National Semiconductor

Application Note OA-27

Low-Sensitivity, Lowpass Filter Design

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Introduction

This App Note covers the design of a Sallen-Key (also called KRC or VCVS [voltage-controlled, voltage-source]) lowpass biquad with low component and op amp sensitivities. This method is valid for either voltage-feedback or current-feedback op amps. Basic techniques for evaluating filter sensitivity performance are included. A filter design example illustrates the method.

Changes in component values over process, environment and time affect the performance of a filter. To achieve a greater production yield, we need to make the filter insensitive to these changes. This App Note presents a design algorithm that results in low sensitivity to component variation.

Lowpass biquad filter sections have the transfer function:

$$\frac{V_o}{V_{in}} \approx \frac{H_o}{1+\left(1/\left(\omega_p Q_p\right)\right)s+\left(1/\omega_p^2\right)s^2}$$

where $s=j\omega$, H_o is the DC gain, ω_p is the pole frequency, and Q_p is the pole quality factor. Both ω_p and Q_p affect the filter phase response, ω_p the filter cutoff frequency, Q_p the peaking, and H_o the gain. For these reasons, we will minimize the sensitivities of H_o, ω_p and Q_p to all of the components (see *Appendix A*).

To achieve the best production yield, the nominal filter design must also compensate for component and board parasitics. For information on filter component pre-distortion, see Reference [5]. SPICE simulations, with good component and board models, help adjust the nominal design point to compensate for parasitics.

See *Appendix A* for an overview of sensitivity analysis, with applications to filter design. See *Appendix B* for useful sensitivity properties and formulas. See the references listed in *Appendix C* for a more complete discussion of sensitivity functions, their applications, and other approaches to improving the manufacturing yield of your filter.

KRC Lowpass Biquad

The biquad shown in Figure 1 is a Sallen-Key lowpass biquad. V_{in} needs to be a voltage source with low output impedance. R_1 and R_2 attenuate V_{in} to keep the signal within the op amp's dynamic range. The Thevenin equivalent of V_{in}, R_1, and R_2 is a voltage source αV_{in}, with an output impedance of R_{12}, where:

$$\alpha = R_2/\left(R_1+R_2\right)$$
$$R_{12} = \left(R_1 \parallel R_2\right)$$

The input impedance in the passband is:

$$Z_{in} = R_1 + R_2 \quad , \quad \omega << \omega_p$$

The transfer function is:

$$\frac{V_o}{V_{in}} \approx \frac{H_o}{1+\left(1/\left(\omega_p Q_p\right)\right)s+\left(1/\omega_p^2\right)s^2}$$

where:

$$K = 1 + R_f/R_g$$
$$H_o = \alpha K$$
$$1/\left(\omega_p Q_p\right) = R_{12}C_5(1-K) + R_3C_4 + R_{12}C_4$$
$$1/\omega_p^2 = R_{12}R_3C_4C_5$$

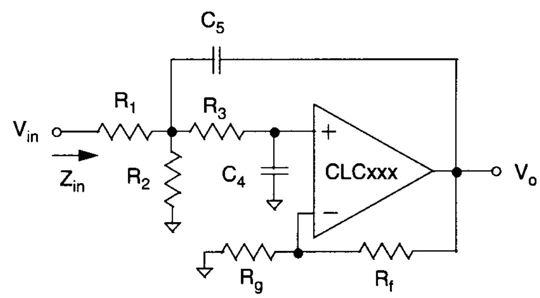

Figure 1: Lowpass Biquad

To achieve low sensitivities, use this design algorithm:

1. Partition the gain for good Q_p sensitivity and dynamic range performance:
   - Use a low noise amplifier before this biquad if you need a large gain
   - Select K for good sensitivity with this empirical formula:

$$K = \begin{cases} 1 & , \; 0.1 \le Q_p \le 1.1 \\ \dfrac{2.2Q_p - 0.9}{Q_p + 0.2} & , \; 1.1 < Q_p < 5 \end{cases}$$

   These values also reduce the op amp bandwidth's impact on the filter response, and increase the bandwidth for voltage-feedback op amps. When $Q_p \ge 5$, the sensitivities of this biquad are very high
   - Set α as close to 1 as possible while keeping the signal within the op amp's dynamic range

2. Select an op amp with adequate bandwidth (f_{3dB}) and slew rate (SR):

$$f_{3dB} \geq 10 f_c$$

$$SR > 5 f_c V_{peak}$$

where f_c is the corner frequency of the filter, and V_{peak} is the largest peak voltage. Make sure the op amp is stable at a gain of $A_v = K$.

3. Select R_f and R_g so that:

$$K = 1 + R_f / R_g$$

For current-feedback op amps, use the recommended value of R_f for a gain of $A_v = K$. For voltage-feedback op amps, select R_f for noise and distortion performance.

4. Initialize the resistance level ($R = \sqrt{R_{12} R_3}$). This value is a compromise between noise performance, distortion performance, and adequate isolation between the op amp outputs and the capacitors.

5. Initialize the capacitance level ($C = \sqrt{C_4 C_5}$), the resistor ratio ($r^2 = R_{12} / R_3$), the capacitor ratio ($c^2 = C_4 / C_5$) and the capacitors:

$$C = 1/(R\omega_p)$$

$$r^2 = 0.10$$

$$c^2 = \max\left(\left(\frac{1 + \sqrt{1 + 4Q_p^2(1 + r^2)(K-1)}}{2 \cdot Q_p \cdot (1 + r^2)/r}\right)^2, \ 0.10\right)$$

$$C_4 = cC$$

$$C_5 = C/c$$

6. Set the capacitors C_4 and C_5 to the nearest standard values.

7. Recalculate C, c^2, R and r^2:

$$C = \sqrt{C_4 C_5}$$

$$c^2 = C_4 / C_5$$

$$R = 1/(C\omega_p)$$

$$r^2 = \left(\frac{2 \cdot cQ_p}{1 + \sqrt{1 + 4Q_p^2(K - 1 - c^2)}}\right)^2$$

8. Calculate R_{12} and the resistors:

$$R_{12} = rR$$

$$R_1 = R_{12}/\alpha$$

$$R_2 = R_{12}/(1-\alpha)$$

$$R_3 = R/r$$

V_{in} can represent a source driving a transmission line, with R_1 and R_2 the source and terminating resistances. For this type of application, make these modifications to the design algorithm:

- Select R_1 and R_2 to properly terminate the transmission line (R_1 includes the source resistance)
- Calculate α and R_{12}
- Adjust C and R so that $R_{12} = rR$

To evaluate the sensitivity performance of this design, follow these steps:

1. Calculate the resulting sensitivities:

α_i	$S_{\alpha_i}^{H_o}$	$S_{\alpha_i}^{\omega_p}$	$S_{\alpha_i}^{Q_p}$
K	1	0	$\left(K \cdot Q_p \cdot \frac{r}{c}\right)$
R_1	$-(1-\alpha)$	$-\frac{\alpha}{2}$	$(\alpha) \cdot \left(Q_p \cdot \frac{c}{r} - \frac{1}{2}\right)$
R_2	$(1-\alpha)$	$-\frac{1-\alpha}{2}$	$(1-\alpha) \cdot \left(Q_p \cdot \frac{c}{r} - \frac{1}{2}\right)$
R_3	0	$-\frac{1}{2}$	$-\left(Q_p \cdot \frac{c}{r} - \frac{1}{2}\right)$
R_f	$\frac{K-1}{K}$	0	$\left((K-1) \cdot Q_p \cdot \frac{r}{c}\right)$
R_g	$-\frac{K-1}{K}$	0	$-\left((K-1) \cdot Q_p \cdot \frac{r}{c}\right)$
C_4	0	$-\frac{1}{2}$	$-\left((K-1) \cdot Q_p \cdot \frac{r}{c} + \frac{1}{2}\right)$
C_5	0	$-\frac{1}{2}$	$\left((K-1) \cdot Q_p \cdot \frac{r}{c} + \frac{1}{2}\right)$

Reducing $\left|S_K^{Q_p}\right|$ lowers the biquad's sensitivity to the op amp bandwidth.

2. Calculate the relative standard deviations of H_o, ω_p and Q_p:

$$\left(\frac{\sigma_X}{X}\right)^2 \approx \sum_i \left(\left|S_{\alpha_i}^X\right| \cdot \frac{\sigma_{\alpha_i}}{\alpha_i}\right)^2$$

In this formula, use:

- The nominal values of H_o, ω_p and Q_p for X
- The nominal values of R_1, R_2, R_3, R_f, R_g, C_4 and C_5 for α_i (*do not use* K since it is not a component)
- The capacitor and resistor standard deviations for σ_{α_i}. For parts with a uniform probability distribution,

$$\sigma_{\alpha_i} = \frac{\max(\alpha_i) - \min(\alpha_i)}{\sqrt{12}}$$

3. If temperature performance is a concern, then estimate the change in nominal values of H_o, ω_p and Q_p over the design temperature range:

$$X(T) \approx X\left(1 + \sum_i \left(S_{\alpha_i}^X \cdot \frac{\alpha_i(T) - \alpha_i}{\alpha_i}\right)\right)$$

In this formula, use:

- The nominal values, at room temperature, of H_o, ω_p and Q_p for X

- The nominal values, at room temperature, of R_1, R_2, R_3, R_f, R_g, C_4 and C_5 for α_i (*do not use* K since it is not a component)
- The nominal resistor and capacitor values at temperature T for $\alpha_i(T)$

4. Estimate the probable ranges of values for H_o, ω_p and Q_p:

$$X \geq (1 - 3 \cdot \sigma_X / X) \cdot \min(X(T))$$

$$X \leq (1 + 3 \cdot \sigma_X / X) \cdot \max(X(T))$$

where X is H_o, ω_p and Q_p.

Design Example

The circuit shown in Figure 2 is a 3rd-order Chebyshev lowpass filter. Section A is a buffered single pole section, and Section B is a lowpass biquad. Use a voltage source with low output impedance, such as the CLC111 buffer, for V_{in}.

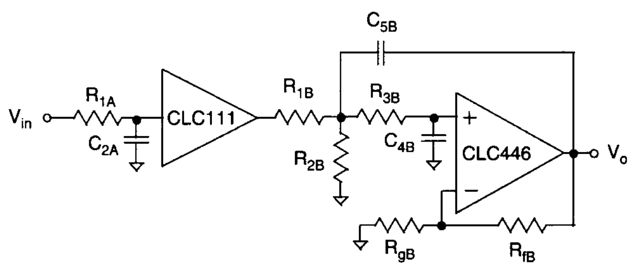

Figure 2: Lowpass Filter

The nominal filter specifications are:

f_c = 50MHz (passband edge frequency)
f_s = 100MHz (stopband edge frequency)
A_p = 0.5dB (maximum passband ripple)
A_s = 19dB (minimum stopband attenuation)
H_o = 0dB (DC voltage gain)

The 3rd-order Chebyshev filter meets our specifications (see References [1-4]). The resulting -3dB frequency is 58.4MHz. The pole frequencies and quality factors are:

Section	A	B
$\omega_p/2\pi$ [MHz]	53.45	31.30
Q_p [ ]	1.706	—

Overall Design:

1. Restrict the resistor and capacitor ratios to:

$$0.1 \leq r^2 \leq 10$$

$$0.1 \leq c^2 \leq 10$$

2. Use 1% resistors (chip metal film, 1206 SMD, 25ppm/°C)
3. Use 1% capacitors (ceramic chip, 1206 SMD, 100ppm/°C)
4. Use standard resistor and capacitor values
5. The temperature range is -40 to 85°C, and room temperature is 25°C

Section A Design:

1. Use the CLC111. This is a closed-loop buffer.
   - f_{3dB} = 800MHz > $10f_c$ = 500MHz
   - SR = 3500V/μs, while a 50MHz, $2V_{pp}$ sinusoid requires more than 250V/μs
   - $C_{ni(111)}$ = 1.3pF (input capacitance)
2. We selected R_{1A} for noise, distortion and to properly isolate the CLC111's output and C_{2A}. The capacitor C_{2A} then sets the pole frequency:

$$1/\omega_p \approx R_{1A} C_{2A}$$

The results are in the table below:

- The Initial Value column shows values from the calculations above
- The Adjusted Value column shows the component values that compensate for $C_{ni(111)}$ and for the CLC111's finite bandwidth (see Comlinear's App Note on filter component pre-distortion [5])
- The Standard Value column shows the nearest available standard 1% resistors and capacitors

Component	Value		
	Initial	Adjusted	Standard
R_{1A}	108Ω	100Ω	100Ω
C_{2A}	47pF	47pF	47pF
$C_{ni(111)}$	—	1.3pF	1.3pF

Section B Design:

1. The recommended value of K_B for Q_p=1.706 is:

$$K_B = \frac{2.2(1.706) - 0.9}{(1.706) + 0.2} = 1.50$$

Set $\alpha_B = H_o/K_B = 0.667$.

2. Use the CLC446. This is a current-feedback op amp
   - f_{3dB} = 400MHz ≈ $10f_c$ = 500MHz
   - SR = 2000V/μs > 250V/μs (see Item #1 in "Section A Design")
   - $C_{ni(446)} = 1.0\text{pF}$ (non-inverting input capacitance)
3. Set R_{fB} to the CLC446's recommended R_f at $A_v = +1.5$:

$$R_{fB} = 348\Omega$$

Then set R_{gB} = 696Ω so that K_B=1.50.

4. Initialize the resistor level for noise and distortion performance:

$$R \approx 200\Omega$$

5. Initialize the capacitor level, resistor and capacitor ratios, and the capacitors:

$$C \approx \frac{1}{(200\Omega) \cdot (2\pi(53.45\text{MHz}))} = 15\text{pF}$$

$$r^2 \approx 0.10$$

$$c^2 \approx \max(0.0983\ ,\ 0.10) = 0.1000$$

$$C_{4B} \approx 4.7\text{pF}$$

$$C_{5B} \approx 47\text{pF}$$

6. Set the capacitors to the nearest standard values:

$$C_{4B} = 4.7pF$$
$$C_{5B} = 47pF$$

7. Recalculate the capacitor level and ratio, and the resistor level and ratio:

$$C = \sqrt{(4.7pF)\cdot(47pF)} = 14.86pF$$
$$c^2 = (4.7pF)/(47pF) = 0.1000$$
$$R = \frac{1}{(14.86pF)\cdot(2\pi(53.45MHz))} = 200.4\Omega$$
$$r^2 = 0.1020$$

8. Calculate R_{12B} and the resistor values:

$$R_{12B} = 64.0\Omega$$
$$R_{1B} = 96.0\Omega$$
$$R_{2B} = 192\Omega$$
$$R_{3B} = 627\Omega$$

The resulting component values are:

Component	Value		
	Initial	Adjusted	Standard
R_{1B}	96.0Ω	78.9Ω	78.7Ω
R_{2B}	192Ω	158Ω	158Ω
R_{3B}	627Ω	582Ω	576Ω
C_{4B}	4.7pF	3.7pF	3.6pF
$C_{ni(446)}$	—	1.0pF	1.0pF
C_{5B}	47pF	47pF	47pF
R_{fB}	348Ω	348Ω	348Ω
R_{gB}	696Ω	696Ω	698Ω

9. The sensitivities for this design are:

α_i	$S_{\alpha_i}^{H_o}$	$S_{\alpha_i}^{\omega_p}$	$S_{\alpha_i}^{Q_p}$
K	1.00	0.00	2.58
R_{1B}	-0.33	-0.33	0.79
R_{2B}	0.33	-0.17	0.40
R_{3B}	0.00	-0.50	-1.19
R_{fB}	0.33	0.00	0.86
R_{gB}	-0.33	0.00	-0.86
C_{4B}	0.00	-0.50	-1.36
C_{5B}	0.00	-0.50	1.36

10. The relative standard deviations of H_o, ω_p and Q_p are:

$$\sigma_{H_o}/H_o \approx 0.38\%$$
$$\sigma_{\omega_p}/\omega_p \approx 0.55\%$$
$$\sigma_{Q_p}/Q_p \approx 1.58\%$$

These standard deviations are based on a uniform distribution, with all resistors and capacitor values being independent:

$$\frac{\sigma_R}{R} \approx \frac{\sigma_C}{C} \approx \frac{1.00\% - (-1.00\%)}{\sqrt{12}} \approx 0.58\%$$

11. The nominal values of H_o, ω_p and Q_p over the design temperature range are:

T [°C]	-40	25	85
H_o [V/V]	1.000	1.000	1.000
$\omega_p/2\pi$ [MHz]	53.88	53.45	53.00
Q_p [ ]	1.706	1.706	1.706

12. The probable ranges of values for H_o, ω_p and Q_p, over the design temperature range, are:

$$0.99 \le H_o \le 1.01$$
$$52.1\text{ MHz} \le (\omega_p/2\pi) \le 54.8\text{MHz}$$
$$1.63 \le Q_p \le 1.79$$

13. Based on the results in #10 and #12, we can conclude that:
    - The DC gain and cutoff frequency change little with component value and temperature changes
    - Q_p has the greatest sensitivity to fabrication changes
    - The greatest filter response variation is in the peaking near the cutoff frequency

Figure 3 shows the results of a Monte-Carlo simulation at room temperature, with 100 cases simulated. These simulations used the "Standard Values" of the components. The gain curves are:

1. Lower 3-sigma limit (mean minus 3 times the standard deviation)
2. Mean value
3. Upper 3-sigma limit (mean plus 3 times the standard deviation)

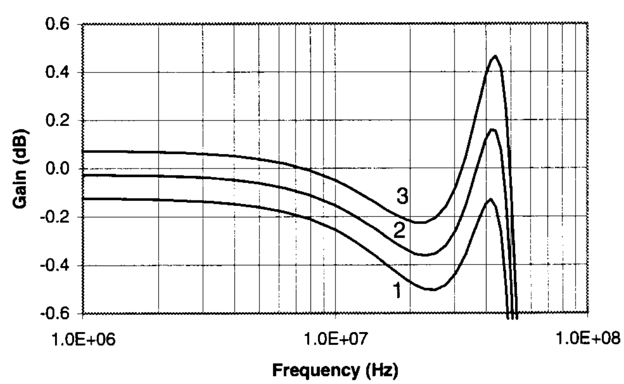

Figure 3: Monte-Carlo Simulation Results

SPICE Models

SPICE models are available for most of Comlinear's amplifiers. These models support nominal DC, AC, AC noise and transient simulations at room temperature.

We recommend simulating with Comlinear's SPICE models to:

- Predict the op amp's influence on filter response
- Support quicker design cycles

Include board and component parasitic models to obtain a more accurate prediction of the filter's response.

To verify your simulations, we recommend bread-boarding your circuit.

Summary

This App Note contains an easy to use design algorithm for a low sensitivity, Sallen-Key lowpass biquad, which works for $Q_p < 5$. It also shows the basics of evaluating filter sensitivity performance.

Designing for low ω_p and Q_p sensitivities gives:

- Reduced filter variation over process, temperature and time
- Higher manufacturing yield
- Lower component cost

A low sensitivity design is not enough to produce high manufacturing yields. The nominal design must also compensate for any component parasitics, board parasitics, and op amp bandwidth (see Comlinear's App Note on filter component pre-distortion [5]). The components must also have low enough tolerance and temperature coefficients.

Appendix A- Sensitivity Analysis Overview

The classic logarithmic sensitivity function is:

$$S_{\alpha_i}^{X} = \frac{\partial(\ln X)}{\partial(\ln \alpha_i)} \;;\quad \alpha_i, X \neq 0$$
$$= \frac{\alpha_i}{X} \cdot \frac{\partial X}{\partial \alpha_i}$$
$$\approx \frac{\Delta X / X}{\Delta \alpha_i / \alpha_i}$$

where α_i is a component value, and X is a filter performance measure (in the most general case, this is a complex-valued function of frequency). The sensitivity function is a dimensionless figure of merit used in filter design.

We can approximate the relative change in X caused by the relative changes in the components α_i as:

$$\frac{\Delta X}{X} \approx \sum_i S_{\alpha_i}^{X} \cdot \frac{\Delta \alpha_i}{\alpha_i}$$

where:

$$\alpha_i,\ X \neq 0$$
$$\left|\frac{\Delta \alpha_i}{\alpha_i}\right|,\ \left|\frac{\Delta X}{X}\right| \ll 1$$

The relative standard deviation of X is calculated using:

$$\left(\frac{\sigma_X}{X}\right)^2 \approx \sum_i \left(\left|S_{\alpha_i}^{X}\right| \cdot \frac{\sigma_{\alpha_i}}{\alpha_i}\right)^2$$

where:

- The summation is over all component values (α_i) that affect X
- All component values (α_i) are physically independent (no statistical correlation)

The nominal value of X is a function of temperature:

$$X(T) = X\left(1 + \frac{X(T) - X}{X}\right)$$
$$\approx X\left(1 + \sum_i \left(S_{\alpha_i}^{X} \cdot \frac{\alpha_i(T) - \alpha_i}{\alpha_i}\right)\right)$$

where:

- X is the nominal value of X at room temperature
- α_i (T) is the nominal value of α_i at temperature T
- X(T) is the nominal value of X at temperature T

To help reduce variation in filter performance:

- Reduce the sensitivity function magnitudes ($\left|S_{\alpha_i}^{X}\right|$), where X is H_o, ω_p and Q_p, and α_i is any of the component values, the gain K, or operating conditions (such as temperature or supply voltage)
- Use components with smaller tolerances
- Use components with lower temperature coefficients

Appendix B- Handy Sensitivity Formulas

Notation:

1. k, m, n = constants
2. α, β = [non-zero] component parameters
3. X, Y = [non-zero] performance measures

Formulas:

1. $S_{\alpha}^{k\alpha^n} = n$
2. $S_{\alpha}^{kX} = S_{k\alpha}^{X} = S_{\alpha}^{X}$
3. $S_{\alpha^k}^{X^m Y^n} = \frac{m}{k} \cdot S_{\alpha}^{X} + \frac{n}{k} \cdot S_{\alpha}^{Y}$
4. $S_{\alpha}^{X} = 1/S_{X}^{\alpha}$
5. $S_{\alpha}^{Y(X(\alpha))} = S_{X}^{Y} S_{\alpha}^{X}$
6. $S_{\gamma}^{X(\alpha,\beta)} = S_{\alpha}^{X} S_{\gamma}^{\alpha} + S_{\beta}^{X} S_{\gamma}^{\beta}$
7. $S_{\alpha}^{X} = \mathrm{Re}\left(S_{\alpha}^{X}\right) + j\,\mathrm{Im}\left(S_{\alpha}^{X}\right)$

   where:

   $$\mathrm{Re}\left(S_{\alpha}^{X}\right) = S_{\alpha}^{|X|}$$
   $$\mathrm{Im}\left(S_{\alpha}^{X}\right) = \arg(X) \cdot S_{\alpha}^{\arg(X)} = \alpha \cdot \frac{\partial(\arg(X))}{\partial(\alpha)}$$

Appendix C- Bibliography

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