while Euler, Peacock, De Morgan, and others have developed it more as a double algebra.

Up to this point i had been regarded as a scalar operator merely, and the corresponding geometry only plane, though attempts had been made without much success to extend the treatment into three-dimensional space. It remained for Hamilton to accomplish this by the simple device of making i a directed operator, or handle, perpendicular to the plane of rotation, which opened the way for any number of similar operators differing in direction, but, as to their other properties, simply square roots of minus one. In order to produce a convenient algebra on this basis, Hamilton was obliged to take the further step of giving to all vectors the properties of $\sqrt{-1}$, and thus the calculus of quaternions was produced, a non-commutative quadruple algebra. These ideas have been generalized still farther by Unverzagt in his "Theorie der goniometrischen und der longimetrischen Quaternionen." In this book the author first develops a trigonometry based on a general instead of a right-angled triangle, and then shows that the operator $j = (-1)^{\frac{\lambda}{\pi}}$ (in which λ is the fundamental angle, taking the place of $\frac{\pi}{2}$) takes in this trigonometry the place of i in De Moivre's theorem generalized. He then takes three units j_1, j_2, j_3, corresponding to Hamilton's i, j, k, and forms a generalized quaternion, based on some angle λ, which reduces to the ordinary system when $\lambda = \frac{\pi}{2}$. The case particularly discussed is that in which $\lambda = 0$.

The theory and laws of linear, associative algebras, which includes quaternions as a particular case, have been thoroughly treated by Peirce in his work bearing that title.

We turn now to the other line along which multiple algebras have been developed. In 1827 Möbius published his "Barycentrische Calcül," in which points are the ultimate units, to which any desired weights may be assigned. He gave the laws of combination of these units so far as addition and subtraction are concerned, but did not proceed to multiplication: in fact, he distinctly states that they can be multiplied only by numbers. He then proceeds to treat analytical geometry on this basis. His treatment of points, so far as it goes, is on the same plan afterwards independently developed by Grassmann.

In 1844, one year after Hamilton's first announcement of his discovery, Grassmann published his "Ausdehnungslehre," which contains a complete and logical exposition of his new algebra for any number of independent units, and hence, geometrically interpreted, for space of any dimensions. This book was so abstract and general in form, and so unlike the ordinary language of mathematics, that it attracted hardly any notice, and the author was obliged to recast and republish it in 1862. Grassman's algebra is non-linear, and only partially associative, so that it differs fundamentally from all those discussed by Peirce. The $\sqrt{-1}$ plays no part whatever in the theory, and Grassman's vector is a vector pure and simple, i.e., a quantity having direction and magnitude, and not, as in quaternions, a *versor*-vector, combining the properties of a vector and of the $\sqrt{-1}$. The fundamental notion of Grassmann's multiplication is extension or generation; the product $p_1 p_2$ is the line generated by a point moving straight from p_1 to p_2, etc.

In this great invention of Grassman we have a multiple algebra which is the natural language of geometry and mechanics, dealing in a manner astonishingly simple, concise, and expressive with these subjects, and certain, it appears to me, to gain constantly in the appreciation of mathematicians as it is more generally understood and used. The fact of its perfect adaptability to n-dimensional space is an additional argument in its favor for those who are interested in that line of investigation.

We have now traced the development of our subject from its elementary beginnings through a long period in which it was in the rhetorical stage, approaching at intervals here and there to the syncopated; then, on the revival of learning in Europe after the dark ages, we have seen its comparatively rapid progress through the syncopated stage to the purely symbolical, when it was at last in a shape suitable for the astonishing progress of the last two hundred years. Finally, in the present century, we have noted the appearance, as in the fulness of time, of multiple algebras from different and independent sources, whose realm is that of the future.

NOTES AND NEWS.

THE astronomers sent to the Sandwich Islands recently on the part of the International Geodetic Association of Europe and the United States Coast and Geodetic Survey, in order to make a more exhaustive study of the changes of latitude, have located their observatories at Waikiki, near Honolulu. It is proposed to observe during the year about sixty-five pairs of stars, chosen on account of their well-determined proper motions, and to make in all not far from twenty-five hundred observations of the latitude. The results, compared with those made simultaneously in Europe and America, will settle definitely the question whether there is a real motion of the pole. At the suggestion of the American representative, the force of gravity will be measured every night that latitude observations are made. This may throw light on one of the theories proposed to explain the changes of latitude, viz., that of large transfers of matter beneath the earth's surface. The new pendulums made at the Coast and Geodetic Survey Office in Washington, and which are similar to those taken to Alaska by Professor Mendenhall last spring, will be employed at Waikiki. They are of fine workmanship, and are capable of detecting changes that do not exceed one hundred-thousandth part of the quantity measured. Besides the observations at the regular station, a number of magnetic determinations will be made at other points in the Islands, — notably at Kealakeakua Bay, where Captain Cook observed the declination more than a hundred years ago, and at Lahaina, where De Freycinet had an observatory for pendulum and magnetic work in 1819. The re-occupation of these points will show the change of the needle during the past century, and will be of great value in determining the secular variation. It is intended also to seize the opportunity now presented to measure the force of gravity on the summit of Mauna Kea (14 000 feet elevation). Observations made at the top of Haleakala (10,000 feet) in 1887 showed conclusively that the mountain was solid. This fact received additional support from the zenith observations at the sea-level north and south of the mountain. The large deviation of the plumb line (29″) brought to light in that work has now been exceeded on Hawaii, where 1′ 26″ has been discovered at the south point of the island (Ka Lae). This fact, recently communicated by Surveyor General Alexander, makes the question of the force of gravity at the summit of Mauna Kea one of double interest, and it is desirable, both from a geological and geodetic standpoint, that pendulum observations be made on top of one of the mountains. Doctor Marcuse, who is from the Royal Observatory at Berlin, observes for latitude on the part of the European association, and Mr. Preston, who made the observations at the summit of Haleakala four years ago, is from the United States Coast and Geodetic Survey, and makes gravity and magnetic determinations. He also, as the representative of the United States, observes for latitude in connection with Dr. Marcuse, in the international geodetic work. The observers had the good fortune to arrive at Honolulu on the day preceding the transit of Mercury (9th of May), and made successful observations of the phenomenon. The second contact was also observed by Mr. Lyons of the government survey. The two interior contacts were noted by local mean time (Waikiki 8″ east of Honolulu) as follows: —

	H.	M.	S.	H.	M.	S.
Mr. Lyons..................	1	26	32	—	—	—
Mr. Preston................		26	53	6	10	50
Dr. Marcuse...............		27	3		11	22

The station was in latitude 21° 16′ 21″ north, and in longitude 157° 49′ 30″ west. The mean observed times of contact are in both cases about a minute less than the computed ones.