ematics, which nothing but publication in full could render intelligible, and that only to the mathematicians among our readers. To such, its full publication in the 'Proceedings' will prove of the greatest value.

The fourth part of the paper was devoted to consideration of some of the applications of multiple algebra. From this we quote the following: "First of all, geometry, and the geometrical sciences which treat of things having position in space, — kinematics, mechanics, astronomy, crystallography, — seem to demand a method of this kind, for position in space is essentially a multiple quantity, and can only be represented by simple quantities in an arbitrary and cumbersome manner. For this reason, and because our spatial intuitions are more developed than those of any other class of mathematical relations, these subjects are especially adapted to introduce the student to the methods of multiple algebra. Here nature herself takes us by the hand, and leads us along by easy steps, as a mother teaches her child to walk. In the contemplation of these subjects, Möbius, Hamilton, and Grassmann formed their algebras, although the philosophical mind of the last was not satisfied until he had produced a system unfettered by any spatial relations. It is probably in connection with these subjects that the notions of multiple algebra are most widely disseminated. Maxwell's 'Treatise on electricity and magnetism' has done so much to familiarize students of physics with quaternion notations, that it seems impossible that this subject should ever again be entirely divorced from the methods of multiple algebra. I wish that I could say as much of astronomy. It is, I think, to be regretted, that the oldest of the scientific applications of mathematics, the most dignified, the most conservative, should keep so far aloof from the youngest of mathematical methods; and standing, as I do to-day, by some chance, among astronomers, although not of the guild, I cannot but endeavor to improve the opportunity by expressing my conviction of the advantages which astronomers might gain by employing some of the methods of multiple algebra. A very few of the fundamental notions of a vector analysis, the addition of vectors and what quaternionists would call 'the scalar part and the vector part of the product of two vectors' (which may be defined without the definition of the quaternion), — these three notions, with some four fundamental properties relating to them, are sufficient to reduce enormously the labor of mastering such subjects as the elementary theory of orbits, the determination of an orbit from three observations, the differential equations which are used in determining the best orbit from an indefinite number of observations by the method of least squares, or those which give the perturbations when the elements are treated as variable. In all these subjects, the analytical work is greatly simplified, and it is far easier to get the best form for numerical calculation than in the use of the ordinary analysis."

Then followed illustrations of the various methods of applying multiple algebra to different classes of problems, and the paper closed as follows: "But I do not so much desire to call your attention to the diversity of the applications of multiple algebra, as to the simplicity and unity of its principles. The student of multiple algebra suddenly finds himself freed from various restrictions to which he has been accustomed. To many, doubtless, this liberty seems like an invitation to license. Here is a boundless field in which caprice may riot. It is not strange if some look with distrust for the result of such an experiment. But the further we advance, the more evident it becomes that this, too, is a realm subject to law. The more we study the subject, the more we find all that is most useful and beautiful attaching itself to a few central principles. We begin by studying 'multiple algebras;' we end, I think, by studying 'multiple algebra.'"

SEAT OF THE ELECTROMOTIVE FORCE.

PROFESSOR BRACKETT'S address was essentially a résumé of the history of the investigations to find the source of the current in galvanic batteries. No attempt was made to settle the question, which has been so long a bone of contention.

The address was so purely historical in its nature, and, withal, was so condensed and concise, that any abstract would be necessarily little more than an index of its contents. Those who are interested in the subject must await its publication in full in the 'Proceedings' of the association.

Galvani's two accidental discoveries were made in 1789: the one was the influence of an electrical machine in causing contractions in a frog's legs, and the other the production of sufficient electricity to cause the contraction by touching two joined strips of copper and zinc to the moist animal tissues. Naturally from these results there arose a theory of the identity of nerve-force and electricity, — the so-called animal variety of electricity. While this controversy, soon to subside, was started among physiologists, a much more

Abstract of an address delivered before the section of physics of the American association for the advancement of science at Buffalo, Aug. 19, 1886, by Prof. C. F. Brackett, of Princeton, vice-president of the section.

violent one has continued to rage among physicists. Is the electricity of the galvanic cell due to chemical action or to contact of dissimilar substances? It is to the history of the attempts to answer this question that the address is devoted.

PROGRESS OF MECHANICAL SCIENCE.

THE recent enlargement of the scope of this section to include all branches of engineering, and the increasing interest manifested in its meetings, warrant my making some remarks as to the true objects of the section, and the means of increasing its usefulness in the future.

In marked contrast with the past, the present age is one of pronounced material development. Formerly the brightest and most gifted men devoted themselves to religion, philosophy, politics, exploration, art; but for the past hundred years the attention of the leading men of the civilized world has been directed to increasing and cheapening those products which minister to the daily life and comfort of man. Farmers, mechanics, and laborers live now more comfortably than did the middle classes of feudal times; the duration of human life has been materially lengthened, and all portions of society recognize the importance of further progress, and the advantage of organization and invention in securing it.

This era of material progress may be said to have commenced with the final perfecting of the steam-engine, which, together with the various attendant machines, takes the place of hand and animal labor, and which has increased and cheapened the production of the necessaries and luxuries of life; and it has pushed the inventor and the engineer to the front rank in modern society. It may be useful to point out the absolute necessity of verbal and written intercourse between investigators and inventors, that the speculation and curiosity of the former may ripen into the effective invention of the latter. Nothing is more remarkable than the multitude of minds and facts which are required for the perfecting of even a simple machine, nor how little the last man may need to add to complete the invention. Facts and natural laws, known for years as curiosities, are taken up by some inventor, who fails in the attempt to render them of practical use; then a second genius lays hold, and, profiting by the mistakes of the first, produces, at great cost, a working machine. Then comes the successful man, who works out the final practical design, and, whether making or losing a fortune, he yet permanently benefits mankind.

The faculties of invention and discovery are generally separate. One set of men observe facts, and deduce laws therefrom; and another set endeavor to turn the results of this observation and deduction to practical account in the production of labor-saving appliances. This section should be the place where these men may meet one another, and profit by the interchange of ideas. Many of the men whom I see before me are devoting their lives to the study of nature, with no desire to make money out of it, but simply to increase human knowledge; and some of their discoveries will eventually be put into practical shapes for the use and convenience of man. History proves, too, that the scientific observers have the safer and happier part. Their success may not be so dazzling as that of some great inventors, but they do not have to bear such bitter trials and disappointments. To deduce natural laws requires mental accuracy in observing and reasoning; to make them useful in doing the world's work requires imagination and ingenuity. Sometimes long years must pass, and generation after generation of inventors wear their lives out, before a needed machine becomes an accomplished success. Evidently, then, the greater the number of minds that can be brought to bear upon a particular problem, the greater is the chance of early success. I believe that it is the particular province of this section of the association to bring these two classes of minds together, and to promote their intercourse, that the discoverer may learn in what direction fresh information is needed, and that the inventor may be advised as to what is already known.

The well-worn history of the steam-engine gives us an instance of an invention which did not spring full-grown from the brain of the inventor. History informs us that it commenced to exist two thousand years ago, in the eolipile of Hero of Alexandria. His treatise remained hidden until translated and printed in 1547; and then Branca, the Italian architect, constructed one for pounding drugs. Hero's book ran through eight editions in different languages, and attracted the attention of a French inventor, who tried vainly to raise water by steam pressure. Then came the Marquis of Worcester, who died a disappointed man after spending $250,000. Then de Morland tried using steam in cylinders, instead of in contact with the water; Papin built a steamboat, only to have it seized and destroyed while on its way to England, and he, too, died broken-hearted and poor; Savery went back to using the steam directly in contact with water; and finally Newcomen

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