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Application Note OA-21

Component Pre-Distortion for Sallen-Key Filters

Kumen Blake

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Introduction

This revision obsoletes the previous revision of this App Note, and covers additional material.

This App Note shows a simple component pre-distortion method that works for many popular Sallen-Key (also called KRC or VCVS [voltage-controlled, voltage-source]) filter sections. This method compensates for voltage-feedback and current-feedback op amps. Several examples illustrate this method.

KRC active filter sections use an op amp and two resistors to set a non-inverting gain of K. Resistors and capacitors placed around this amplifier provide the desired transfer function. The op amp's finite bandwidth causes K to be a function of frequency. For this reason, KRC filters typically operate at frequencies well below the op amp's bandwidth ($f << f_{3dB}$).

"Pre-distortion" compensates for the op amp's finite bandwidth by modifying the nominal resistor and capacitor values. The pre-distortion method in this App Note compensates for the op amp's group delay, which is approximately constant when $f << f_{3dB}$.

One possible design sequence for KRC filters is:

1. Design the filter assuming an ideal op amp (K is assumed constant over frequency)
   - Select components for low sensitivities
   - Do a worst case analysis
   - Do a temperature analysis
2. Pre-distort the resistors and capacitors to compensate for the op amp's group delay
3. Compensate for parasitic elements

Filter Component Pre-distortion

This section outlines a simple pre-distortion method that works for many popular Sallen-Key filters using current-feedback or voltage-feedback op amps. Other more general pre-distortion methods are available (see Reference [4]) which require more design effort.

To pre-distort your filter components:

1. Calculate the op amp's delay:

$$\tau_{oa} \approx -\frac{1}{f_c}\cdot\frac{\phi(f_c)}{360°}$$

   where $\phi(f)$ is the op amp phase response in degrees, and f_c is the cutoff frequency (passband edge frequency) of your filter.
   - Subtract the phase shift caused by your measurement jig from any measured value of $\phi(f_c)$
   - The group delay is specified at f_c because it has the greatest impact on the filter response near that frequency
   - Other less accurate estimates of the op amp delay at f_c are:
     - Step response propagation delay
     - $1/(2\pi f_{3dB})$
2. The time delay around the filter feedback loop ("electrical loop delay") adds to the op amp delay. For this reason,
   - Make the filter feedback loop as physically short as possible
   - If you need greater accuracy in the following calculations, use the electrical loop delay (τ_{eld}) instead of the op amp delay (τ_{oa}):

$$\tau_{eld} \leftarrow \tau_{oa}$$

   See *Appendix B* for information on calculating τ_{eld}.
3. Replace K in the filter transfer function with a simple approximation to the op amp's frequency response
   - Start with a simple, single pole approximation:

$$K \leftarrow K/(1+\tau_{oa}s)\ ,\quad s = j\omega$$

   - Alter the approximation to K and simplify:
     - *Do not create new terms* (a coefficient times a new power of s) in the transfer function *after simplifying*
     - Convert $(1+\tau_{oa}s)$ to the exponential form (a pure time delay) when it multiplies, or divides, the entire transfer function
     - Do not change the gain at $\omega \approx \omega_p$ in allpass sections
     - The most useful alterations to K are:

$$\frac{K}{1+\tau_{oa}s} \approx K\cdot\frac{1-(\tau_{oa}/2)s}{1+(\tau_{oa}/2)s}$$
$$\approx K(1-\tau_{oa}s)$$
$$\approx Ke^{-\tau_{oa}s}$$

     All of these approximations are valid when:

$$\omega << 1/\tau_{oa}$$

4. Use an op amp with adequate bandwidth (f_{3dB}) and slew rate (SR):

$$f_{3dB} \geq 10f_H$$
$$SR > 5f_H V_{peak}$$

where f_H is the highest frequency in the passband of the filter, and V_{peak} is the largest peak voltage. This increases the accuracy of the pre-distortion algorithm. It also reduces the filter's sensitivity to op amp performance changes over temperature and process. Make sure the op amp is stable at a gain of $A_v = K$.

Appendix A contains examples using transfer functions. The next section will apply the results from *Appendix A*.

KRC Lowpass Biquad

The biquad shown in Figure 1 is a Sallen-Key lowpass biquad. V_{in} needs to be a voltage source with low output impedance. R_1 and R_2 attenuate V_{in} to keep the signal within the op amp's dynamic range. Using Example 3 in *Appendix A*, we can show:

$$\frac{V_o}{V_{in}} \approx \frac{H_o}{1+\left(1/(\omega_p Q_p)\right)s+\left(1/\omega_p^2\right)s^2} \cdot e^{-\tau_{oa}s}$$

$$\omega,\ \omega_p << 1/\tau_{oa}$$

where:

$$\alpha = R_2/(R_1+R_2)$$
$$K = 1+R_f/R_g$$
$$H_o = \alpha K$$
$$R_{12} = (R_1 \parallel R_2)$$
$$1/(\omega_p Q_p) = R_{12}C_5(1-K)+R_3C_4+R_{12}C_4$$
$$1/\omega_p^2 = R_{12}R_3C_4C_5 + K\tau_{oa}R_{12}C_5$$

After selecting α and R_{12}, calculate R_1 and R_2 as:

$$R_1 = R_{12}/\alpha$$
$$R_2 = R_{12}/(1-\alpha)$$

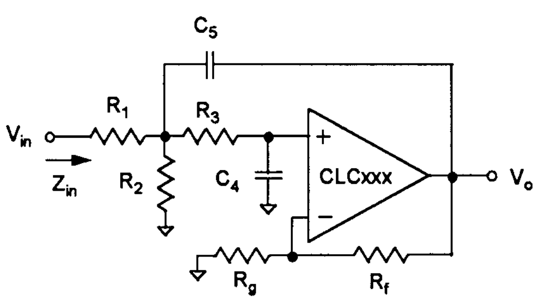

Figure 1: Lowpass Biquad

To pre-distort this filter:

1. Design the filter assuming K constant ($\tau_{oa} = 0$). Use low values for K so that:
   - τ_{oa} will have less impact on the biquad's response
   - For voltage-feedback op amps, τ_{oa} will be smaller ($\tau_{oa} \approx$ K divided by the gain-bandwidth product)
2. Recalculate the resistors and capacitors using the pre-distorted values of ω_p and Q_p ($\omega_{p(pd)}$ and $Q_{p(pd)}$) that will compensate for τ_{oa}:

$$1/\omega_{p(pd)}^2 = 1/\omega_{p(nom)}^2 - K\tau_{oa}R_{12}C_5 = R_{12}R_3C_4C_5$$

$$1/\left(\omega_{p(pd)}Q_{p(pd)}\right) = 1/\left(\omega_{p(nom)}Q_{p(nom)}\right) = R_{12}C_5(1-K)+R_3C_4+R_{12}C_4$$

   where $\omega_{p(nom)}$ and $Q_{p(nom)}$ are the nominal values of ω_p and Q_p
3. Repeat step 2 until $\omega_p \approx \omega_{p(nom)}$ and $Q_p \approx Q_{p(nom)}$, where:

$$1/\omega_p^2 = 1/\omega_{p(pd)}^2 + K\tau_{oa}R_{12}C_5$$

$$1/\left(\omega_p Q_p\right) = 1/\left(\omega_{p(pd)}Q_{p(pd)}\right)$$

Design Example

The circuit shown in Figure 2 is a 3rd-order Chebyshev lowpass filter. Section A is a buffered single pole section, and Section B is a lowpass biquad. Use a voltage source with low output impedance, such as the CLC111 buffer, for V_{in}.

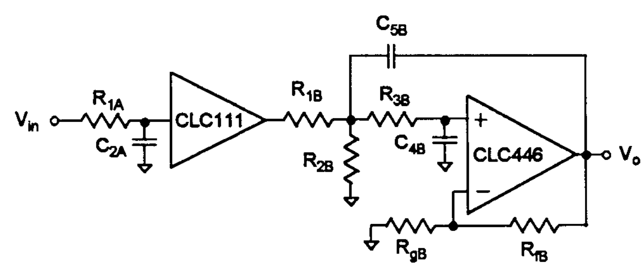

Figure 2: Lowpass Filter

The nominal filter specifications are:

f_c = 50MHz (passband edge frequency)
f_s = 100MHz (stopband edge frequency)
A_p = 0.5dB (maximum passband ripple)
A_s = 19dB (minimum stopband attenuation)
H_o = 0dB (DC voltage gain)

The 3rd-order Chebyshev filter meets our specifications (see References [1-4]). The resulting -3dB frequency is 58.4MHz. The pole frequencies and quality factors are:

Section	A	B
$\omega_p/2\pi$ [MHz]	53.45	31.30
Q_p [ ]	1.706	—

Overall Design:

1. Use the CLC111 for section A. This is a closed-loop buffer
   - f_{3dB} = 800MHz > $10f_c$ = 500MHz
   - SR = 3500V/µs, while a 50MHz, $2V_{pp}$ sinusoid requires more than 250V/µs
   - $\tau_{oa} \approx$ 0.28ns at 50MHz
   - $C_{ni(111)}$ = 1.3pF (input capacitance)

2. Use the CLC446 for section B. This is a current-feedback op amp
   - f_{3dB} = 400MHz ≈ 10f_c = 500MHz
   - SR = 2000V/μs > 250V/μs (see Item #1)
   - τ_{oa} ≈ 0.56ns at 50MHz
   - $C_{ni(446)}$ = 1.0pF (non-inverting input capacitance)
3. Use 1% resistors (chip metal film, 1206 SMD, 25ppm/°C)
4. Use 1% capacitors (ceramic chip, 1206 SMD, 100ppm/°C)
5. Use standard resistor and capacitor values
6. See Reference [6] for the low-sensitivity design of this biquad.

Section A Pre-distortion:

We selected R_{1A} for noise, distortion and to properly isolate the CLC111's output and C_{2A}. The pole is then set by C_{2A}. The pre-distorted value of R_{1A}, that also compensates for $C_{ni(111)}$, is (see Example 1 in *Appendix A*):

$$R_{1A} = \left(1/\omega_p - \tau_{oa}\right)/\left(C_{2A} + C_{ni(111)}\right)$$

The resulting components are in the table below:

- The Initial Value column shows the values before pre-distortion
- The Adjusted Value column shows the values after pre-distortion, and adjusting C_{2A} for $C_{ni(111)}$
- The Standard Value column shows the nearest available standard 1% resistor and capacitor values

Component	Value		
	Initial	Adjusted	Standard
R_{1A}	108Ω	100Ω	100Ω
C_{2A}	47pF	47pF	47pF
$C_{ni(111)}$	—	1.3pF	1.3pF

Section B Pre-distortion:

1. The design started with these values:

   $\omega_{p(nom)}$ = 2π(53.45MHz)
   $Q_{p(nom)}$ = 1.706
   K_B = 1.50
   α_B = 0.667
   C_{4B} + $C_{ni(446)}$ = 4.7pF
   C_{5B} = 47pF

2. Iteration 0 shows the initial design results. Iterations 1-3 pre-distort R_{12B} and R_{3B} to compensate for the CLC446's group delay:

Iteration	0	1	2	3
$\omega_{p(pd)}$ / 2π [MHz]	53.45	63.21	60.65	61.21
$Q_{p(pd)}$ [ ]	1.706	1.443	1.503	1.490
R_{12B} [Ω]	64.00	50.17	53.32	52.63
R_{3B} [Ω]	627.0	571.9	584.9	581.9

Iteration	0	1	2	3
$K\tau_{oa}R_{12B}C_{5B}$ [ns²]	2.527	1.981	2.105	2.078
ω_p / 2π [MHz]	47.15	55.18	53.08	53.53
Q_p [ ]	1.934	1.653	1.718	1.703

3. The resulting components are:

Component	Value		
	Initial	Adjusted	Standard
R_{1B}	96.0Ω	78.9Ω	78.7Ω
R_{2B}	192Ω	158Ω	158Ω
R_{3B}	627Ω	582Ω	576Ω
C_{4B}	4.7pF	3.7pF	3.6pF
$C_{ni(446)}$	—	1.0pF	1.0pF
C_{5B}	47pF	47pF	47pF
R_{fB}	348Ω	348Ω	348Ω
R_{gB}	696Ω	696Ω	698Ω

Figures 3 and 4 show simulated gains for the following conditions:

1. Ideal (Initial Values, τ_{oa} = 0)
2. Without Pre-distortion (Initial Values, $\tau_{oa} \neq 0$)
3. With Pre-distortion (Standard Values, $\tau_{oa} \neq 0$)

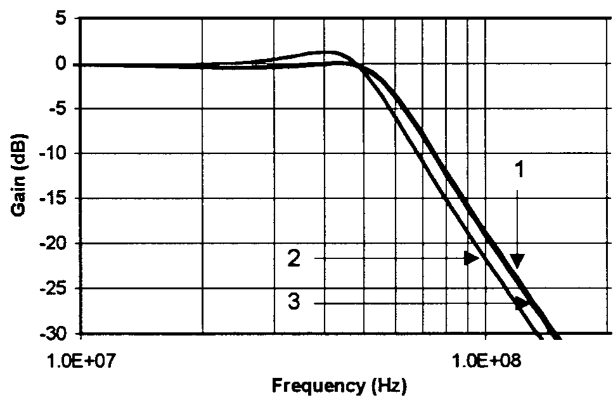

Figure 3: Simulated Filter Magnitude Response

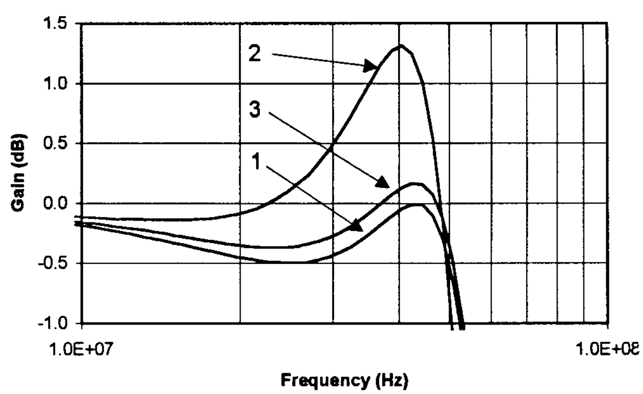

Figure 4: Simulated Filter Magnitude Response in the Passband

SPICE Models

SPICE models are available for most of Comlinear's amplifiers. These models support nominal DC, AC, AC noise and transient simulations at room temperature.

We recommend simulating with Comlinear's SPICE models to:

- Predict the op amp's influence on filter response
- Support quicker design cycles

Include board and component parasitics to obtain a more accurate prediction of the filter's response, and to further improve your design.

To verify your simulations, we recommend bread-boarding your circuit.

Summary

This App Note demonstrates a component pre-distortion method that:

- Works for popular Sallen-Key filter sections
- Is quick and simple to use
- Shows the op amp's effect on the filter response
- Gives reasonable op amp selection criteria

Appendix A and the *Design Example* section contain illustrations of this method.

Appendix A- Transfer Function Examples

Example 1:

Single pole section, K in the numerator:

$$\frac{V_o}{V_{in}} \approx \frac{K}{1+\left(1/\omega_p\right)s}$$

$$1/\omega_p = \tau_1$$

where τ_1 is a time constant set by resistors and capacitors.

To include the op amp's group delay, substitute for K and simplify:

$$\frac{V_o}{V_{in}} \approx \frac{1}{1+(\tau_1)s} \cdot \frac{K}{1+\tau_{oa}s}$$

$$\approx \frac{K}{1+\left(1/\omega_p\right)s} \quad ; \quad \omega,\ \omega_p << 1/\tau_{oa}$$

$$1/\omega_p = \tau_1 + \tau_{oa}$$

Notice that:

- There are no new powers of s in the transfer function
- Changing the resistor and capacitor values can compensate for τ_{oa}
- The approximation is reasonably accurate when $f << f_{3dB}$

To pre-distort this filter section, recalculate the resistors and capacitors using the equation:

$$\tau_1 = 1/\omega_p - \tau_{oa}$$

Example 2:

Single pole allpass section, K times the numerator:

$$\frac{V_o}{V_{in}} \approx \frac{1-\left(1/\omega_z\right)s}{1+\left(1/\omega_p\right)s} \cdot K$$

$$1/\omega_p = \tau_1$$

$$1/\omega_z = \tau_2$$

where τ_1 and τ_2 are time constants set by resistors and capacitors. This section operates as an allpass filter when:

$$\tau_1 = \tau_2$$

To include the op amp's group delay, substitute for K and simplify. Since this is an allpass transfer function, the approximation to K does not change gain at $\omega=\omega_p$:

$$\frac{V_o}{V_{in}} \approx \frac{1-(\tau_2)s}{1+(\tau_1)s} \cdot \frac{1-(\tau_{oa}/2)s}{1+(\tau_{oa}/2)s} \cdot K$$

$$\approx \frac{1-\left(1/\omega_z\right)s}{1+\left(1/\omega_p\right)s} \cdot K$$

$$\omega,\ \omega_p,\ \omega_z << 1/\tau_{oa}$$

$$1/\omega_z = \tau_2 + \tau_{oa}/2$$

$$1/\omega_p = \tau_1 + \tau_{oa}/2$$

Notice that:

- There are no new powers of s in the transfer function
- The gain at ω_p does not change (this is an allpass section)
- Changing the resistor and capacitor values can compensate for τ_{oa}
- The approximation is reasonably accurate when $f << f_{3dB}$

To pre-distort this filter, recalculate the resistors and capacitors using the equations:

$$\tau_2 = 1/\omega_z - \tau_{oa}/2$$

$$\tau_1 = 1/\omega_p - \tau_{oa}/2$$

Example 3:

Biquad section, s term in the denominator that includes K:

$$\frac{V_o}{V_{in}} \approx \frac{1}{1+\left(1/\left(\omega_p Q_p\right)\right)s+\left(1/\omega_p^2\right)s^2}$$

$$1/\left(\omega_p Q_p\right) = \tau_1 + K\tau_2$$

$$1/\omega_p^2 = \tau_3^2$$

where τ_1, τ_2 and τ_3 are time constants set by resistors and capacitors.

To include the op amp's group delay, substitute for K and simplify:

$$\frac{V_o}{V_{in}} \approx \frac{1}{1+\left(\tau_1+K(1-\tau_{oa}s)\cdot\tau_2\right)s+\left(\tau_3^2\right)s^2}$$

$$\approx \frac{1}{1+\left(1/\left(\omega_p Q_p\right)\right)s+\left(1/\omega_p^2\right)s^2}$$

$$\omega,\ \omega_p << 1/\tau_{oa}$$

$$1/\left(\omega_p Q_p\right) = \tau_1 + K\tau_2$$

$$1/\omega_p^2 = \tau_3^2 - K\tau_2\tau_{oa}$$

Notice that:

- There are no new powers of s in the transfer function
- Changing the resistor and capacitor values can compensate for τ_{oa}
- The approximation is reasonably accurate when $f << f_{3dB}$

To pre-distort this filter:

1. Design the filter assuming K constant (τ_{oa} = 0).
2. Recalculate the resistors and capacitors using the pre-distorted values of ω_p and Q_p ($\omega_{p(pd)}$ and $Q_{p(pd)}$) that will compensate for τ_{oa}:

$$1/\omega_{p(pd)}^2 = 1/\omega_{p(nom)}^2 + K\tau_2\tau_{oa}$$
$$= \tau_3^2$$
$$1/\left(\omega_{p(pd)} Q_{p(pd)}\right) = 1/\left(\omega_{p(nom)} Q_{p(nom)}\right)$$
$$= \tau_1 + K\tau_2$$

   where $\omega_{p(nom)}$ and $Q_{p(nom)}$ are the nominal values of ω_p and Q_p
3. Repeat step 2 until $\omega_p \approx \omega_{p(nom)}$ and $Q_p \approx Q_{p(nom)}$, where:

$$1/\omega_p^2 = 1/\omega_{p(pd)}^2 - K\tau_2\tau_{oa}$$
$$1/\left(\omega_p Q_p\right) = 1/\left(\omega_{p(pd)} Q_{p(pd)}\right)$$

Example 4:

Biquad section, s^2 term in the denominator multiplied by K:

$$\frac{V_o}{V_{in}} \approx \frac{1}{1+\left(1/\left(\omega_p Q_p\right)\right)s+\left(1/\omega_p^2\right)s^2}$$

$$1/\left(\omega_p Q_p\right) = \tau_1$$

$$1/\omega_p^2 = K\tau_2^2$$

where τ_1 and τ_2 are time constants set by resistors and capacitors.

To include the op amp's group delay, substitute for K and simplify:

$$\frac{V_o}{V_{in}} \approx \frac{1}{1+\left(\tau_1\right)s+\left(\tau_2^2\cdot K/\left(1+\tau_{oa}s\right)\right)s^2}$$

$$\approx \frac{e^{\tau_{oa}s}}{1+\left(1/\left(\omega_p Q_p\right)\right)s+\left(1/\omega_p^2\right)s^2}$$

$$\omega,\ \omega_p << 1/\tau_{oa}$$

$$1/\left(\omega_p Q_p\right) = \tau_1 + \tau_{oa}$$

$$1/\omega_p^2 = K\tau_2^2 + \tau_1\tau_{oa}$$

Notice that:

- The ($1+\tau_{oa}s$) factor in the numerator was converted to the exponential form, which represents a constant group delay
- There are no new powers of s in the transfer function
- Changing the resistor and capacitor values can compensate for τ_{oa}
- The approximation is reasonably accurate when $f << f_{3dB}$

To pre-distort this filter:

1. Design the filter assuming K constant (τ_{oa} = 0).
2. Recalculate the resistors and capacitors using the pre-distorted values of ω_p and Q_p ($\omega_{p(pd)}$ and $Q_{p(pd)}$) that will compensate for τ_{oa}:

$$1/\omega_{p(pd)}^2 = 1/\omega_{p(nom)}^2 - \tau_1\tau_{oa}$$
$$= K\tau_2^2$$
$$1/\left(\omega_{p(pd)} Q_{p(pd)}\right) = 1/\left(\omega_{p(nom)} Q_{p(nom)}\right) - \tau_{oa}$$
$$= \tau_1$$

   where $\omega_{p(nom)}$ and $Q_{p(nom)}$ are the nominal values of ω_p and Q_p
3. Repeat step 2 until $\omega_p \approx \omega_{p(nom)}$ and $Q_p \approx Q_{p(nom)}$, where:

$$1/\omega_p^2 = 1/\omega_{p(pd)}^2 + \tau_1\tau_{oa}$$
$$1/\left(\omega_p Q_p\right) = 1/\left(\omega_{p(pd)} Q_{p(pd)}\right) + \tau_{oa}$$

Appendix B- Electrical Loop Delay

τ_{eld} can be calculated as:

$$\tau_{eld} = x\cdot\sqrt{\varepsilon_r\mu_r}/c + \tau_{oa}$$

where:

- x is the distance around the filter feedback loop, excluding the op amp
- ε_r is the equivalent relative permittivity of the PCB trace
- μ_r is the equivalent relative permeability of the PCB trace
- *c* is the speed of light in free space (3.00×10^8 m/s)
- τ_{oa} is the op amp group delay at f_c

For a typical printed circuit board, $\sqrt{\varepsilon_r\mu_r} \approx 2.0$. This gives:

$$\tau_{eld} \approx x\cdot(0.067\ \text{ns/cm}) + \tau_{oa}$$

where x is in centimeters, and τ_{oa} is in nanoseconds.

Appendix C- Bibliography

[1] R. Schaumann, M. Ghausi and K. Laker, *Design of Analog Filters: Passive, Active RC, and Switched Capacitor*. New Jersey: Prentice Hall, 1990.

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[3] A. Williams and F. Taylor, *Electronic Filter Design Handbook*. McGraw Hill, 1995.

[4] S. Natarajan, *Theory and Design of Linear Active Networks*. Macmillan, 1987.

[5] M. Steffes, "Simplified Component Value Pre-distortion for High Speed Active Filters," Comlinear Application Note, OA-21, Rev. A, March 1993 (no longer available).

[6] K. Blake, "Low-Sensitivity, Lowpass Filter Design," *Comlinear Application Note*, OA-27, July 1996.

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National Semiconductor Corporation
1111 West Bardin Road
Arlington, TX 76017
Tel: 1(800) 272-9959
Fax: 1(800) 737-7018

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Ocean Centre, 5 Canton Road
Tsimshatsui, Kowloon
Hong Kong
Tel: (852) 2737-1600
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