

sion to meet in reference to rocks of a different composition a few years ago.¹ M. Renard's line of argument would prove that a dike in conglomerate had the same origin as the conglomerate itself, — would prove, that, when sandstones and lava-flows are interbedded, both have a common origin. In any volcanic district we have mingled in inextricable confusion lava-flows, ashes, scoriae, dikes, and sedimentary rocks: are these all of common origin because they are associated? Is a lava-flow, buried by the seashore sands, of like origin with the sand? In our older rocks we have dikes cutting in every direction: are they the same as the rocks they cut?

The only proof regarding the origin of associated rocks is the relation that they bear to one another: the mere fact of association in itself is no proof.

In another respect M. Renard's argument is faulty, inasmuch as it assumes that all crystalline schists are of sedimentary origin. Eruptive are, as a rule, more subject to alteration than sedimentary rocks; therefore, in proportion to their abundance, they are more commonly found as metamorphic rocks than the others. One of the common metamorphosed characters of eruptive rocks is a schistose structure, and the mere fact that a rock shows such a structure affords no proof of its origin. The writer has seen a well-marked schist cutting in a dike directly across the stratification of a conglomerate, — it was, of course, a metamorphosed basic, eruptive rock, — and he has seen numerous other examples of a similar character.

The best evidence regarding the origin of the olivine rocks is in behalf of their eruptive characters, as M. Renard points out: on the other side, positive evidence seems to be wanting, it being rather a matter of personal opinion than facts. In such cases as those examined by Professor Bonney, and the one studied by the present writer on Lake Superior, the facts and evidence in behalf of their eruptive origin are clear and explicit. So far, then, as the mineralogical constitution of the St. Paul's rocks go, it points rather towards an eruptive than a sedimentary origin for them.

Indeed, did it not, it is difficult to see how any denudation could take place so far down in the sea, as is here required, when, as M. Renard admits, there is no evidence that any sinking has occurred.²

The writer would therefore hold that the St. Paul's rocks offer no evidence in favor of

their being the remains of a lost Atlantis; but rather that they are of eruptive origin, like the other Atlantic islands, although probably of earlier date than the prevailing rocks upon the latter.

M. E. WADSWORTH.

THE PASCAL HEXAGRAM.

THE Royal academy of Belgium in 1879, and again in 1881, offered its prize for a solution of the following question: "To extend as much as possible the theories of the points and lines of Steiner, Kirkman, Cayley, Salmon, Hesse, Bauer, to the properties which are, for higher plane curves and for surfaces and curves in space, the analogues of the theorems of Pascal and Brianchon (see, for these last, the writings of MM. Cremona, P. Serret, and Folie)." The only contestant for the prize in 1881 was Professor Veronese of the university of Padua, whose work on the subject of the original theorems was already well known. To the paper submitted by him, the Belgian academy, advised by its committee, consisting of MM. Folie, Catalan, and de Tilly, declined to award the prize; and the paper has, in consequence, been published in full in the *Annali di matematica* (xi., Dec., 1882, 143 p.) with the report of M. Folie, and a commentary thereon by Veronese. It is a controversy of unusual liveliness for a mathematical one. Before entering upon its merits, we shall give a summary of the memoir of Professor Veronese.

The extensions of the properties of the Pascal hexagram hitherto proposed have been special, and not general, and hence are incapable of being carried farther. When, for instance, the six perfectly arbitrary points on the conic are replaced by six generatrices of the hyperboloid, three must be taken from one system, and three from the other; and one gets, with this restriction, only a single pair of lines, corresponding to one conjugate pair of the twenty Steiner points. Cremona's extension to a cubic in space, on the other hand, can be obtained by simple projection from the hexagram in a plane conic. To develop these special, uninteresting, easy results would not be, according to Veronese, to answer the proposed question; so, leaving them one side, he proceeds to the application of a different method, — the theory of substitutions. His method is, in brief, to represent the six points on a conic by six values of a parameter, whose permutations give, from any figure whatever which they represent, seven hundred and twenty figures of the same kind, or a divisor of 720. If, for

¹ Proc. Bost. soc. nat. hist., 1880, xx. 470-479.

² See also Prof. A. Geikie, *Nature*, 1882, xxvii. 25, 26.

instance, the parameters are the homogeneous co-ordinates of a point in a five-dimensional flat, one gets, by permuting them, seven hundred and twenty points, which correspond in twelves to the sixty Pascal lines. The analogy is precise; for the two figures have the same algebraic base, namely, the substitutions. In his former paper, Veronese forms, for convenience, out of the six fundamental points, fifteen triangles, and, out of the sixty Pascal lines and Kirkman points, six configurations *II*, consisting each of ten Pascal lines and ten Kirkman points, poles and polars with respect to a conic. He finds, that, in a five-fold flat, to the triangles correspond fifteen surfaces of the second order in four dimensions; to the sixty Pascal lines, sixty surfaces of the fourth order in three dimensions; to the twenty Steiner points, twenty surfaces of the sixth order in two dimensions; and, to the six figures *II*, six configurations *II*, represented in the theory of groups by the six remarkable six-valued functions found by Serret (*Liouville*, 1850). As a sample of the vast multitude of propositions given concerning these figures and spaces, we may take the following: the seven hundred and twenty points obtained by permuting the six co-ordinates form a hundred and twenty cycles of six points on rational curves of the fifth order. They lie in sixes on conics in twenty-four hundred planes, which pass by hundred and twenties through the twenty intersections of the space unity with the faces of the fundamental pyramid. They are in twenty-fours in four hundred and fifty threefold spaces, which go by thirties through the intersections of the space unity with the fifteen threefold faces of the fundamental pyramid; and in hundred and twenties on thirty-six fourfold spaces, which go by sixes through the intersections of the space unity with the six fourfold faces of the fundamental pyramid. Such properties as these are simple and interesting in space of high degrees; but it is well to utilize them also for space of two and three dimensions, which Veronese does by means of his method of projection (*Math. ann.*, xix.). Thus for every complete tetrahedron, pentagon, and hexagon, in space of three dimensions, he gets configurations of points, lines, and curves, like those of the Pascal hexagram, and so for every triangle, quadrilateral, pentagon, and hexagon of the plane; and he remarks that the same method might be applied to configurations determined by any value of n in a space of $n - 1$ or less dimensions. Another geometrical interpretation of the groups of substitutions of six letters is given by six

linear complexes of lines in involution two and two (*Klein, math. ann.*, ii.). They determine fifteen surfaces of the second order, whose intersections are sixty curves of the fourth order corresponding to the sixty Pascal lines. There is also a theorem analogous to the Pascal theorem for a rational quartic in fourfold space.

M. Folie, in his report on this paper, complains that the contestant has refused to understand the question in the plain sense in which it was proposed; that he should have started out from the propositions which in M. Folie's book, 'Sur les fondements d'une géométrie supérieure Cartésienne,' are said to be analogous to the Pascal properties, namely, that in a plane cubic curve opposite sides of two quadrilaterals cut in a line, and that in a cubic surface opposite faces of two tetrahedrons cut in four lines in a plane; that, after having extended the question as far as possible in this direction, it was open to him to take another point of view, and even that which he has taken, though that is perhaps least of all susceptible of generalization. This work, he says, is remarkable and highly original, and would have deserved the prize had it been the aim of the academy simply to call forth a work of that description; but its object was to engage young geometers in the way already opened in his own memoirs, and to provoke them to researches which should complete those of the Belgian school of geometers, according to the expression of M. Chasles. This the author has not done: the question, hence, remains unattacked, and will continue to be retained upon the programme of the academy. Veronese, in reply, very pertinently inquires why it was not equally incumbent upon the contestant to follow in the way marked out by the Italian and the French schools, by Cremona and by Serret, and maintains that the prize is wrongly withheld on account of his having followed a new and original way instead of that which M. Folie professes to have pointed out to the geometers of the future. He admits that his results are not very susceptible of generalization, for the reason that they are already so extremely general. He complains that M. Folie has given no idea of the contents of his paper, — the usual task of a *rapporteur*, — and that, in each instance in which he refers to it, he fails to understand it. M. Folie says, for example, that Veronese has applied his method to cubics in space because he could, but not to plane curves or surfaces of order higher than the second, because his method was not there applicable; while, in fact, Veronese obtains

his results for curves in space not at all by application of his method, but by simple projection from the Pascal hexagram. M. Folie objects to Veronese's using the term 'involution' instead of 'cyclic homography'; but an examination of the table of contents might have shown him that Veronese devotes a section of his paper to cyclic homographies, and he gives simply a natural extension to the ordinary meaning of the term 'involution.' But, worst of all, M. Folie makes a singular slip in the enunciation of the original question, for there are no points or lines in the figure which are known as the points or lines of either Hesse or Bauer. At the end, Veronese turns the tables upon his opponent, and points out several striking inconsistencies in his memoirs, and several instances of his peculiar 'art of phrasing': as, "The greater part of these [M. Folie's] theorems had not yet been discovered, in spite of the depth and penetration of geometers;" "To deduce the corollaries from them would be an enterprise which would require, perhaps, years of labor;" "It is a field which I have cleared, and in which those who follow will find an ample harvest of discoveries."

In conclusion, we can but share the regret expressed by the direction of the *Annali*, that academies should so frequently provide unwisely for the advancement of science, either by proposing subjects which are too special, or by compelling authors to follow in their solution a direction determined *a priori*.

CHRISTINE LADD FRANKLIN.

OCCURRENCE OF AMBER NEAR TRENTON, N.J.

At the April meeting of the Trenton natural history society, the occurrence of amber in the bed of Crosswicks Creek was referred to, and no one of those present reported success in searching for it. The authority for its occurrence rests wholly, I believe, upon the statement in Comstock's *Mineralogy* (Boston, 1827), that it occurs 'near Trenton, N.J.,' and, again, "that found near Trenton occurs in small grains, and rests on lignite, or carbonated wood, or even penetrates it" (p. 297). I have several times met with small grains or pebbles of the mineral in the bed of Crosswicks Creek, and in 1860 found a mass as large as a pea, which I gave to the late W. S. Vaux, Esq., of Philadelphia. These small grains of amber, found in the bed of the creek, are undoubtedly derived from the beds of clay which are exposed in the bluff forming the southern bank of the

creek. Clays of the same character and age (cretaceous) occur nearer Trenton than Crosswicks Creek; and in them, also, occurs much fossil wood. In and on this, grains of amber are not uncommon. They are usually very small, and difficult to detect. The fossil wood in this cretaceous clay is soft and very 'recent' in appearance, and burns with an uncertain, flickering flame. The scanty traces of amber found with this — derived, I suppose, from it — is the fossilized sap of the trees now found in these deposits of clay.

CHARLES C. ABBOTT.

THE TOTAL SOLAR ECLIPSE OF MAY 6.

THE U. S. S. *Hartford*, which sailed from Callao, Peru, March 22, with the American and English astronomers on board, arrived at Caroline Island April 20, sixteen days before the date of the eclipse. The island is in reality a chain of small islands of coral formation, encircling a lagoon; the length of the enclosure being about seven miles and a half, and the breadth one mile and a half. The land is low, but supports an excellent growth of grass and other vegetation, including a number of coconut-trees. There are no permanent inhabitants; but the island is leased by an English firm which deals in guano, cocoanuts, and other products of this and similar Pacific islands. An agent of this firm visits the island occasionally, and superintends the work of those employed. Seven persons were found living on the island for the time being, having been brought there from Tahiti two months before. These were four men, one woman, and two children. There were two large frame houses in excellent condition, besides several smaller houses, which furnished comfortable accommodations for the party, and also for the French astronomers, who arrived two days later in the *L'Eclairer*. The latter party was composed of the following scientific men: M. Janssen of Meudon; M. Tacchini of Rome; M. Palisa of Vienna, formerly of Pola; M. Trouvelot of Meudon, formerly of Cambridge, Mass.; and M. Pasteur, photographer, also of Meudon.

The landing of the heavy cases containing the instruments was accomplished with difficulty, as even the small ship's boats could not come within several hundred feet of the shore, which was composed of rough coral rock. The cases were taken from the boats by men standing in about two feet of water, and carried to the shore, thence across several hundred feet of coral rock to the land, and about a quarter of a mile farther to the site selected for the ob-