day when the advance of American geology seemed to depend on state surveys is passing, and will soon pass away. They did good skirmish-work, and deserve to be remembered for many gifts to science; but the problems in scientific geology are now too large to be solved within the limits of a state. Scarce a state in this country has a question that can be properly considered from work done within its limits alone. In the future the state surveys can find their best place, not in efforts to develop general scientific problems, but rather in economic questions, which can always be localized, and in the work of bringing to the notice of the people whom they serve such matters of pure science as may naturally concern them. Other forms of research would better be left to the general government surveys, or to the studies of independent geologists.

It is now pretty well ascertained that our states are unwilling to support permanent scientific establishments on such a scale as will enable them to do good scientific work, but they will pay some one or two men to keep a sharp lookout for any utilities that may be discovered. Fortunately nature so mingles the 'utile' and the 'dulce,' that some good to science will come out of this care for profit, which is to be in the future the task of the state surveyor.

M. HERMITE'S LECTURES.

Cours de M. Hermite, professé pendant le 2e semestre 1881–82. Redigé par M. ANOYER, *élève de l'École normale supérieure. Second tirage revu par* M. HERMITE *(Librarie scientifique).* Paris, *A. Hermann*, 1883.

THIS work of M. Hermite fills, in great part, a decided gap in mathematical literature, and affords a means to American mathematical students, at least, of overcoming a difficulty that of late has become rather serious. With the exception of those who have had the opportunity of listening to the lectures of Hermite or Weierstrass on the theory of functions of a complex variable, all students interested in that subject must have experienced a great deal of difficulty in reading the more modern memoirs which deal with it. Some such book as Durége's, or Neumann's, on Riemann's theory, is very much wanted on what may, with propriety, be called the Weierstrass-Hermite theory of functions. The necessity for such a treatise is steadily increasing, as any one will readily see by looking over the last few volumes of *Crelle-Borchardt*, the *Mathematische annalen*, the *Annali di matematica*, or the two numbers which have already appeared of *Mittag-Leffler's acta mathematica*. The present work by M. Hermite does not profess to be such a treatise. In fact, it is not a treatise at all, but, as its title implies, simply the course of lectures given at the Sorbonne by M. Hermite, and treating of quite an extended list of subjects. The principal topics discussed are the quadrature and rectification of curves, the determination of the areas and volumes of curved surface, the general theory of functions of a complex variable, and the application of this theory to the study of the Eulerian integrals and the elliptic functions.

The first five chapters are devoted to geometry, and contain applications which are chosen with a view to what is contained in the succeeding chapters. Since, for the rectification of conics and the quadrature of plane cubics, it is necessary to consider integrals of the form $\int f(xy)\,dx$, where $f(xy)$ is a rational function of x and y, and y is the square root of a quartic function of x, the author takes up this general integral, and gives Legendre's reduction to the normal forms of the elliptic integrals, and also some of Tchebychef's results concerning the cases where the elliptic integrals are reducible to algebraico-logarithmic functions.

The next three chapters are taken up with an exposition of the more elementary properties of functions of a complex variable, the author giving an account of Darboux's investigations relatively to the integral $\int_a^b \mathrm{F}(x) f(x)\,dx$, where $\mathrm{F}(x)$ is, between the limits, always positive, $f(x)$ is a continuous function of the form $\phi(x) + i\,\psi(x)$, and where a and b are real. Another method, due to Weierstrass, for integrals of this nature, is also indicated.

In the next four chapters the immediate consequences of Cauchy's theorem are developed, and an account given of Weierstrass's and Mittag-Leffler's investigations in the theory of uniform functions, including their decomposition of a holomorphic function into prime factors, and their general expression for a uniform function with an infinite number of poles, or of essential singular points, the last being due almost solely to Mittag-Leffler.

The next three chapters deal with the Eulerian integrals, and include Prym's expression for $\Gamma(x)$, and Weierstrass's expression for $\frac{1}{\Gamma(x)}$, and a demonstration by M. Hermite of Laplace's formula for the approximate calculation of $\Gamma(x)$, where x is a very large integer.

The next two chapters refer to functions which are discontinuous along a line, — Appell's and Tannery's series, and Poincarré's example of a function having an *espace lacunaire*. As preliminary to Cauchy's theorem concerning the number of roots of a polynomial contained in the interior of a contour, the expression is given by a line-integral of roots of an equation contained within a given contour. Then follows Cauchy's theorem, the establishment of Lagrange's series, Eisenstein's theorem upon series whose co-efficients are commensurable, and which satisfy an algebraical equation, and the enunciation of Tchebychef's theorem upon series with rational co-efficients, which may represent functions composed of algebraic, logarithmic, and exponential functions.

The next chapter treats of multiform functions arising from the integration of uniform and of multiform functions, and of the means of reducing them to uniform functions by systems of cuts (*conpures*).

The remaining five chapters treat entirely of the doubly-periodic functions. After first showing the multiple values of the elliptic integrals of the first kind which correspond to the different paths traced out by the variable, and establishing the double periodicity of the inverse functions to this integral, he defines a function, $\Phi(x)$, which conducts to the analytical expressions for the doubly-periodic functions. The function $\Phi(x)$ is defined by the equations, —

$$\Phi(x+a) = \Phi(x)$$
$$\Phi(x+b) = \Phi(x)\,\mathrm{exp.}\left[-\frac{ki\pi b}{a}(2x+b)\right],$$

where k is an integer. Then follows the investigation of the elliptic functions, including, of course, Jacobi's Θ, H, and Z functions, the definition of Weierstrass's functions, Appell's expression for doubly-periodic uniform functions in the case where they possess essential singular points, and, finally, a demonstration by M. Goursat of Fuch's theorem concerning the definite integrals K and K', considered as functions of the modulus.

It is perhaps to be somewhat regretted that the book is lithographed instead of printed in the usual manner; but this is of comparatively little consequence, as the writing is very clear and legible. Thanks are certainly due to M. Andoyer, the editor, for the trouble which he must have taken in elaborating what would seem to have been merely a set of notes on M. Hermite's lectures. The whole matter has been revised by M. Hermite, and the aggregate result of his and M. Andoyer's labors is a book which is a decided acquisition to mathematical literature. It is to be hoped that M. Hermite will see fit to go more fully into the subject of the functions of a complex variable, and that of elliptic functions, at a future time, and give to the world a treatise which will be more satisfactory than even the present very valuable work.

T. CRAIG.

WEEKLY SUMMARY OF THE PROGRESS OF SCIENCE.

ASTRONOMY.

New measures of Saturn's rings. — O. Struve gives the results of a series of measurements of the rings of Saturn at Pulkowa during August and September, 1882, compared with a similar series, also taken by himself, with the same instrument, and at the same time of the year in 1851. In a memoir on the subject in 1851, he seeks to prove, that, while the outer diameter of the rings remains constant, the inner is continually shortening, basing his conclusions on the observations and drawings from Huygens's time. If the conclusion were correct, and the contraction constant, the measures of 1882 should have given a perceptibly shorter inner diameter than those of 1851. The inner diameter of the dark ring seems to be slightly shorter than in 1851, but the difference is not nearly so large as the theory calls for. The dark ring seems, however, to have changed since 1851. Then it seemed divided by a dark streak, the inner part being entirely separate from the bright ring. In 1882, all trace of this division had disappeared, and the dark ring seemed to be merely a faint continuation of the bright ring. — (*Astr. nachr.*, No. 2498.) M. McN. [688

Formation of the tails of comets. — Mr. Rumford suggests that the repulsive force which is unmistakably manifested in the formation of comets' tails may be due, not to any electric action, or any imagined impulse of solar radiations, but merely to evaporation. A small particle from which evaporation is taking place on the side next the sun will be driven backward with a velocity continually accelerated; and, when more than half of the mass of the particle has been evaporated, the velocity of the residue may be much greater than the average velocity with which the gaseous molecules are driven off from the heated body. In the case of hydrogen at a temperature of 70° or 80° F., the velocity thus acquired might be greater than a hundred thousand miles a day. If we suppose the evaporating material to be gases which have been liquefied by the cold of space (carbon dioxide and volatile hydrocarbons), it becomes easy to account for a powerful repulsive action at distances from the sun even much greater than that of the earth. The writer suggests that the comet's light may be in part due to the 'bombardment' of precipitated particles by the evaporated molecules in the condition called by Crookes 'the fourth state of matter'; so that, "without electrical discharges,