Application Note OA-29

Low-Sensitivity, Highpass Filter Design With Parasitic Compensation

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Introduction

This Application Note covers the design of a Sallen-Key highpass biquad. This design gives low component and op amp sensitivities. It also shows how to compensate for the op amp's bandwidth (pre-distortion) and parasitic capacitances. A design example illustrates this method. These biquads are also called KRC or VCVS [voltage-controlled, voltage-source].

Changes in component values over process, environment and time affect the performance of a filter. To achieve a greater production yield, the filter needs to be insensitive to these changes. This Application Note presents a design algorithm that results in low sensitivity to component variation. See [6] for information on evaluating the sensitivity performance of your filter.

To achieve the best production yield, the nominal filter design must also compensate for component and board parasitics. The components are pre-distorted [5] to compensate for the op amp bandwidth. This Application Note expands the pre-distortion method in [5] to include compensation for parasitic capacitances. This method is valid for either voltage-feedback or current-feedback op amps.

Parasitic Compensation

To pre-distort your filter components and compensate for parasitic capacitances:

1. Use the method in [5] to include the op amp's effect on the filter response. The result is a transfer function of the same order whose coefficients include the op amp group delay (τ_{oa}) evaluated at the passband edge frequency (f_c).
2. For all parasitic capacitances in parallel with capacitors:
   - Add the capacitors together
   - Simplify the resulting coefficients
   - Use the sum of time constants form for the coefficients when possible
3. For all parasitic capacitances in parallel with resistors:
   - Replace the resistor R_x in the filter transfer function with the parallel equivalent of R_x and C_p:

     $$\frac{R_x \leftarrow R_x}{(1+R_xC_ps)},\ s = j\omega$$

   - Alter this impedance to a convenient form and simplify:
   - *Do not create new terms* (a coefficient times a new power of s) in the transfer function *after simplifying*
   - The most useful approximations are:

     $$\frac{R_x}{(1+R_xC_ps)} \approx R_x(1-R_xC_ps)$$

     $$\approx R_x e^{-R_xC_ps}$$

     These approximations are valid when:

     $$\frac{\omega << 1}{(R_xC_p)}$$

   - Convert (1+ R_xC_ps) to the exponential form (a pure time delay) when it multiplies, or divides, the entire transfer function
   - Do not change the gain at $\omega \approx \omega_p$ in allpass sections
   - When simplifying, discard any terms that are products of the error terms ($K\tau_{oa}$ and R_xC_p); they are negligible
   - Use the sum of time constants form for the coefficients when possible
4. Use an op amp with adequate bandwidth (f_{3dB}) and slew rate (SR):

   $$f_{3dB} \geq 10f_H$$
   $$SR > 5f_HV_{peak}$$

where f_H is the highest frequency in the passband of the filter, and V_{peak} is the largest peak voltage. This increases the accuracy of the pre-distortion algorithm. It also reduces the filter's sensitivity to op amp performance changes over temperature and process. Make sure the op amp is stable at a gain of A_v = K.

KRC Highpass Biquad Design

The biquad shown in Figure 1 is a Sallen-Key highpass biquad. V_{in} needs to be a voltage source with low output impedance.

The transfer function is:

$$\frac{V_o}{V_{in}} \approx \frac{H_\infty\left(\frac{1}{\omega_p^2}\right)s^2}{1+\left(\frac{1}{(\omega_pQ_p)}\right)s+\left(\frac{1}{(\omega_p^2)}\right)s^2}$$

where:

$$K = 1 + \frac{R_f}{R_g}$$

$$H_\infty = K$$

$$\frac{1}{(\omega_p Q_p)} = R_5 C_1 + R_5 C_3 - R_4 C_3 (K-1)$$

$$\frac{1}{\omega_p^2} = R_4 R_5 C_1 C_3$$

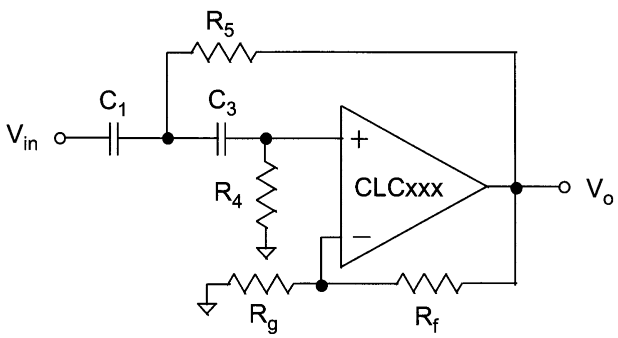

Figure 1: Highpass Biquad

To achieve low sensitivities, use this design algorithm:

1. Partition the gain for good Q_p sensitivity and dynamic range performance:
   - Use a low noise amplifier before this biquad if you need a large gain
   - Select K with this empirical formula:

   $$K = \begin{cases} 1, & 0.1 \le Q_p \le 1.1 \\ \dfrac{2.2\,Q_p - 0.9}{Q_p + 0.2}, & 1.1 < Q_p < 5 \end{cases}$$

   These values also reduce the op amp band width's impact on the filter response. This biquad's sensitivities are too high when $Q_p \ge 5$
2. Select an op amp with adequate bandwidth (f_{3dB}) and slew rate (SR):

   $$f_{3dB} \ge f_H$$
   $$f_{3dB} \ge 10 f_c$$
   $$SR > 5 f_H V_{peak}$$

   where f_H is the highest signal frequency, fc is the corner frequency of the filter, and V_{peak} is the largest peak voltage. Make sure the op amp is stable at a gain of $A_v = K$.
3. For current-feedback op amps, use the recommended value of R_f for a gain of $A_v = K$. For voltage-feedback op amps, select R_f for noise and distortion performance. Then set R_g for the correct gain:

   $$R_g = \frac{R_f}{(K-1)}$$
4. Initialize the resistance level $\left(R = \sqrt{R_4 R_5}\right)$.

   Increasing R will:
   - Increase the output noise
   - Reduce the distortion
   - Improve the isolation between the op amp outputs and C_1 and C_3
   - Make the parasitic capacitances a larger fraction of C_1 and C_3
5. Initialize the capacitance level $\left(C = \sqrt{C_1 C_3}\right)$, and the component ratios $\left(c^2 = \frac{C_3}{C_1} \text{ and } r^3 = \frac{R_5}{R_4}\right)$:

   $$c = \frac{1}{(\omega_p R)}$$

   $$c^2 = 0.10$$

   $$r^2 = \max\left\{0.10, \left(\frac{1 + \sqrt{1 + 4Q_p^2 \left(1 + c^2\right)(K-1)}}{2 \cdot Q_p \cdot \left(1 + c^2\right) / c}\right)\right.$$
6. Recalculate c^2 and initialize the capacitors:

   $$c^2 = \left(\frac{2 \cdot r \cdot Q_p}{1 + \sqrt{1 + 4Q_p^2 \left(K - 1 - r^2\right)}}\right)^2$$

   $$C_1 = \frac{C}{c}$$

   $$C_3 = cC$$
7. Set C_1 and C_3 to the nearest standard values.
8. Recalculate C, c^2, R and r^2:

   $$C = \sqrt{C_1 C_3}$$

   $$c^2 = \frac{C_3}{C_1}$$

   $$R = \frac{1}{(\omega_p C)}$$

   $$r^2 = \left(\frac{1 + \sqrt{1 + 4Q_p^2 \left(1 + c^2\right)(K-1)}}{2 \cdot Q_p \cdot \left(1 + c^2\right) / c}\right)^2$$
9. Calculate the resistors:

   $$R_4 = \frac{R}{r}$$

   $$R_5 = rR$$

The component sensitivity formulas are in the table below. The sensitivities to $\alpha_i = K$ are a measure of this biquad's sensitivity to the op amp group delay [5]. To evaluate this biquadís sensitivity performance, use the method in [6].

α_i	$S^{H_\infty}_{\alpha_i}$	$S^{\omega_p}_{\alpha_i}$	$S^{Q_p}_{\alpha_1}$
C_1	0	$-\frac{1}{2}$	$-\left(Q_p \cdot \frac{r}{c} - \frac{1}{2}\right)$
C_3	0	$-\frac{1}{2}$	$\left(Q_p \cdot \frac{r}{c} - \frac{1}{2}\right)$
R_4	0	$-\frac{1}{2}$	$\left((K-1) \cdot Q_p \cdot \frac{c}{r} + \frac{1}{2}\right)$
R_5	0	$-\frac{1}{2}$	$-\left((K-1) \cdot Q_p \cdot \frac{c}{r} + \frac{1}{2}\right)$
R_f	$\frac{K-1}{K}$	0	$\left((K-1) \cdot Q_p \cdot \frac{c}{r}\right)$
R_f	$-\frac{K-1}{K}$	0	$-\left((K-1) \cdot Q_p \cdot \frac{c}{r}\right)$
K	1	0	$\left(K \cdot Q_p \cdot \frac{c}{r}\right)$

KRC Highpass Biquad Parasitic Compensation

To pre-distort this biquad, and compensate for the [parasitic] non-inverting input capacitance of the op amp (C_{ni}), do the following (see *Appendix A* for the derivation of the formulas):

1. Start the iterations by ignoring the parasitics:

$$\tau^2 = 0$$
$$\tau_4^2 = 0$$

2. Estimate the pre-distorted values of ω_p and Q_p ($\omega_{p(pd)}$ and $Q_{p(pd)}$) that will compensate for τ_{oa} and C_{ni}:

$$\omega_{p(pd)} = \frac{\omega_{p(nom)}}{\sqrt{1 - \tau_4^2 \omega_{p(nom)}^2}}$$

$$Q_{p(pd)} = \frac{Q_{p(nom)}}{\left(\frac{\omega_{p(pd)}}{\omega_{p(nom)}} - Q_{p(nom)} \tau_2 \omega_{p(pd)}\right)}$$

where $\omega_{p(nom)}$ and $Q_{p(nom)}$ are the nominal values of ω_p and Q_p

3. Recalculate the resistors and capacitors using $\omega_{p(pd)}$ and $Q_{p(pd)}$:

$$\frac{1}{\omega_{p(pd)}^2} = R_4 R_5 C_1 C_3$$

$$\frac{1}{\left(\omega_{p(pd)} Q_{p(pd)}\right)} = R_5 C_1 + R_5 C_3 - R_4 C_3 (K-1)$$

The *Design Example* accomplishes this by recalculating R and r^2, then R_4 and R_5:

$$R = \frac{1}{\left(\omega_{p(pd)} C\right)}$$

$$r^2 = \left(\frac{1 + \sqrt{1 + 4Q_{p(pd)}^2 \left(1 + c^2\right)(K-1)}}{2 \cdot Q_{p(pd)} \cdot \left(1 + c^2\right) / c}\right)^2$$

$$R_4 = \frac{R}{r}$$

$$R_5 = rR$$

4. Calculate the resulting parasitic correction factors:

$$\tau_2 = R_4 C_{ni}$$

$$\tau_4^2 = K \tau_{oa} R_4 C_3 + R_4 R_5 \left(C_1 + C_3\right) C_{ni}$$

5. Calculate the resulting filter response parameters ω_p and Q_p:

$$\omega_p = \frac{\omega_{p(pd)}}{\sqrt{1 + \tau_4^2 \omega_{p(pd)}^2}}$$

$$Q_p = \frac{Q_{p(pd)}}{\left(\frac{\omega_p}{\omega_{p(pd)}} + Q_{p(pd)} \tau_2 \omega_p\right)}$$

6. Repeat steps 2-5 until:

$$\omega_p = \omega_{p(nom)}$$

$$Q_p = Q_{p(nom)}$$

7. Estimate the high frequency gain:

$$H_\infty \approx \frac{K}{\left(1 + \tau_4^2 \omega_{p(pd)}^2\right)}$$

If this reduces the gain too much, then repartition the gain.

Design Example

The circuit shown in Figure 2 is a 3rd-order Butterworth highpass filter. Section A is a buffered single pole section, and Section B is a highpass biquad. Use a voltage source with low output impedance, such as the CLC111 buffer, for V_{in}.

The nominal filter specifications are:

f_c = 50MHz	(passband edge frequency)
f_s = 10MHz	(stopband edge frequency)
f_H = 200MHz	(highest signal frequency)
A_p = 3.0dB	(maximum passband ripple)
A_s = 40dB	(minimum stopband attenuation)
H_∞ = 0dB	(passband voltage gain)

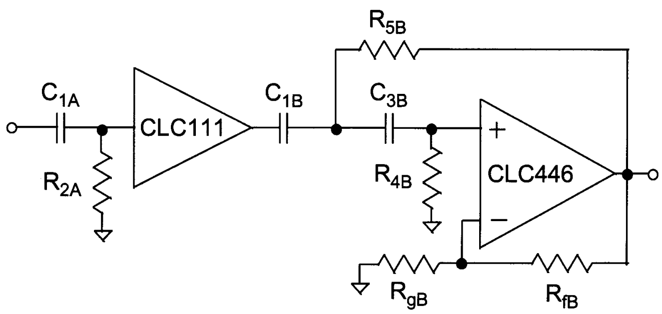

Figure 2: Highpass Filter

The 3rd-order Butterworth filter [1-4] meets our specifications. The pole frequencies and quality factors are:

Section	A	B
$\omega_p/2\pi$ [MHz]	50.00	50.00
Q_p [ ]	–	1.000

Overall Design:

1. Restrict the resistor and capacitor ratios to:
   $0.1 \le c^2, r^2 \le 10$
2. Use 1% resistors (chip metal film, 1206 SMD)
3. Use 5% capacitors (ceramic chip, 1206 SMD)
4. Use standard resistor and capacitor values

Section A Design and Pre-distortion:

1. Use the CLC111. This is a closed-loop buffer.
   - f_{3dB} = 800MHz > f_H = 200MHz
   - f_{3dB} = 800MHz > $10f_c$ = 500MHz
   - SR = 3500V/µs, while a 200MHz, $2V_{pp}$ sinusoid requires more than 1000V/µs
   - $\tau_{oa} \approx$ 0.28ns at 10MHz
   - $C_{ni(111)}$ = 1.3pF (input capacitance)
2. Select R_{2A} for noise, distortion and to properly isolate the CLC111's output and C_{1A}. The pre-distorted value of R_{2A}, that also compensates for $C_{ni(111)}$, is [5]:

$$R_{2A} = \frac{\left(\dfrac{1}{\omega_p - \tau_{oa}}\right)}{\left(C_{1A} + C_{ni(111)}\right)}$$

The results are in the table below:

- The Initial Value column shows ideal values that ignore any parasitic effects
- The Adjusted Value column shows the component values that compensate for $C_{ni(111)}$ and CLC111ís group delay (τ_{oa})
- The Standard Value column shows the nearest standard 1% resistors and 5% capacitors

Component	Initial	Value Adjusted	Standard
C_{1A}	30pF	30pF	30pF
R_{2A}	106Ω	92.8Ω	93.1Ω
$C_{ni(111)}$	–	1.3pF	1.3pF

Section B Design:

1. Since Q_p=1.000, set K_B to 1.00
2. Use the CLC446. This is a current-feedback op amp
   - f_{3dB} = 400MHz > f_H = 200MHz
   - f_{3dB} < $10f_c$ = 500MHz; the design will be sensitive to the op amp group delay
   - SR = 2000V/µs > 1000V/µs (see Item #1 in "Section A Design")
   - $\tau_{oa} \approx$ 0.56ns at 10MHz
   - $C_{ni(446)}$ = 1.0pF (input capacitance)
3. Use the CLC446's recommended R_f at A_v = 1.0:
   R_{fB} = 453Ω
   Then leave R_{gB} open so that K_B = 1.00
4. Initialize the resistor level:
   $R \approx 100\Omega$
5. Initialize the capacitor level, and the component ratios:
   $$C \approx \frac{1}{2\pi(50.00\text{MHz})\cdot(100\Omega)} = 31.83\text{pF}$$
   $$c^2 \approx 0.1000$$
   $$r^2 \approx \max\{0.10, 0.0826\} = 0.1000$$
6. Recalculate c^2 and initialize the capacitors:
   $c^2 \approx 0.127$ $C_{1B} \approx 89.3\text{pF}$ $C_{3B} \approx 11.3\text{pF}$
7. Set the capacitors to the nearest standard values:
   $C_{1B} \approx 91\text{pF}$ $C_{3B} \approx 11\text{pF}$
8. Recalculate the capacitor level and ratio, and the resistor level and ratio:
9. Calculate the resistors:
   $R_{4B} = 324\Omega$ $R_{3B} = 31.2\Omega$
10. The sensitivities for this design are:
    $$C = \sqrt{(91\text{pF})\cdot(11\text{pF})} = 31.64\text{pF}$$
    $$c^2 = \frac{(11\text{pF})}{(91\text{pF})} = 0.1209$$
    $$R = \frac{1}{2\pi(50.00\text{MHz})\cdot(31.64\text{pF})} = 100.6\Omega$$
    $$r^2 = 0.1056$$

α_i	$S^{H_\infty}_{\alpha_i}$	$S^{\omega_p}_{\alpha_i}$	$S^{Q_p}_{\alpha_1}$
C_{1B}	0.00	-0.50	-0.39
C_{3B}	0.00	-0.50	0.39
R_{4B}	0.00	-0.50	0.50
R_{5B}	0.00	-0.50	-0.50
R_{fB}	0.00	0.00	0.00
R_{gB}	0.00	0.00	0.00
K	1.00	0.00	1.12

Section B Pre-distortion:

1. The design gives these values:
   $\omega_{p(nom)} = 2\pi(50.00\text{MHz})$
   $Q_{p(nom)} = 1.000$
   $K_B = 1.00$
   $C_{1B} = 91\text{pF}$
   $C_{3B} = 11\text{pF}$
2. Iteration 1 shows the initial design results. Iterations 2-4 pre-distort R_{4B} and R_{5B} to compensate for the CLC446's group delay, and for $C_{ni(446)}$:

Iteration #		1	2	3	4
$\frac{\omega_{p(pd)}}{2\pi}$	[MHz]	50.00	59.73	56.81	57.54
$Q_{p(pd)}$	[ ]	1.000	0.9320	0.9561	0.9505
R	[Ω]	100.6	84.22	88.54	87.42
r^2	[Ω]	0.0962	0.1108	0.1053	0.1065
R_{4B}	[Ω]	324.3	253.0	272.9	267.9
R_{5B}	[Ω]	31.21	28.03	28.73	28.53
τ_2	[ns]	0.324	0.253	0.273	0.268
τ_4	[ns]	1.741	1.511	1.575	1.559
$\frac{\omega_p}{2\pi}$	[MHz]	43.87	51.96	49.52	50.13
Q_p	[ ]	1.034	0.984	1.003	0.999

The midband gain estimate is:

$H_\infty \approx 0.770$ [V/V]. Iteration 1
≈ 0.759 [V/V]. Iteration 4

The simulations gave a lower value for H_∞. Increasing K could help overcome this loss, but would also increase the sensitivities.

3. The resulting components are:

Component	Initial	Value Adjusted	Standard
C_{1B}	91pF	91pF	91pF
C_{3B}	11pF	11pF	11pF
$C_{ni(446)}$	–	1.0pF	1.0pF
R_{4B}	324Ω	268Ω	267Ω
R_{5B}	31.2Ω	28.5Ω	28.7Ω
R_{fB}	453Ω	453Ω	453Ω
R_{gB}	∞	∞	∞

Figures 3 and 4 show simulated gains. The curve numbers are:

1. Ideal (Initial Design Values, $\tau_{oa} = 0$, $C_{ni} = 0$)
2. Without pre-distortion (Initial Design Values, $\tau_{oa} \neq 0$, $C_{ni} = 0$)
3. With pre-distortion (Pre-distorted Values, $\tau_{oa} \neq 0$, $C_{ni} = 0$)

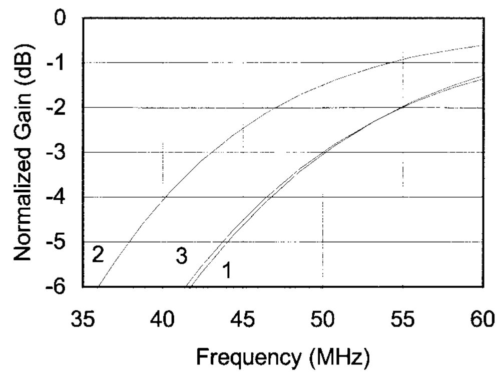

Figure 3: Simulated Filter Magnitude Response

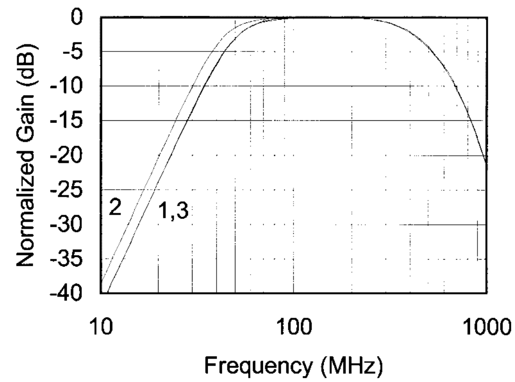

Figure 4: Simulated Filter Magnitude Response

SPICE Models

SPICE models are available for most of Comlinear's amplifiers. These models support nominal DC, AC, AC noise and transient simulations at room temperature.

We recommend simulating with Comlinear's SPICE models to:

- Predict the op amp's influence on filter response
- Support quicker design cycles

Include board and component parasitic models to obtain a more accurate prediction of the filter's response.

To verify your simulations, we recommend bread-boarding your circuit.

Summary

This Application Note contains an easy to use design algorithm for a low sensitivity, Sallen-Key highpass biquad. Designing for low ω_p and Q_p sensitivities gives:

- Reduced filter variation over process, temperature and time
- Higher manufacturing yield
- Lower component cost

A low sensitivity design is not enough to produce high manufacturing yields. This Application Note shows how to compensate for the op amp bandwidth, and for the [parasitic] input capacitance of the op amp. This method also applies to any other component or board parasitics. The components must also have low enough tolerance and temperature coefficients.

Appendix A – Derivation of Pre-distortion and Parasitic Capacitance Compensation Formulas

To pre-distort this filter, and compensate for the [parasitic] input capacitance of the op amp (C_{ni}):

1. Use the method in [5] to include the op amp's effect on the filter response. The result is:

$$\frac{V_o}{V_{in}} \approx \frac{H_\infty\left(\frac{1}{\omega_p^2}\right)s^2}{1+\left(\frac{1}{(\omega_p Q_p)}\right)s+\left(\frac{1}{\omega_p^2}\right)s^2}\cdot e^{-\tau_{oa}s}$$

   where the op amp group delay (τ_{oa}) is evaluated at the passband edge frequency (f_c), and:

$$\frac{1}{(\omega_p Q_p)} = R_5C_1 + R_5C_3 - R_4C_3(K-1)$$

$$\frac{1}{\omega_p^2} = R_4R_5C_1C_3 + K\tau_{oa}R_4C_3$$

$$K = \frac{1+R_f}{R_g}$$

$$H_\infty = K$$

2. Since C_{ni} is in parallel with R_4, replace R_4 with the parallel equivalent of R_4 and C_{ni}:

$$R_4 \leftarrow \frac{R_4}{(1+R_4C_{ni}s)}$$

$$\frac{V_o}{V_{in}} \approx \frac{H_\infty\left(\frac{R_4C_3(R_5C_1+K\tau_{oa})}{1+R_4C_{ni}s}\right)s^2\cdot e^{-\tau_{oa}s}}{\left(1+\left(\frac{R_4C_3(1-K)}{1+R_4C_{ni}s}+R_5(C_1+C_3)\right)s+\left(\frac{R_4C_3(R_5C_1+Kt_{oa})}{1+R_4C_{ni}s}\right)s^2\right)}$$

3. After simplifying, we obtain:

$$\frac{V_o}{V_{in}} \approx \frac{H_\infty\left(\frac{1}{\omega_p^2}\right)s^2}{1+\left(\frac{1}{(\omega_p Q_p)}\right)s+\left(\frac{1}{\omega_p^2}\right)s^2}\cdot e^{-\tau_{oa}s}$$

   where:

$$\frac{1}{(\omega_p Q_p)} = t_1 + t_2$$

$$\tau_1 = R_5C_1 + R_5C_3 - R_4C_3(K-1)$$

$$\tau_2 = R_4C_{ni}$$

$$\frac{1}{\omega_p^2} = \tau_3^2 + \tau_4^2$$

$$\tau_3^2 = R_4R_5C_1C_3$$

$$\tau_4^2 = K\tau_{oa}R_4C_3 + R_4R_5(C_1+C_3)C_{ni}$$

$$K = \frac{1+R_f}{R_g}$$

$$H_\infty = \frac{K\cdot(\tau_3^2)}{(\tau_3^2+\tau_4^2)}$$

Appendix B – Bibliography

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