form;" that is to say, by means of the elementary functions of analysis. But though the importance of this problem for practical purposes must be acknowledged, the problem itself, understood in this form, is in general an impossible one.

The modern theory, inaugurated by Briot and Bouquet's and Fuchs's discoveries, has reversed the whole problem. It considers the differential equation (together with a proper number of initial conditions) as defining a function, and proposes to derive directly from the differential equation the characteristic properties of its integrals, true to the general principle of the theory of functions, that the essential thing about a function is not its form, which usually may be varied in many ways, but the totality of its characteristic properties.

It is in particular the theory of linear differential equations that has been very fully considered from this standpoint; and there is scarcely any branch of mathematical science that has attracted a more general attention in our day, and in which more important discoveries have been made, than the theory of linear differential equations. Still every one who wished to become familiar with it, and who had to work his way through the vast and difficult literature on the subject, has keenly felt the want of a systematic exposition uniting the numerous researches scattered in the different mathematical journals and publications of learned societies.

To meet this want, and to give an account of the theory as it stands to-day, is the object of the "Treatise on Linear Differential Equations," by Professor Thomas Craig of Johns Hopkins University. The first volume, which is to be followed by a second one, is entitled "Equations with Uniform Co-efficients," and deals principally with Fuchs's theory and the investigations immediately connected with it. The rich material has been carefully sifted, and is presented in a clear and intelligible language in the most natural order of ideas.

An introductory chapter gives the general properties of a system of linear differential equations of a more formal character, among others the well-known theorems on systems of independent particular integrals.

Next follows an elegant exposition of the theory of linear differential equations with constant co-efficients, where the reader will find, besides Euler's solution, an account of various ingenious methods due to Cauchy, Hermite, and others.

After these preparations, we are led, in Chapter III., into the very centre of the modern theory; viz., the determination of the form of the integrals in the region of a critical point. It is first shown, that, if the differential equation be written in the form

$$\frac{d^n y}{dx^n} + p_1 \frac{d^{n-1} y}{dx^{n-1}} + \ldots + p_n y = 0,$$

the critical points of any one of its integrals are always found among the critical points of the system of co-efficients, $p_1, p_2 \ldots p_n$. Then Fuchs's theorems concerning the form of the integrals in the region of a critical point are developed with all the details about "groups of integrals" added by Hamburger, Floquet, and others.

A particular integral is said to be regular in a critical point a, if it remains finite for $x=a$ after multiplication by some proper power of $x - a$; and, in order that all the integrals may be regular in a, it is necessary and sufficient that $(x-a)^\alpha p^\alpha$ $(\alpha = 1, 2 \ldots n)$ be holomorphic in a. Chapter IV. contains an account of Frobenius's elegant treatment of this case, and gives a simple criterion for the non-appearance of logarithms.

The next chapters are devoted to that important class of differential equations (called regular equations) all of whose integrals are regular in all the critical points; and the fertility of the general methods is abundantly shown in the application to the equation of the second order, in particular that with three critical points, which, on account of its high importance, is very fully treated, with many interesting results concerning Riemann's P-function, spherical harmonics, Bessell's functions, etc.

The differential equation of the hypergeometric series, to which the above equation can always be reduced, takes such a central place in recent mathematical researches that it well deserves to be considered with all detail, as is done in Chapter VII., which contains a reproduction of Goursat's "Thesis on the Hypergeometric Series."

The theory of irregular integrals is still in a very imperfect state. Chapter IX. gives an account of Frobenius's and Thomé's researches, and the same subject is treated in Chapter X. by the elegant method of decomposition of a differential quantic into symbolic prime factors. Interesting special classes of irregular equations will be found in the chapters on Halphen's equations, and on equations with doubly periodic co-efficients.

The two remaining chapters might, it seems to us, as well have been reserved for the second volume, where the same subjects will be more fully dwelt upon. Still the two conceptions of group and of invariant of a differential equation which they develop are of so fundamental importance that they can scarcely be introduced too soon.

If the co efficients of a linear differential equation are uniform functions of x, any system of n independent particular integrals submit to a homogeneous linear substitution when the variable point x describes any closed path in its plane. The entire system of substitutions obtained in this way forms a group, called the "group of the differential equation."

The notion of "invariant" of a linear differential equation, on the other hand, arises when the given equation is transformed into another of the same form by the introduction of two new variables, and its definition is analogous to that of an invariant of an algebraic quantic.

We must confine ourselves to these few indications, and refer the reader to the book itself for further information. Only then will he obtain an adequate idea of the thoroughness and completeness with which the subject has been treated. As far as we are able to judge, no investigation of any importance has been omitted, and the justice and conscientiousness with which continually reference to the original papers is given are a characteristic feature of this most valuable book, which, we are sure, will contribute a great deal to spread the knowledge of this important discipline.

We look forward with much interest to the appearance of the second volume, which will contain, among other things, an exposition of the theory of linear differential equations with algebraic integrals, and of Poincaré's theory of Fuchsian groups and Fuchsian functions.

AMONG THE PUBLISHERS.

THE Bulletin of the Ohio Agricultural Experiment Station for October, 1889, is Vol. I, No. 1 of a technical series, and contains three articles by Clarence M. Weed, — "Preparatory Stages of the 20-Spotted Ladybird," "Studies in Pond Life," and "A Partial Bibliography of Insects affecting Clover."

— The opening article in the December number of *Outing*, "Wabun Anung," by F. Houghton, is a clear description of a tour in the region of the Great Lakes. Another article is the "Merits and Defects of the National Guard," by Lieut. W. R. Hamilton. We note further the "Game of Curling," by James Hedley; "Wheeling through the Land of Evangeline;" "Game Protection;" "Instantaneous Photography," by W. I. Lincoln Adams; "Women and their Guns;" "The Yale Stroke;" "Alligator Shooting in Florida;" and "Na-ma-go-os," a fishing sketch.

— John Wiley & Sons have just published "A Hand-Book for Sugar Manufacturers and their Chemists," by Guilford L. Spencer of the United States Department of Agriculture. The volume contains practical instruction in sugar-house control, the diffusion process, selected methods of analysis, reference tables, etc. The essential requirements of a thorough chemical control and superintendence of a sugar-factory are carefully described, and only such analytical processes are given as relate to sugar-house products and the waste residues when necessary to a complete control. Technical chemical terms have as far as possible been avoided. The little book ought to stimulate our sugar-manufacturers and their chemists to more extensive investigations and more thorough work.

— Messrs. Ginn & Co. announce for publication early in December the first volume of a serial entitled "Harvard Studies in Classical Philology," edited by a committee of the classical instructors of Harvard University. It is the expectation that one volume,

containing about two hundred octavo pages, will be issued each year. The contributors will be, for the most part, instructors in the university, or graduates of the same, but contributions of other scholars will not be absolutely excluded. Any correspondence respecting contributions should be addressed to Professor James B. Greenough, Professor John Williams White, or Professor F. D. Allen, Cambridge, Mass. Subscriptions (one dollar, four marks, or five francs a volume) may be sent to Otto Harrassowitz Leipzig, Germany; Ginn & Co., 57 & 59 Ludgate Hill, London E.C., England; or the latter firm at Boston, New York, or Chicago

— With the December number the *Magazine of American History* completes its twenty-second volume. The frontispiece to the current issue is a portrait of Lord Brougham; and the opening paper by the editor is a sketch of his early career during the infancy of our Republic, with pen-pictures of his contemporaries and surroundings, the establishment of the *Edinburgh Review*, and the marriage of its editor in New York City. The second illustrated paper is a "Tribute to Hooper C. Van Voorst," the late president of the Holland Society, by George W. Van Siclen. The third contribution is "The Story of Brave, Beautiful Margaret Schuyler," an historic ballad from the pen of Judge Charles C Nott of Washington. Curiously interesting is the article following, of R. W. Shufeldt, "The Drawings of a Navajo Artist," illustrated with the Indian pencil; as is also the "Acrostic by John Quincy Adams," in facsimile, from Ella M. M. Nave. "The Sciota Purchase in 1787," by Col. E. C. Dawes of Cincinnati, and the "Private Contract Provision in Ordinance of 1787," by Hon. W. P. Cutler, are important contributions to the number. These are ably written, and will doubtless serve to correct many errors in recent histories of Ohio. "Joseph Hawley, the Northampton Statesman," is the theme of a paper by Charles Lyman Shaw; "Fort Perrot, Wisconsin," is from T. H. Kirk; "First Editions of the Bible printed in America," from Clement Furgeson; and "Gen. Grant and the French," from Theodore Stanton of Paris. This magazine is steadily exerting an educational and healthful influence in all departments of literature and study.

LETTERS TO THE EDITOR.

***Correspondents are requested to be as brief as possible. The writer's name is in all cases required as proof of good faith.*

The editor will be glad to publish any queries consonant with the character of the journal.

On request, twenty copies of the number containing his communication will be furnished free to any correspondent.

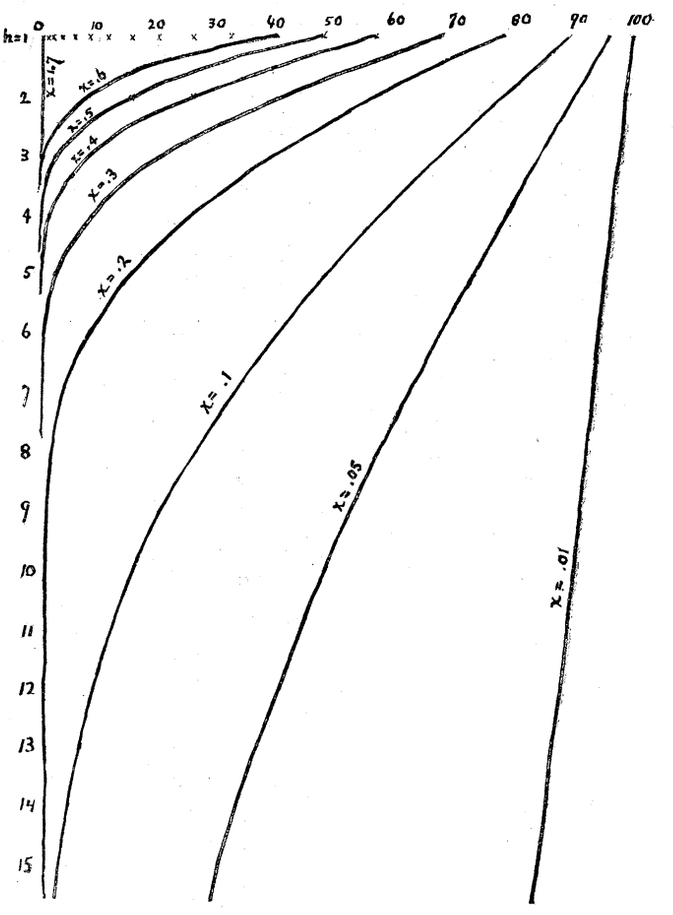
Intelligence of Ants.

I SEND you the following regarding ants, by Mr. W. E. Bosworth of this city, written out at my request, which seems to me an interesting and at the same time somewhat rare observation. It is almost exactly similar to the account by McCook of the sleeping of harvesting ants, of Texas, as quoted in G. J. Romanes' "Animal Intelligence," p. 84. I do not recall any other instance given of the sleeping of ants. "At different times, and for more than one season, I was favorably situated to see the movements of quite a large colony of small black ants, as they passed to and fro in their busy haste over a board floor, going, as I supposed, for their supply of water, which was in the direction of a small stream close by. While watching their quick, eager movements, there were several along the line that attracted my attention, as they remained in one place so long that I concluded they must be dead; and although they were directly on the line of march, and in the way of the others, these passed on, paying no attention to them whatever. At another time I noticed that one of the ants supposed to be dead got up, and walked off as lively as the rest; and, while watching this one, another one close by began to slow up, seemed to totter in his gait, and finally came to a dead halt. After seeing this, it occurred to me that the one had just waked up, and the other had just gone to sleep. In order to test the matter, and gratify my curiosity, I concluded to experiment on some of them. With a fine straw they were gently rubbed on the back. This mild treatment did not make the slightest impression on them; but a sharp push seemed to take them completely by surprise, and to fully arouse them. For an instant they seemed lost, circulating around, running up and down, but finally starting off with the rest. This was repeatedly tried with the same result. Their movements on being disturbed very forcibly reminded me of a child when suddenly waked out of sound sleep." JAS. LEWIS HOWE.

Louisville, Ky., Nov. 21.

Galton's Bodily Efficiency Diagram and the Marking System.

FRANCIS GALTON'S bodily efficiency diagrams (*Nature*, Oct. 31, 1889) can evidently be applied to the rating, on an arbitrary scale, of all sorts of things besides physical measurements and tests. For instance: the annexed diagram represents, by Galton's method, the rating of errors as the measure of precision gradually rises. The data were taken from the table on p. 12 of Merriman's "Least Squares" (first edition). The curves are drawn in general for values of x differing by .1; the ordinates in all cases being values of h, and the abscissas the rating on a scale of 100. The diagram shows at a glance how in all cases the rating of the same



error decreases as the measure of precision increases, but how, for very large and very small errors (see the curves $x = .01$ and $x = 1.7$), the measure of precision affects the rating little.

The rating of any errors which are distributed roughly according to the probability curve, as they are, for instance, in every school examination, ought to conform in general to these curves, and I think teachers usually strive to have it do so, either consciously or instinctively. If the error is flagrant, the question containing it is marked zero, or nearly so. The discrepancies in the marks of different teachers, or in the marks of the same teacher at different times, seem due to the different measures of precision mentally adopted. The curves show that these variations of the measure of precision affect most the rating of mediocre work, and this also accords with the experience of teachers. Now, of course the errors of each scholar have their own probability curve and their own value of h, which perhaps might be calculated from a long series of examination-papers. It would probably differ for different subjects. The custom, then, of marking good and poor scholars on different scales has a reason. The only question is, whether these scales can be so systematized as to be quite just, and whether it