SCIENCE.—SUPPLEMENT.

FRIDAY, JANUARY 1, 1886.

THE STUDY OF GEOMETRY.

WE have a pernicious habit in this country of supposing, that, because in a republic all men are born equal as to their rights, they are also born equal as to their abilities. We have a different theory in regard to horses: we know that a race-horse is altogether different from a dray-horse, and we give him a totally different kind of life from the beginning. We have no trouble in recognizing him: we simply inquire who were his ancestors, and our expectation as to his qualities is carefully based upon the answer to that question. It would, perhaps, be a good plan if the young of the human species were divided into two groups at an early age, — one large and one small; one composed of those of whom nothing more than plain living is expected, and the other composed of the race-horses, of those whose ancestors, or whose chance endowments, give reason to hope that they may give some aid to learning or to culture.

There is, at all events, no reason why all young people should be taught geometry in the same way. For most children, a form of reasoning so abstract is not only repulsive, but very nearly impossible of comprehension. A little may be done for them (or for their descendants) by giving them a small dose of geometry, made as plain and easy and direct as it can be made; but they do not need to know every thing that can be done with the straight line and circle. Life is short, and the whole content of geometry as known to Euclid is long. For most children in schools, a good specimen of the kind of reasoning, and a fair knowledge of the principal results, are all that is desirable. For such, a geometry like Wentworth's serves a very good purpose.

But it is a pity that the kind of geometry a person is taught should depend upon his geographical position near this or that kind of a school. Any one whose destiny is to do difficult thinking in after-life should have a different kind of early training: he should dwell long among the geometrical concepts, should become thoroughly imbued with the bare and rigid form of reasoning, and should have the results as familiar as his mother-tongue. It is a serious loss to him if he is made to run over the subject with uncouth haste. Students of this kind will find their natural guide in such a text-book as Newcomb's or Halsted's.[1] In neither is it the aim to give the most rapid and cursory system possible. Both are written from the stand-point of the modern idea that the geometry of this world is not the only possible geometry, and that it is mere matter of accident that two parallel lines do not approach each other, and that two straight lines do not enclose a space. Both have felt the influence of the syllabus of the English association for the improvement of geometrical teaching. The idea of figure is shorn of its material content, and limited to its bounding lines or surfaces. The sum of two right angles is not regarded as a purely imaginary idea with no reality corresponding to it, but the 'straight angle' is allowed to play its natural part. In Professor Newcomb's book, nis favorite idea is carried out of leading up to new and strange conceptions by very slow and gradual steps: Mr. Halsted's is intended for boys[2] of much more highly developed minds. There are no concessions to youthful weakness. It is also intended for boys of well-developed taste in the art of book-making. It presents a splendor of paper and of margin which is far removed from the republican simplicity of our ancestors.

The ancients believed that the geometrical concepts came down from heaven, but that the chief end of geometry was to measure the earth. We admit now that the concepts are, in the first instance, of the earth and earthy; but we have given an enormous development to the geometry of pure position, and have made it as remote from all possibility of application as the theory of numbers itself. It is in consonance with this development that in both these books measurement is given somewhat the position of an appendix to the subject, instead of being made to appear as the end towards which all the propositions lead up.

Mr. Halsted does an excellent thing in giving an introductory chapter on logic. When pure reasoning is about to become the student's daily occupation for many months, it is a pity not to give him a general view of the processes involved at the start. It

[1] *The elements of geometry.* By GEORGE BRUCE HALSTED. New York, *Wiley*, 1885. 8°.

[2] As a synonyme for 'student of geometry,' one should, however, say *girl* with the understanding that boys are to be included. Geometry is chiefly studied in the high schools, and the high-school graduates number three girls to every boy. If geometry is as good a specific against bad reasoning as is commonly supposed, logicalness will soon become a feminine instead of a masculine characteristic.

is very curious to find a compendium of logic with the syllogism left out. Hamlet is even less necessary to his play than the syllogism to logic. It is true, however, that the syllogism is an easy matter compared with inversion and contra-position. There is hardly a boy who is not greatly surprised to find that when he has proved that an isosceles triangle has two equal angles, it still remains to be proved that a triangle having two equal angles is isosceles. As De Morgan has pointed out, Euclid himself was apparently not aware that it follows every time from *A implies B* that *non-B implies non-A.*

In regard to 'his rule of inversion,' when three or more propositions are involved, Mr. Halsted has fallen into a slight inaccuracy. In the first place, if the term 'contradictory' is to be applied to three terms at all, it should be used in the same sense as when applied to two terms; the three terms should together cover the whole field, and they should not overlap. The word is a bad one for this purpose, however, and it is just as well to keep the two properties — that of being exhaustive and that of being incompatible — distinct.

In the second place, there is a redundancy in the rule as given by Mr. Halsted. From the three propositions,[1]

X implies x,
Y implies y,
Z implies z,

it may be inferred that

x implies X,
y implies Y,
z implies Z,

provided that *the subjects cover the whole field, and the predicates are incompatible.* It is not necessary that the subjects should be known to be incompatible, though it follows from the premises given that they are so, but also that the predicates are exhaustive. From the first two we have

X Y implies x y;

and, since there is no x y, there cannot be any X Y either.

It is very well worth while to have formulated the reasoning involved, instead of going through all the separate steps every time there is occasion for it, as the usual books on geometry do.

The conclusion does not follow if it is given that the subjects are incompatible, and that the predicates together fill the universe. The nature of the argument is most clearly seen in space. Lange believes that the logical laws of thought are derived from space-conceptions. Suppose there is a table painted in various colors, but so that

the red is all in the violet,
the yellow is all in the blue,
and the orange is all in the green;

[1] The letters stand for either terms or propositions.

and suppose, also, that the red, the yellow, and the orange together cover the whole table, and that the violet, the blue, and the green do not overlap: it follows that

red = violet,
yellow = blue,
orange = green.

To show how a somewhat complicated argument can be simplified by having this type of reasoning at command, we add a real illustration from algebra. In Descartes' method of solution of the biquadratic equation, the following relations are seen to hold between its roots and those of the auxiliary cubic: —

Roots of the biquadratic.		*Roots of the cubic.*
All real	*implies*	All real and positive.
Two real (unequal)	*implies*	One positive, two imaginary.
Two real (equal)	*implies*	One positive, two equal negative.
All imaginary	*implies*	One positive, two unequal negative.

But the division on the left is exhaustive, and the classes on the right are mutually exclusive: hence, by a purely logical *tour de force,* these propositions can all be inverted, and the desired inferences *from* the roots of the cubic *to* the roots of the biquadratic can be obtained at once.

Mr. Halsted's reviewers have pointed out before that he is deficient in a certain natural and becoming modesty. 'Two formative years' of his life is too high-sounding a phrase to be applied to any but a very great mathematician, like Professor Cayley, for instance.

CEREBRAL EXCITABILITY AFTER DEATH.

THE problems of brain physiology are so complex, and our means of studying them, especially in the human subject, so insufficient, that it is not to be wondered at if rather out-of-the-way and venturesome experiments are sometimes undertaken by the anxious physiologist; as, witness the actual stimulation of the exposed brain in a patient whose death seemed certain. Such an experiment is not apt to be repeated; and a few French physicians have now wisely set to work to study the results of stimulating the cerebrum, exciting the sense-organs, and subjecting the whole body to a vigorous examination in the case of criminals who have suffered death by decapitation.[1] Such investigations are not new; but the results have been, as a rule, either entirely negative, or brought out only a few rather obvious facts. In the experiments about to be described, the methods

[1] *Revue scientifique,* Nov. 28. By J. V. LABORDE.