0.064
74.4	1459.7	− 0.070

Then the number of cases which are required by the theory may be found from the above table, while the observed number of cases are found by computing $U - 3\,M$, $U - 2.5\,M$, etc., and counting the number of cases below these points. By this process we find the following results: —

	$M = 73.8.$			$M = 74.0$			$M = 74.2$			$M = 74.4.$		
	Observation.	Theory.	△	Observation.	Theory.	△	Observation.	Theory.	△	Observation.	Theory.	△
$U - 3.0\,M$	0.0	0.2	– 0.2	0.0	0.2	– 0.2	0.0	0.2	– 0.2	0.0	0.2	– 0.2
$U - 2.5\,M$	0.0	0.8	– 0.8	0.0	0.9	– 0.9	0.0	0.9	– 0.9	0.0	0.9	– 0.9
$U - 2.0\,M$	0.9	2.9	– 2.0	1.8	3.0	– 1.2	1.8	3.1	– 1.3	1.8	3.2	– 1.4
$U - 1.5\,M$	6.4	8.2	– 1.8	7.3	8.5	– 1.2	8.2	8.7	– 0.5	9.0	8.9	+ 0.1
$U - 1.0\,M$	18.9	18.9	– 0.0	19.8	19.5	+ 0.3	19.8	20.0	– 0.2	19.8	20.3	– 0.5
$U - 0.5\,M$	36.7	35.2	+ 1.5	37.5	36.0	+ 1.5	40.2	36.5	+ 3.7	41.1	37.0	+ 4.1
U	59.0	53.9	+ 5.1	59.0	55.7	+ 3.3	59.0	56.4	+ 2.6	59.0	57.0	+ 2.0
$U + 0.5\,M$	76.8	72.6	+ 4.2	76.8	74.0	+ 2.8	76.8	74.8	+ 2.0	76.8	75.3	+ 1.5
$U + 1.0\,M$	84.0	86.3	– 2.3	85.0	87.3	– 2.3	85.9	87.8	– 1.9	85.9	88.2	– 2.3
$U + 1.5\,M$	92.9	94.6	– 1.7	92.9	95.1	– 2.2	92.9	95.4	– 2.5	92.9	95.6	– 2.7
$U + 2.0\,M$	99.1	98.3	+ 0.8	99.1	98.5	+ 0.6	99.1	98.6	+ 0.5	99.1	98.7	+ 0.4
$U + 2.5\,M$	99.1	99.6	– 0.5	99.1	99.6	– 0.5	99.1	99.7	– 0.6	99.1	99.7	– 0.6
$U + 3.0\,M$	100.0	99.9	– 0.1	100.0	99.9	+ 0.1	100.0	99.9	+ 0.1	100.0	99.9	+ 0.1
$\Sigma\,\triangle^2$		62.90			35.55			37.76			39.24	

We find, therefore, the following series of values corresponding best to the series of observations: —

$$M = 74.0; \quad U = 1457.1; \quad \frac{c\,M}{\sqrt{2\,\pi}} = -\,0.057.$$

It is clear that this method gives the more satisfactory results the greater the number of observations. If the number of observations is small, a slight change in the value of M may change any single value so much, that the regularity of the series $\Sigma \triangle^2$ is so much affected that the point where this sum becomes a minimum cannot be determined very accurately, although it may be possible to find it very nearly by assuming a sufficiently long series of M on both sides of the probable value and applying graphical methods for finding the minimum. The differences between the average of all statures and the stature of the average child of a certain age is quite considerable. I have computed these values for the ages of 11, 12, and 13 years, of girls.

Girls: 11 years.	Stature, Average:	1370.0	$U = 1386.9$	$\triangle = +16.9$
" 12 "	" "	1446.6	1457.1	+ 10.5
" 13 "	" "	1494.2	1506.5	+ 12.3

As might have been expected, the statures during a period when the rate of growth is decreasing, are higher than the averages of all statures. This difference will continue until the adult stage is reached. It becomes also probable that the average individual does not grow as long as the tables of averages seem to indicate.

FRANZ BOAS.

Clark University, Worcester, Mass., April 25.

THE BROOKLYN INSTITUTE AND POLITICAL SCIENCE.

THE Brooklyn Institute of Arts and Sciences is an institution that has earned a national reputation for its unique and successful educational work. Founded in 1824, it began five or six years ago, under the direction of Professor Franklin W. Hooper, a career of greatly increased usefulness and influence. To-day it has nineteen hundred subscribing members, organized in twenty-five departments of work, a property valued at $250,000, and an annual income from membership fees of upward of $11,000.

The membership of the institute, while it includes a considerable number of distinguished specialists in the various departments, is largely made up of people of general culture, and of young men and women who, without being able to continue their studies in college, are intelligent and thoughtful, and interested in one or more departments of study. The largest and, considering the standing of its members in the community, the most influential of all the departments of the institute is that of political and economic science, organized in December, 1889, with Professor Richmond Mayo-Smith, the specialist in statistical science of Columbia College, as its first president.

This department has already done a most excellent work in Brooklyn, in its department meetings, its courses of lectures upon subjects in political science, and in the addresses of distinguished speakers, given under its auspices, upon occasions of wide popular interest. It is largely to the stimulating influence of this work during the last three years, that the proposition, recently made to the department, to establish a school of political science, is due. Excellent as the lectures and anniversary meetings of the department have been, the members now demand something more systematic and specialized.

The plan proposed contemplates the ultimate establishment of a fully equipped school of political science with elementary and advanced courses in civil government, political economy, social science, and history, at nominal rates for tuition. The proposition to establish such a school was enthusiastically received at the recent annual meeting of the department; the only question now is as to the proper ways and means for putting the plan into practice.

It is evident that there are grave difficulties in the way of the successful carrying out of such a project. The lack of uniformity in the acquirements of the membership of the institute, and the influences tending to interfere with a faithful attendance upon courses once begun are not so great obstacles as the difficulty of finding instructors with the qualifications requisite for this particular work. The executive committee of the department, to whom the whole matter was entrusted with power to act according to their judgment in the matter, will not be likely to move hastily. Should sufficient encouragement be offered in the way of a moderate endowment, the school may be opened in the fall, and courses in some of the above mentioned subjects offered for 1892–93.

PREPARATION FOR THE STUDY OF MEDICINE.[1]

INCOMPLETE is a discussion of this subject that does not include a consideration of the great value of an elementary knowledge of Latin and Greek.

I here most seriously disclaim any attempt to prove that devotion to Latin and Greek for the purpose of reading the literature of these languages is either requisite or even desirable as a preparation for the study of medicine. The field of modern literature and of modern science has become so vast and important that the average student will find neither time nor relative profit in the attempt to *master* the ancient classics.

I do, however, earnestly advocate the study of the rudiments — I mean simply the rudiments — of Latin and Greek, as most valuable labor-saving instruments in acquiring an English, a scientific and a medical education.

I ask indulgence, if I dwell somewhat at length on this portion of my subject, for I think we are in danger of losing sight of the many and great benefits, which every true student will receive from a judicious study of some things in

[1] Address of President E. L. Holmes Rusk, Medical College, Chicago.