manner not unlike that usually followed, and the belief is expressed that the moon has an effect upon the atmosphere which would appear by a proper tabulation of barometric observations.

The above summary is sufficient to give an idea of the scope of the work. It is introduced to the public by Dr. Julius Hann, who remarks, with regard to deductive investigations, "Even where results derived deductively find no immediate application in nature, since the actual conditions are never so simple as those which must furnish the basis of the conclusions reached, yet they are of great interest and value in advancing knowledge, since they increase our insight into the nature of phenomena, and open the path upon which, in the course of time, we shall attain to their complete understanding."

The execution of the author's design, however, is not wholly satisfactory. On account of the fragmentary nature of the work, it is often difficult to understand the bearing of the subjects discussed, or to see what use can be made of the formulae derived. It is also not always easy to follow the author in his argument, and consequently the general effect upon the reader is one of disappointment. The treatise does not merit the title which is given it, though it may furnish useful suggestions to those who are investigating the subjects which it discusses.

HISTORY OF LAND-HOLDING.

The early history of land-holding among the Germans. By DENMAN W. ROSS. Boston, *Soule & Bugbee*, 1883. 8 + 274 p. 8°.

THIS work of Mr. Ross starts from the principle of individual ownership and isolated farmsteads, as the primitive usage of the Germanic nations. The evidence for this the author finds in the sixteenth chapter of the Germania of Tacitus, in which he explains the *vici* to be villages, not of free tribesmen, as is generally assumed, but of serfs. Of community of ownership he finds no evidence, either in Caesar or Tacitus. In the period of the barbarian laws, too, the facts which have usually been understood to point to common or collective ownership he explains as meaning *undivided* property. He has no difficulty in proving the general prevalence of the principle of individual ownership at this latter period, so far as the laws and other documents of the period afford any evidence. That ownership is common wherever it appears in these documents, is as a rule temporary, and subject to individual claims, seems also fully established. The gap in the evidence is as to the two or three centuries which intervened between Tacitus and the barbarian codes, — a gap which is of no importance, if his interpretation of Tacitus is correct, but which leaves room, if that interpretation be not accepted, for the development of free village-communities in this interval, which may then, in some cases, have survived to a later period, by the side of the system of individual ownership which we must accept as the prevalent one for this period.

After developing these general principles, Mr. Ross proceeds (p. 26) to show how the isolated household may, in the course of a few generations, have developed into a clan-village; here, again, into a community of ownership which is not really corporate in character, but is on its way to divided and individual ownership (p. 38). The rules and usages of the inheritance and transfer of land are described with great fulness, after which the usages which appear to tell in favor of an original collective ownership — the rights of *vicini* to exclude strangers, to purchase in preference to strangers, and to inherit in case of lack of heirs — are discussed. Certainly these usages, which, it must be admitted, may accompany a system of private ownership, are, nevertheless, most easily explained on the assumption of a *previous* condition of collective ownership. We cannot think the explanation given on p. 52 to be wholly satisfactory.

The breaking-up of the clan-system is next considered, this being effected especially by female inheritance, adoptions, and alienations. An important topic is the founding of free colonies, off-shoots of the clan-communities, but modelled upon the serf-communities; and their organization and management are described with great fulness and lucidity. The relation between these free villages and the serf-villages — clan-villages of proprietors and of tenants — is discussed; and there is much here that would apply equally well to the village-community theory. They are indeed essentially the same in character with those assumed by that theory, only that they are represented by Mr. Ross as a later outgrowth instead of a primitive organization. The essay (which occupies 109 pages) ends with some brief considerations upon immunity, primogeniture, etc. The conclusions of the essay are supported by a mass of 'Notes and references,' occupying about 130 pages, and containing copious extracts from documents. There is a full index. This book is every way a thorough piece of

work, which certainly places the village-community theory upon the defensive, and overthrows a considerable part of its assumptions; and, apart from its controversial character, as a 'history of land-holding' it possesses the highest value.

WEEKLY SUMMARY OF THE PROGRESS OF SCIENCE.

MATHEMATICS.

Hyperelliptic integrals. — The full title of this paper by M. Staude is "Geometrische deutung der additionstheoreme der hyperelliptischen integrale und functionen erster ordnung im systeme der confocalen flächen zweiten grades." Only a brief notice of M. Staude's paper is possible in this place, although its importance makes it worthy of a much more extended one. The paper is divided into five chapters. In the first chapter the author considers the geometric significance of the symmetric algebraic functions of two independent variables, and the differentials of the integral functions of an hyperelliptic form (*gebilde*) of deficiency (*geschlecht*). The second chapter treats of the representation of the *gebilde* in systems of confocal surfaces by aid of hyperelliptic functions, and opens by the introduction of certain transcendental parameters in place of the usual elliptic co-ordinates. An expression is also given of the homogeneous point co-ordinates in space in terms of products of the double theta-functions, and also of homogeneous plane co-ordinates in space by aid of products of two double theta-functions. The third chapter is of particular interest from a purely geometrical point of view. In this the author considers the relations of the addition theorem for hyperelliptic integrals to systems of confocal surfaces, treating particularly the reduction of given sums of three integrals to sums of two integrals of the same kind. The fourth and fifth chapters have not yet appeared, but the author mentions their contents. Chapter four is to treat of the ray-systems of common tangents to two confocal surfaces; and chapter five is to be devoted to a geometrical interpretation of Abel's addition theorem, by aid of which the reduction of the sum of any four of the integrals in question to the sum of two integrals of the same kind is arrived at by a purely geometrical process. — (*Math. ann.*, xxii.) T. C. [471

Discontinuous groups of linear substitutions. — The complete title of M. Picard's paper is "Sur une classe de groupes discontinus de substitutions linéaires et sur les fonctions de deux variables indépendantes restant invariable par ces substitutions." The theory of the elliptic functions has given the first example of a uniform function of a variable which does not change for a group of an infinite number of linear non-permutable substitutions effected upon the variable. The modular functions, i.e., the functions arising from considering the modulus as given by the ratio of the two periods, was for the first considered by M. Hermite. M. Poincaré has treated in his theory of the Fuchsian functions, in all its generality, the subject of functions of one variable which are reproduced by a group of an infinite number of linear substitutions. M. Picard, in the present memoir, proposes to consider functions of *two* independent variables which may be considered as analogous to the elliptic modular functions. He shows, first, that the Abelian functions do not conduct to functions entirely analogous to the modular functions, and illustrates this by the Abelian functions of the first order. But by taking the case of the Abelian functions of the second order, i.e., of three variables, he has found an indication of the desired extension, and hopes in a future paper to enter more fully into the subject of functions of two variables which are analogous to the modular functions. The present paper is interesting as pointing out the difficulties, and indicating the manner of overcoming them, in an entirely new department of the theory of functions. — (*Acta math.*, i.) T. C. [472

PHYSICS.

Target-shooting. — From Liagre's theory that errors in target-shooting are compounded of errors in sighting and in levelling, each of which follow independently the law of error, it was shown by Mr. C. H. Kummell that shots of equal probability are arranged in ellipses, which can be reduced to circles of shots uniformly distributed, the integration being much simplified by using the reduced distances and directions. Sir J. Herschel's 'even-chance circle' (ellipse, more generally), the one hit or missed with equal probability, can be deduced from the shots actually found in any given circle (ellipse), the most reliable result being given by the one containing the greatest number of shots, whose radius (mean semi-diameter) is the *most probable* shot. The number of shots falling within this ellipse should be about thirty-nine and one-half per cent. The equations between the even-chance shot (ρ), the most probable shot (ε), and the average shot (r_0), are —

$$\rho = \varepsilon\sqrt{2\,l\,2}, \quad r_0 = \varepsilon\sqrt{\frac{\pi}{2}}.$$

In determining these from the sums of squares of the vertical and horizontal co-ordinates of the separate shots, the number that miss the target should be considered. The probable position of centre and axes should not be calculated from the observations, unless the true positions are unknown. A target of ninety shots at eight hundred yards' range, by the Irish team at Creedmoor in 1874, gave discrepancies of less than five per cent between observation and theory, in the number of shots within successive rings. One of fifty pistol-shots, at fifty yards' range, showed a similar agreement. — (*Phil. soc. Wash., math. sect.; meeting* Nov. 21.) [473