gathers in support of the hypothesis of physiological selection, on the segregation of the fit. Domesticated varieties cannot show much evidence for physiological selection, because breeders keep their strains separate artificially, and this kind of variation is not in their interest. They do show very strongly, however, how important it is to prevent intercrossing with the parent forms if the varietal form is to maintain itself. It is hardly possible that a species could be formed without the prevention of intercrossing with other forms : it is even difficult to imagine any single variation so intensely useful as to resist the swamping effects of free intercrossing. In the natural state the variation in question would not be noticed until the process were over ; and so, as is the case with natural selection, the process cannot be directly observed. But it can be proved that the kind of variation which the theory requires does occur in nature and under domestication. If the season of flowering or pairing were advanced or retarded (and changes in the environment would frequently produce the result), the conditions for physiological selection would be given.

But physiological selection will be best shown in what may be termed 'spontaneous variability of the reproductive system.' Of this fact we have evidence in *individuals* (e.g., Mr. Darwin observes that "it is by no means rare to find certain males and females which will not breed together, though both are known to be perfectly fertile with other males and females"), in *races* (e.g., under domestication, "the yellow and white varieties (of Verbascum), when crossed, produce less seed than the similarly colored varieties" — *Darwin*), in *species* (for, as the distinction between varieties and species is of degree only, and as the main distinction is as regards mutual sterility, every instance of sterility between parent and varietal forms is evidence of the action of physiological selection).

Dr. Romanes then proceeds to show that "the facts of organic nature are such as they ought to be, if it is true that physiological selection has played any considerable part in their causation ;" and to do this he shows that the three cardinal objections to the theory of natural selection — namely, sterility, intercrossing, and inutility — find a ready explanation in the hypothesis of physiological selection. In this evidence it is brought out that in all probability the variation in the reproductive system is the primitive and distinctive one in the formation of species, and not that it was developed as secondary to another specific distinction in any other part of the organism. In addition, it is shown that the theory is capable of explaining why species have multiplied, and have not become transmuted in a linear series, and that the large body of favorable evidence furnished by the geographical distribution of organic life is perhaps the strongest argument for the truth of the theory. For the details of these points, reference must be made to the original paper.

A word as to the relation of the theories of natural and of physiological selection. It has already been noticed that the kind of evidence on which each depends is alike ; that the former deals with the origin of genera, families, orders, and classes, even more than that of species, while the latter relates to species alone ; that the former perpetuates useful distinctions alone, while the latter takes up the non-adaptive kind. It remains to add, that the two theories are in no way opposed to one another, but are complementary and co-operative. Without physiological selection, natural selection would be overcome by the adverse influences of free intercrossing : without natural selection, physiological selection could perpetuate no differences of specific type other than those of mutual sterility and trivial details of structure, form, or color.

In conclusion, Dr. Romanes suggests the following experimental verification of his theory, and asks the co-operation of observers in different geographical areas. The experiment consists in taking well-marked natural varieties of plants, and testing the relative degrees of fertility, first within themselves, and next towards one another ; in continuing the process "in successive years over a number of natural varieties, by carefully conducted artificial fertilization, and by counting the seeds and tabulating the results."

LAUNHARDT'S MATHEMATICAL ECONOMICS.

PROFESSOR LAUNHARDT has made what seems to us quite a notable contribution to the literature of mathematical economics in the volume before us. Whatever may be thought of the importance of investigations of this nature, it cannot be denied that the works of the principal writers on the mathematical theory of political economy — Cournot, Walras, Jevons, and perhaps others — are marked by insight as well as ingenuity, and in many respects by true scientific method as well as scientific form. They have nothing in common with that pseudo-science which we occasionally find endeavoring to conceal its emptiness behind a breastwork of mathematical formulas.

Professor Launhardt bases the theory of political economy on the Walras-Jevons idea of utility in

Mathematische begründung der volkswirthschaftslehre. By WILHELM LAUNHARDT. Leipzig, *Engelmann*, 1885. 8°.

relation to value. This may be indicated with sufficient precision in a brief space. One of the first points noticed by economists in the theory of value is that the exchange values of different commodities are not at all proportioned to their utilities. The theory advanced by Jevons — and Walras's is substantially identical with it — points out, that while it is true that the aggregate utility of the whole amount of a given kind of commodity has no relation to its exchange value, yet in a certain sense commodities do exchange in the ratio of their utilities. The total utility of different amounts of the same commodity is not proportional to the amount: as successive equal increments are added to the existing quantity, they add less and less to the aggregate utility. Now, what the theory asserts is, that the exchange value of any commodity is determined by the utility which would result from the addition of a small quantity of it to the amount already possessed. Thus commodities do not, indeed, exchange in the ratio of their *total* utility, but they do exchange in the ratio of their *final* utility; that is, of the utility of the last small portion produced, or, what is the same thing, of the next small portion that might be produced. The total utility, u, of the whole quantity, x, of a given commodity, is, then, given by an equation,

$$u = f(x),$$

which may be called the utility-equation; and the exchange value of the commodity is proportional to the derivative of u with respect to x. We might conceivably obtain the form of the utility-equation of any article from a study of its commercial statistics; but this has not been done for any commodity, and it may be doubted whether it ever can be done — with even the lowest tolerable degree of accuracy — unless, possibly, in some very peculiar cases. We do know, however, in practically every case, that $f(x)$ increases with x, but increases at a diminishing rate; that it is 0 when $x = 0$, and reaches a maximum for some value of x. This last point might at first sight be doubted, for it is equivalent to saying that for every commodity there is a point beyond which the quantity on hand cannot be increased without its becoming a nuisance; but it is plain that such a point does in general exist, though it may be very far beyond the quantity actually possessed.

What Launhardt has added to the work of his predecessors is chiefly the discussion of a large number of applications of the general theory, — a discussion which was in most instances made possible only by a special and arbitrary assumption concerning the form of the utility-equation. Since the function $ax - bx^2$ (where a and b are positive constants) is a very simple function, possessing the properties above mentioned as belonging to the utility-function, — viz., it is 0 when x is 0, then increases but at a diminishing rate, and reaches a maximum at a certain point, — Launhardt adopts it, stating at the outset that he would employ it for purposes of illustration, but insensibly falling into the way of deducing from the assumption of its sufficiency the greater part of his theorems. That the form is not sufficiently general for even the roughest approximation, despite the fact that the choice of different coefficients, a and b, gives a wide range for the different characters of different commodities, one may easily convince himself. The derivative of $ax - bx^2$ is $a - 2bx$: accordingly, the exchange value of a unit of any commodity would be a linear function of the entire quantity of that commodity available; so that, if we consider any three quantities, x_1, x_2, x_3, such that x_2 is the arithmetical mean of x_1 and x_3, the exchange value of the article when the quantity is x_2 would necessarily be a mean between its values when the quantity is x_1 and x_3. This is certainly not even approximately true for commodities in general; and this consideration alone would be sufficient to justify us in not accepting the form $ax - bx^2$ as sufficiently general for purposes of investigation. Indeed, as already stated, the author seems to have had no deliberate intention of so using it.

We have dwelt at some length on this point, because the most striking conclusions in the first section of the book — that devoted to exchange — are dependent upon it. One or two theorems of this kind may be quoted, and they will also serve to indicate the nature of the questions discussed by the author. The theorems are printed in italics, as embodying the net outcome of the mathematical investigations which precede them.

"When the merchant is so placed that he can fix his rate of profit at the point most advantageous to him, he obtains two-thirds of the entire economic gain accomplished by the exchange, or twice as much as the producer and consumer together.

"The most advantageous duty is therefore equal to one-third the difference between the price which the domestic goods would bring if there were no importation, and the price at which the foreign goods could be sold with no profit to the producer."

The simplicity of these results is equalled by their unreliability. It is not very surprising that a simple result should be reached from a mathematical hypothesis so much simpler than the facts warrant, even for the purposes of the purest theory; but, in spite of the small value of the results, the methods of arriving at them, often ingenious and depending on a refined analysis of the subject-matter, seem to us of decided interest to any who may be considering the part which mathematical methods are capable of taking in the development of economic science. We cannot here enter upon a discussion of this general question; but we may be permitted to say that we do not look forward to their giving important direct aid in the investigation of the fundamental questions of economics, though they may, when the science has reached a more advanced stage, be useful in the more minute discussion of special problems. In a certain indirect and incidental way, we think that mathematical inquiry may be useful even to the fundamental theory; for the necessity under which the mathematician lies, of clearly and exactly comprehending his premises, will doubtless in some instances bring about a more accurate view of economic phenomena. Upon the mathematical economists themselves, this necessity of accurate definition is apt to act in a most harmful manner, as their writings abundantly prove. When they have got hold of a notion which lends itself to mathematical treatment, the temptation is very great to unduly extend its province. Jevons's theory of utility in relation to value is a conspicuous example of these merits and defects. While the accurate analysis of some features of the phenomena of value which was a necessary preliminary to the mathematical discussion has been useful to economists in general, the results reached by the mathematical theory are open to the gravest objections; and this quite apart from any subsidiary defects, such as those occurring in some of Launhardt's discussions, as pointed out above. In the mathematical development of the theory, its exponents overlook two capital points, — first, that, under a *régime* of separation of employments, the direct utility of a product to its producer has little or no significance; secondly, that, when an addition to the amount of a given commodity supplies with it a new class of individuals who formerly could not possess it, the utility thus arising is very different — and, if measurable at all, its amount follows a very different law — from that which arises from an increase in the quantity possessed by those who were already provided with the commodity.

We have not left ourselves space to speak of other points, some of them very interesting, in the section on exchange, nor to make more than a passing mention of the other two sections, on production and transportation respectively. On the subject of money, the author takes, in our opinion, a very erroneous view. In the section on transportation, the mathematical premises come nearer than almost anywhere else to a representation of the actual problem: a large part of the questions there discussed are, in fact, such as are necessarily considered in an essentially mathematical way, though doubtless with little scientific method, by railroad managers. A satisfactory idea of the book can only be obtained by reading it. For the benefit of those who may contemplate doing so, we may state that a knowledge of the first elements of the differential calculus will make the little volume of two hundred pages sufficiently easy reading.

THE POPULATION OF MEDIAEVAL CITIES.

SOCIAL science has certain problems of reconstructing past conditions out of fragmentary remains, which are analogous to that reconstruction of terrestrial life and conditions which has been the triumph of modern natural science. History does not now content itself with a mere narration of events, but strives to portray the whole social condition of the people, — to give a vivid picture of society as it existed at the time. Modern historical writing has accomplished this to a greater or less extent, and the result is that our histories are histories of the people rather than of dynasties.

In one particular, however, this reproduction is incomplete. The historians do not give us exact statistical details of the relations of population, industry, commerce, etc., without which any description of a modern community would be considered entirely incomplete. It is impossible for them to do so, because such statistical investigations are entirely modern, most of them reaching back only to the beginning of this century. In former times there were no statistical bureaus, no census of the people, no returns of trade and commerce. There was no demand for such information, either for governmental or scientific purposes. It is notorious that ancient and mediaeval writers had no sense for numbers. The figures they give of the strength of armies or the population of cities are mere estimates, and on the face of them are often obvious exaggerations. One of the most difficult problems the historian has before him, is to weigh the statements of different writers as to the number of people concerned in any event, and very few purely literary historians have the requisite scientific training for such work.

The pure historian must here appeal to the professional statistician for help. The acute and learned work of which we give the title is an example of what German industry can accomplish

Die volkszahl deutscher städte zu ende der mittelalters und zu beginn der neuzeit. Von J. JASTROW. Berlin, *Gaertner*, 1886. 8°.