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ANALYSIS AND DESIGN OF THE OP AMP CURRENT SOURCE

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A voltage controlled current source utilizing an operational amplifier is discussed. Expressions for the transfer function and output impedances are developed using both the ideal and non-ideal op amp models. A section on analysis of the effects of op amp parameters and temperature variations on circuit performance is presented.

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INTRODUCTION

Many times when one is trying to implement a specific circuit function, the need for a high quality, low cost, voltage-controlled current source becomes alarmingly apparent. The operational amplifier current source discussed here, meets the above requirements admirably. Although this circuit has been included in op amp applications literature for many years, until now it has received very little attention. It is the purpose of this application note to present the circuit along with the analysis and advantages of the design.

In the following section, the general expressions for both the basic transfer equation and the output impedance will be developed using both the ideal and the non-ideal op amp model. It will then be shown that the simplified equations, derived with the ideal op amp model, give a very accurate description of actual circuit performance and may be used in most applications with virtually no error.

Also included is an alternate configuration of the op amp current source and its respective advantages and disadvantages.

Finally, a section is included on error analysis of the circuit due to circuit parameters and temperature. Integral with this section is a discussion on statistical analysis. A type of analysis using worst case "statistics" is introduced, giving the reader an insight into actual performance one could expect using typical circuit parameter values.

A DIFFERENTIAL INPUT OP AMP CURRENT SOURCE

Theory

Figure 1 shows the schematic for a voltage-controlled current source using an operational amplifier and four resistors. This circuit has the advantage that it is capable of either sinking or supplying load current. In addition, the controlling voltage may be applied to either input, or differentially.

When an ideal op amp is assumed, the transfer equation of the circuit shown in Figure 1 is:

$$I_L = \frac{E2\,\frac{R3}{R4} - E1\,\frac{R2}{R1}}{R3 + R_L\left(\frac{R3}{R4} - \frac{R2}{R1}\right)} \qquad (1)$$

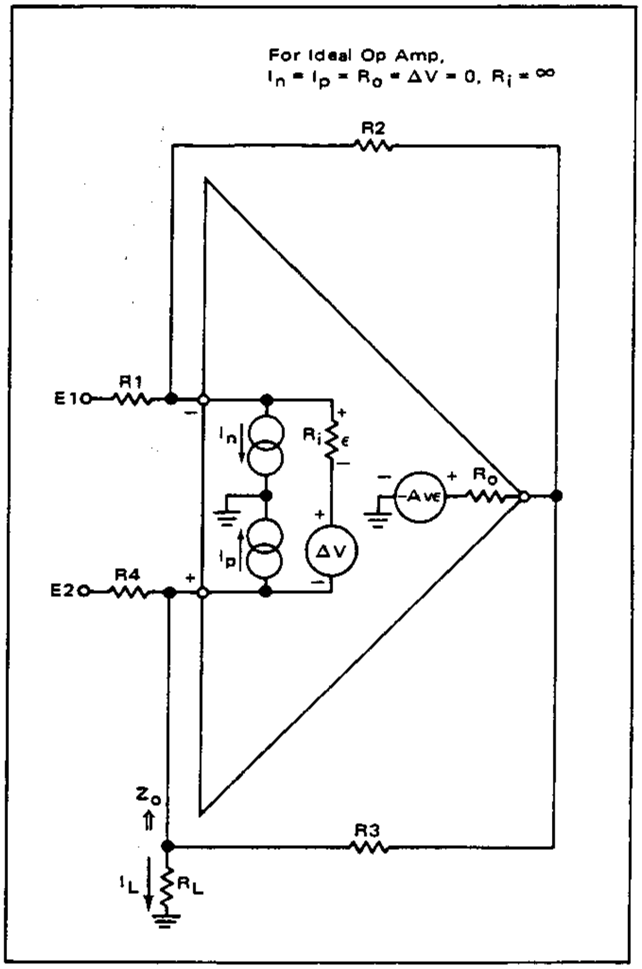

FIGURE 1 — Op Amp Current Source with Equivalent Circuit of Op Amp

While the output impedance is given by:

$$Z_o = \frac{R3}{\frac{R3}{R4} - \frac{R2}{R1}} \qquad (2)$$

It should be noted from the equations above that the operation of the circuit as a current source (i.e., a very high output impedance) is greatly dependent on the matching of the ratios of R3 to R4 and R2 to R1. If the ratios are equal, the output impedance becomes infinite producing an ideal current source. In practice, this condition is difficult to achieve. It can, however, be approached, providing a very high output impedance. It will be shown later that in 68% of all cases, one can expect $|Z_o|$ to be greater than $R3/1.31\,\epsilon$, where ϵ is the tolerance of the resistors.

Equations 1 and 2 have assumed an ideal operational amplifier. When the non-ideal op amp parameters are taken into account, the equations become:

$$I_L = \frac{E2\frac{R3}{R4} - E1\frac{R2}{R1} + \Delta V\left(1 + \frac{R2}{R1}\right) - I_nR1 - I_nR3}{R3 + R_L\left(\frac{R3}{R4} - \frac{R2}{R1}\right) + \frac{1 + \frac{R2}{R1} - \frac{R2}{R1} - \frac{R3}{R1}}{A_V + \frac{R3}{R_i} - \frac{R_o}{R_i}}\left((R_o + R3)\left(1 + \frac{R_L}{R4}\right) - \frac{R_LR_o}{R1} - R_L\right)}$$

$$+ \frac{\frac{1 + \frac{R2}{R1} - \frac{R2}{R_i} - \frac{R3}{R_i}}{A_V + \frac{R3}{R_i} - \frac{R_o}{R_i}}\left((R_o + R3)\left(I_p - \frac{E2}{R4}\right) - R_o\left(\frac{E1}{R1} - \frac{\Delta V}{R1} - I_n\right)\right)}{R3 + R_L\left(\frac{R3}{R4} - \frac{R2}{R1}\right) + \frac{1 + \frac{R2}{R1} - \frac{R2}{R_i} - \frac{R3}{R_i}}{A_V + \frac{R3}{R_i} - \frac{R_o}{R1}}\left((R_o + R3)\left(1 + \frac{R_L}{R4}\right) - \frac{R_LR_o}{R1} - R_L\right)} \quad (3)$$

$$Z_o = \frac{R3 + \frac{1 + \frac{R2}{R1} - \frac{R2}{R_i} - \frac{R2}{R_i}}{A_V + \frac{R3}{R_i} - \frac{R_o}{R_i}}}{\frac{R3}{R4} - \frac{R2}{R1} + \frac{\left(1 + \frac{R2}{R1} - \frac{R2}{R_i} - \frac{R3}{R_i}\right)\left(\frac{R_o + R3}{R4} - \frac{R_o}{R1} - 1\right)}{A_V + \frac{R3}{R_i} - \frac{R_o}{R1}}} \quad (4)$$

Where:

- A_V is the open loop voltage gain
- ΔV is the input offset voltage
- I_p is the non-inverting input bias current
- I_n is the inverting-input bias current
- R_o is the output impedance of the op amp
- R_L is the differential input impedance

Fortunately, Equations (3) and (4) need never be used since some of the terms can be neglected. Substituting typical op amp parameters, such as those of the MC1741, into expressions (3) and (4) gives:

$$I_L = \frac{E2\frac{R3}{R4} - E1\frac{R2}{R1} + \Delta V\left(1 + \frac{R2}{R1}\right) - I_nR2 - I_pR3}{R3 + R_L\left(\frac{R3}{R4} - \frac{R2}{R1}\right) + 0.400\,\Omega} + 4.00 \times 10^{-11}\text{ A} \quad (5)$$

$$Z_o = \frac{R3 + 0.396 \times 10^{-4}\,\Omega}{\frac{R3}{R4} - \frac{R2}{R1} + 0.198 \times 10^{-10}} \quad (6)$$

The magnitudes of the numerical terms as given in Equations (5) and (6) are virtually insignificant. The error introduced by neglecting these terms is much less than the uncertainty in I_L and Z_o due to the tolerance factors of even 0.001% resistors!

Therefore, Equations (7) and (8) give a very accurate representation of the performance of the operational amplifier current source.

$$I_L = \frac{E2\frac{R3}{R4} - E1\frac{R2}{R1} + \Delta V\left(1 + \frac{R2}{R1}\right) I_nR2 - I_pR3}{R3 + R_L\left(\frac{R3}{R4} - \frac{R2}{R1}\right)} \quad (7)$$

$$Z_o = \frac{R3}{\frac{R3}{R4} - \frac{R2}{R1}} \quad (8)$$

Before proceeding further, a couple of things should be noted about the exact expressions. In Equation (4), the parameters E1, E2, R_L, ΔV, I_p, and I_n are not present. This means that except for a very small term including A_V, R_o, and R_i, the output impedance is determined only by the external resistors and not the quality of the op amp. And, as has been previously shown, the term that is dependent on the op amp parameters is extremely small. This is an important result because it allows the use of lower quality op amps without degradation in output impedance.

The circuit transfer equation as given in Equation (7) shows that the load current, I_L, may be different from zero when the input voltages are zero, due to the offset voltage (ΔV) and input bias currents (I_p and I_n) of the op amp. Let this offset load current be represented by I_o. It is given by:

$$I_o = \frac{\Delta V\left(1 + \frac{R2}{R1}\right) - R2I_n - R3I_p}{R3 + R_L\left(\frac{R3}{R4} - \frac{R2}{R1}\right)} \quad (9)$$

Equation (9) shows that there is some value of ΔV such that I_o will be identically equal to zero. With most operational amplifiers, this value of offset voltage is well within the offset adjustment range of the op amp. This adjustment may be made with no degradation in performance of the rest of the circuit.

Equation (10) shows an expression for the load current written as the sum of two terms; the first is the current due to the input voltages, while the second is a constant due to the offset voltage and input bias currents.

$$I_L = \frac{E2\frac{R3}{R4} - E1\frac{R2}{R1}}{R3 + R_L\left(\frac{R3}{R4} - \frac{R2}{R1}\right)} + I_o \quad (10)$$

As stated above, I_O may be adjusted to zero. However, in some applications (such as some dual slope A/D systems) a small constant current is often desirable.

The requirement for a very high output impedance places severe restrictions on the values of R1 thru R4; namely that the ratio of R2 to R1 be as close to the ratio of R3 to R4 as possible. In most applications, all four resistors would be of equal value and with as close a tolerance as is practical. This condition will give the highest practical output impedance.

Now that it has been shown that the ratios of R2 to R1 and R3 to R4 must be very close, this fact can be used to simplify Equation (7) further. We see from Equations (7) and (8) that each time the resistors are selected to give a very high Z_O, the R_L term in the denominator of Equation (7) is very small and is truly insignificant compared to the magnitude of R3. This enables us to simplify Equation (7) to:

$$I_L = \frac{E2\frac{R3}{R4} - E1\frac{R2}{R1}}{R3} + \frac{2\,\Delta V}{R3} - I_n\frac{R2}{R3} - I_p \qquad (11)$$

or:

$$I_L = \frac{E2\frac{R3}{R4} - E1\frac{R2}{R1}}{R3} + I_O \qquad (12)$$

Where:

$$I_O = \frac{2\,\Delta V}{R3} - I_n\frac{R2}{R3} - I_p \qquad (13)$$

The Equations (11), (12) and (13) will be used as the basic transfer equations for the rest of this analysis on the op amp current source.

CURRENT SOURCE WITH FEEDBACK

Another configuration of the op amp current source is shown in Figure 2. The feedback resistor, R_F, is included to increase the maximum voltage that may be developed across the load.

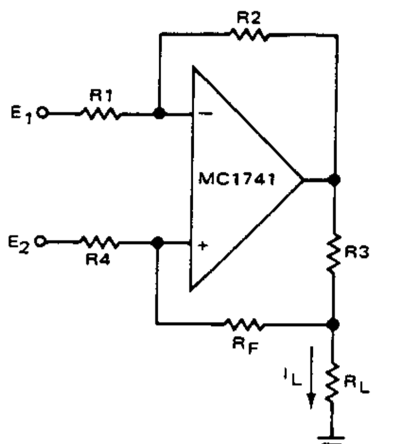

FIGURE 2 – Alternate Op Amp Current Source Approach

The maximum voltage that can be developed across the load will be determined by either the maximum common mode input voltage of the op amp or the maximum output voltage swing.

With the circuit shown in Figure 1, the voltage developed across the load is applied to the op amp as a common-mode input voltage. This voltage must always be less than the maximum common-mode voltage that the op amp can tolerate or damage to the op amp may result.

The addition of the feedback resistor, as shown in Figure 2, decreases the amount of the load voltage appearing at the non-inverting input terminal. If it is assumed that the current into the non-inverting input is negligible, the resistors R_F and R4 form a simple voltage divider. Thus the voltage on the non-inverting input terminal is simply:

$$V_{in} + I_L R_L \frac{R4}{R_F + R4} = V_L \frac{R4}{R_F + R4} \qquad (14)$$

By choosing appropriate values for R_F and R4, it can be insured that the common-mode voltage will not be exceeded. It should be noted here that most of the newer op amps have common mode voltage limits nearly equal to the supply voltage. In this case the maximum voltage that could be developed across the load is limited by the maximum voltage swing of the op amp.

The circuit shown in Figure 2 can also eliminate the problem caused by the limited output voltage swing. If we look at the transfer equation of the circuit shown in Figure 2 we have ideally:

$$I_L = \frac{\frac{R3}{R4}E2 - \frac{R2}{R1}E1}{R3 + R_L\left[1 - \frac{R4}{R4 + R_F}\left(1 + \frac{R2}{R1}\right) + \frac{R3}{R4 + R_F}\right]} \qquad (15)$$

The output impedance is given by:

$$Z_O = \frac{R3\left(\frac{R_F}{R4} + 1\right)}{\frac{R3 + R_F}{R4} - \frac{R2}{R1}} \qquad (16)$$

From Figure 2, it can be seen that all of the current flowing in R_L and R_F must flow through R3. To decrease the voltage swing at the output of the op amp, for a given load current, one must simply reduce the value of R3.

This remedy may also be used with the circuit shown in Figure 1, but it has undesirable side effects. Namely R4 must be reduced proportionally causing more current flow through R3 and R4. This extra current through R3 will cause the op amp to supply more current and consequently dissipate more power.

With the circuit shown in Figure 2 the value of R3 can be reduced without increasing the current. This is easily done by selecting appropriate values for R4 and R_F. Therefore the load voltage swing capability can be increased without increasing the voltage swing on the output of the op amp.

There are, however, restrictions on the values of the resistors. By using Equations (15) and (16) it can be seen that in order for Z_O to be infinite, R1, R2, R3, R4, and R_F must be related by:

$$\frac{R3 + R_F}{R4} = \frac{R2}{R1} \qquad (17)$$

As shown by Equations (15) and (16) both I_L and Z_O are directly proportional to R3. In the case of I_L, this simply means that different scale factors exist for E1 and E2 and they may be adjusted accordingly.

The relationship for Z_O requires that a more accurate set of resistor values be used. Ideally, R3 has no effect on the output impedance as the denominator of Equation (16) is zero. In actual circuits, however, seldom does the denominator reduce to zero, so the output impedance is directly proportional to R3. This means that for the same output impedance, a smaller value for R3 requires a respectively smaller denominator which demands a correspondingly more accurate set of resistors.

ERROR ANALYSIS

It is evident that any deviation in the value of I_L from the desired value will be directly dependent upon the respective uncertainties of the parameters determining I_L: Therefore, the load current may be written as:

$$I_L = I_{LO} \pm \delta I_L \qquad (18)$$

where I_{LO} is the desired value and δI_L is the amount I_L differs from the desired value. One way of getting δI_L is with the derivative. Differentiating implicitly gives:

$$dI_L = \frac{\partial I_L}{\partial E1} dE1 + \frac{\partial I_L}{\partial E2} dE2 + \frac{\partial I_L}{\partial R1} dR1 + \frac{\partial I_L}{\partial R2} dR2 + \ldots (19)$$

If we use Equation (11) for the function of I_L, this results in:

$$dI_L = \frac{dE2}{R4} - \frac{R2\,dE1}{R1R3} + \frac{R2E1\,dR1}{(R1)^2R3} - \left[\frac{E1}{R1R3} + \frac{I_n}{R3}\right] dR2 + \left[\frac{E1R2}{R1(R3)^2} - \frac{2\,\Delta V}{(R3)^2} - \frac{I_nR2}{(R3)^2}\right] dR3 - \frac{E2\,dR4}{(R4)^2} + \frac{2d\,\Delta V}{R3} - \frac{R2\,dI_n}{R3} - dI_p \qquad (20)$$

Replacing the derivative terms in Equation (20) by their respective uncertainties results in:

$$\delta I_L = \frac{1}{R4}\delta E2 - \frac{R2}{R1R3}\delta E1 + \frac{R2E1}{(R1)^2R3}\delta R1 - \left(\frac{E1}{R1R3} + \frac{I_n}{R3}\right)\delta R2 + \left(\frac{E1\,E2}{R1(R3)^2} - \frac{2\,\Delta V}{(R3)^2} - \frac{I_nR2}{(R3)^2}\right)\delta R3 - \frac{E2}{(R3)^2}\delta R4 + \frac{\partial}{R3}\delta\Delta V - \frac{R2}{R3}\delta I_n - \delta I_p \qquad (21)$$

As mentioned earlier, in most applications resistors R1 thru R4 will be of equal value nominally. Let this nominal value be represented by R and the tolerance by ϵ. The exact value of each resistor will be R plus or minus some small amount ϵR. Where $\delta R \leqslant \epsilon R$. Allowing each resistor to have a different value we have:

$$R1 = R \pm \delta R1,\ R2 = R \pm \delta R2, \text{etc.}$$

If ϵ is small, we can replace each resistor in Equation (21) by the nominal value without introducing appreciable error in δI_L. This allows Equation (21) to be written as:

$$\delta I_L = \frac{1}{R}\delta E2 - \frac{1}{R}\delta E1 + \frac{E1}{R^2}\delta R1 - \left(\frac{E1}{R^2} + \frac{I_n}{R}\right)\delta R2 + \left(\frac{E1}{R^2} - \frac{2\,\Delta V}{R^2} - \frac{I_n}{R}\right)\delta R3 - \frac{E2}{R2}\delta R4 + \frac{2}{R}\delta\Delta V - \delta I_n - \delta I_p \qquad (22)$$

Equation (22) may be used to determine worst case error. As the name implies, worst case error is that condition where all terms are of magnitude and sign such as to give the largest possible absolute value for δI_L. This condition implies that every δ term be at the maximum value allowed by the uncertainty limits and of proper sign so that all terms are additive. When these values are substituted into Equation (22), we get a maximum value for δI_L. This value is the absolute worst case value, under no condition will I_L differ from the desired value by more than δI_L (worst case).

Usually the worst case value of δI_L is an unrealistically pessimistic approach. A value for δI_L is needed that will give a better estimation of circuit performance. One such indication of performance is the standard deviation, which by definition, gives a set of limits that encloses the actual value of δI_L for 68% of all cases. Equation (23) gives an expression for $\sigma\delta I_L$ in terms of the worst case limits of the parameters.

$$\sigma\delta I_L = \left[\left(\frac{0.433}{R}\right)^2 \delta E1^2 + \left(\frac{0.433}{R}\right)^2 \delta E2^2 + \left(\frac{0.433E1}{R^2}\right)^2 \delta R1 + \left(\frac{E1}{R^2} - \frac{I_n}{R}\right)^2 (0.433\delta R2)^2 + \left(\frac{E1}{R^2} - \frac{2\,\Delta V}{R^2} - \frac{I_n}{R}\right)^2 (0.433\delta R3)^2 + \left(\frac{0.433E2}{R^2}\right)^2 \delta R4^2 + \left(\frac{0.666}{R}\right)^2 \delta\Delta V^2 + (0.333\delta I_n)^2 + (0.333\delta I_p)^2\right]^{1/2} \qquad (23)$$

where the δ terms are the worst case values of the parameters. Note that if the worst case limits of the parameters are not symmetrically placed about their nominal values, a separate upper and lower value of δI_L must be computed. When this is the case, $\sigma\delta I_L$ is no longer a standard deviation and the significance of $\sigma\delta I_L$ changes. For more information on this and for the derivation of Equation (23), see the Appendix to this note.

Thus far, in this analysis of I_L errors, no mention has been made of temperature drift. Changes in I_L due to temperature drift of the determining parameters are present and an expression of this function would be very desirable. The analytical expression for dI_L/dT is easily obtained from Equation (20), the implicit differentiation of the I_L equation. Dividing each term in Equation (20) by dT gives the desired dI_L/dT.

$$\frac{dI_L}{dT} = \frac{1}{R4}\frac{dE2}{dT} - \frac{R2}{R1R3}\frac{dE1}{dT} + \frac{R2E1}{(R1)^2R3}\frac{dR1}{dT} - \left(\frac{E1}{R1R3} + \frac{I_n}{R3}\right)\frac{dR3}{dT} + \left(\frac{E1R2}{R1R3} - \frac{2\Delta V}{(R3)^2} - \frac{I_nR2}{(R3)^2}\right)\frac{dR3}{dT} - \frac{E2}{(R4)^2}\frac{dR4}{dT} + \frac{2}{R3}\frac{d\Delta V}{dT} - \frac{R2}{R3}\frac{dI_n}{dT} - \frac{dI_p}{dT} \quad (24)$$

where the d/dT terms are the respective parameters' rates-of-change with respect to temperature. The worst case value of dI_L/dT is easily obtained. If desired, Equation (24) may be treated like Equation (22) to find a value of dI_L/dT which is more probable than the worst case value.

Ideally, the op amp current source has an infinite output impedance. In this case, any error in the value of Z_O is meaningless as infinity plus or minus anything is still infinite. Differentiating Equation (8) implicitly results in

$$dZ_O = \frac{\frac{R3}{R2}dR1 - \frac{R3R1}{R2^2}dR2 - \frac{R1}{R2}dR3 + \frac{R3^2}{R4^2}dR4}{\left(\frac{R3}{R4} - \frac{R4}{R1}\right)^2} \quad (25)$$

From Equation (25) it can be seen that if R3/R4 = R2/R1 then $dZ_O = \infty$. However, at the same time the value of Z_O is also infinity. As stated earlier, this condition conveys no useful information.

Since the circuit is used as a current source and the output impedance should be as large as possible, all that is important is the minimum value of Z_O. To investigate this condition, let each resistor be nominally R plus or minus some small amount ϵR, where ϵ is the tolerance of the resistors. Substituting these values for the resistors into Equation (8) gives:

$$Z_O = \frac{R(1 \pm \epsilon 3)}{\frac{1 \pm \epsilon 3}{1 + \epsilon 4} - \frac{1 \pm \epsilon 2}{1 + \epsilon 1}} \quad (26)$$

The minimum value Z_O can easily be obtained by selecting signs such that the denominator is maximized and the numerator is at the minimum. If the ϵ factors are equal, Equation (26) reduces to:

$$Z_O(\text{min}) = R\frac{1 + \epsilon}{\frac{1 + \epsilon}{1 - \epsilon} - \frac{1 - \epsilon}{1 + \epsilon}} = \frac{R}{4}\left(\frac{1}{\epsilon} + 1 - \epsilon - \epsilon^2\right) \quad (27)$$

or:

$$Z_O(\text{min}) = \frac{R}{4\epsilon} \quad (28)$$

Again the worst case value is unrealistically pessimistic. The probability that all the resistors will be of proper sign and magnitude to give Z_O (min) is remote. Using the analysis method outlined in the Appendix results in:

$$Z_O'(\text{min}) = \frac{R}{1.31\,\epsilon} \quad (29)$$

Where Z_O' (min) is the lower 68% confidence limit for Z_O and ϵ is the tolerance of the resistors.

In most circumstances the rates-of-change with temperature of all the resistor values is about the same. If this is true, the rate-of-change of Z_O with temperature is very close to zero, since dZ_O/dT is a function of only the resistors and their temperature derivations. If needed, dZ_O/dT could be obtained by the same method that was used to determine Z_O (min), however, the approximation of dZ_O/dT to zero will be very close.

SUMMARY

From the preceding discussion, it is apparent that the operational amplifier current source is a high quality current source in all respects. Not only can the current be accurately controlled but the required high output impedance is easily achieved.

Due to the capability of the op amp output to swing either positive or negative, there are no polarity restrictions on any of the voltages or currents in the transfer equation. This gives the circuit a great deal of versatility. Not only can the circuit sink and source load current, it can be controlled by either the sum or difference of two or more voltages and the input voltage may be applied differentially.

Another important feature of the op amp current source is that the transfer equation remains a linear function as the load current is decreased through zero. In other words, as well as working in either direction, the circuit exhibits no "crossover" distortion through zero. One should note, as shown by Equation (12), that unless the offset current, I_O, has been adjusted out, the input voltage may not be exactly zero when the load current is identically zero. This fact, however, does not introduce any non-linearity into the transfer function, only a small amount of offset current.

It has already been shown that the quality of the op amp used in the circuit is unimportant. The changes in load current due to changes in op amp parameters are small. This means that the supply rejection of the circuit is basically that of the op amp being used, which is usually on the order of 30 microvolts per volt or better.

Since the load current is virtually independent of the op amp parameters, changes in load current due to temperature are due only to the external resistor values and input voltages.

Still another feature of the op amp current source is its ability to present a constant output impedance under changing conditions of both the load current and the voltage across the load. The output impedance is a function of the external resistors only.

In summary, the operational amplifier has many advantages over conventional types of current sources. Some of these are briefly listed here.

1) Has the ability to either sink or source load current.
2) Can use a voltage of either polarity to control the load current.
3) Input voltage may be applied differentially.
4) Current may be controlled by the sum or difference of two voltages.
5) Circuit can supply a constant current of either polarity, independent of the controlled current.
6) Controlled current value is determined by external resistors and voltages and not by op amp parameters.
7) Has easily achievable high output impedance.
8) Temperature drift of controlled current is inherently very low and determined primarily by external parameters.
9) Has excellent linearity over entire input-voltage range.
10) Output impedance remains constant with changes in either load current or input voltage.
11) Has the ability to supply current into or out of a grounded load.

The potential applications of the op amp current source are tremendous because of its versatility, high accuracy, and low cost. In addition, the circuit is easy to design and use.

One typical application of the op amp current source is in the dual slope type of analog-to-digital converter. It is very suitable to this application because of its high accuracy and ability to either sink current from or source current to the integrator. Figure 3 shows a typical front-end of a dual slope DVM using the op amp current source.

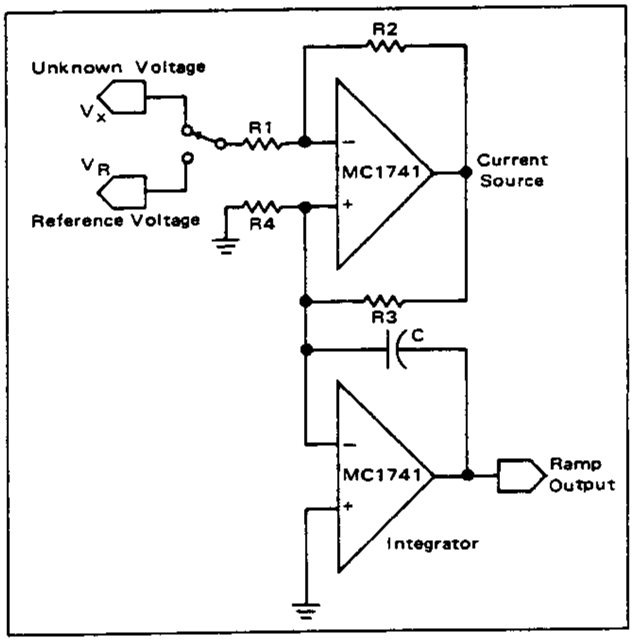

FIGURE 3 – Current Source used in Front End of DVM Which uses a Dual Slope A/D Converter

The operational amplifier current source absolutely has no peers as a voltage-controlled current source where the load current needs to approach or pass through zero. Some conventional types of current sources work to zero output current but not without a sacrifice in both accuracy and linearity.

One should not get the impression that the op amp current source should only be used as a variable-current source. On the contrary, the op amp current source is unquestionably the finest circuit available for general purpose usage.

APPENDIX A

Derivation of δI_L

In circuit analysis, the problem is frequently encountered of determining the uncertainty in a parameter which is a function of two or more independent quantities when given the respective uncertainties of those quantities. For example, suppose it is necessary to find the uncertainty in Z from the uncertainties in X and Y. Where:

$$Z = F(X, Y) \quad \text{(A1)}$$

If X and Y have Gaussian distributions, then obviously Z will have also. The variance, and hence the standard deviation of Z can be determined from the respective variances of X and Y by a theorem of elementary statistics, namely:

$$\sigma Z = \left[\left(\frac{\partial Z}{\partial X}\right)^2 \sigma X^2 + \left(\frac{\partial Z}{\partial Y}\right)^2 \sigma Y^2\right]^{1/2} \quad \text{(A2)}$$

Where σ^2 is the variance and σ represents the standard deviation. This equation may be expanded to as many variables as desired, provided they are independent.

In most applications of the op amp current source, the only information known about the parameters determining δI_L is their nominal values and a set of worst case limits for each. A useful relationship between these worst case limits and a statistical description of δI_L is needed.

The problem is confounded by the fact that seldom do all of the parameters determining δI_L have Gaussian or normal distribution curves. This being the case, the dependent quantity, δI_L, cannot have a true normal distribution curve. In this case, the standard deviation of δI_L is not a true standard deviation because the term is defined only for a normal or Gaussian distribution. The fact that the term standard deviation is undefined does not prevent us from finding a set of limits such that 68% of all values of δI_L will fall between them. These limits will be defined as $I_{LO} \pm \sigma\delta I_L$, where $\sigma\delta I_L$ would be the standard deviation of δI_L if the function δI_L had a true normal curve. The only consequence of δI_L not having exactly a normal curve is that our value for $\sigma\delta I_L$ cannot be doubled with the expectation that 95% of our values for δI_L will fall between $I_{OL} \pm 2\,\sigma\delta I_L$. In most cases I_L will closely approximate a Gaussian curve and the ±2 Sigma limits will be close to 95%, but this cannot be assumed without an actual description of the distribution curve.

The operational amplifier parameters have Gaussian distributions. It is an easy matter to determine their respective standard deviations from their worst case limits. Knowing that 99% of all values fall between the nominal value $\pm 3\sigma$, we simply set the $\pm 3\sigma$ limits equal to the worst case values and solve for σ. Using this relationship, the σ values of the op amp parameters are represented by one-third the worst case limit.

The resistors present a unique problem because no information on the shapes of the actual distribution curves is available. One way to circumvent this problem would be to use a type of worst case analysis. With this method one would assume a worst case distribution and determine the limits enclosing 68% of all possible values. These limits would then be used in Equation (A-2) to determine the standard deviation of δI_L. Any other allowable distribution's set of 68% confidence limits would have to fall within those obtained using our "worst case" distribution.

Now the only problem is to select a "worst case" curve and define a way to separate allowable distributions from those which are not allowable. Figure A-1 shows a few typical resistor distribution curves and the distributions of the respective δR_S, where δR is the difference between the nominal value and the actual value.

Ideally, a "worst case" curve would be one which allows every possible distribution and still gives a narrow set of 68% confidence limits. Clearly, these conditions are contradictory and some compromise must be made.

There is, however, a very logical curve to select. The rectangular distribution, where each value between the end limits has an equal probability, allows the absolute maximum amount of disorder to be given to the resistor value, while simultaneously providing an easily defined set of 68% confidence limits. These limits are simply those values equally spaced either side of the nominal value which enclose 68% of the area under the δR distribution curve.

Selection of the rectangular distribution also gives us an easy definition of what an "allowable" distribution must be. Namely, that it must be symmetrical about the center value and there can be one and only one inflection point between the end limits. If the above conditions are met there is no possible distribution such that the 68% confidence limits are further apart than those obtained with the rectangular distortion.

Note that the above conditions eliminate the skewed distribution where the worst case limits are not equally spaced about the nominal value. A skewed distribution may, however, be handled if we compute a separate upper and lower $\sigma\delta I_L$.

Using simple geometry it can be shown that 68% of the area under the δR curve shown in Figure A-1f is between the vertical lines 0.433 X either side of the center value, where X is the worst case value.

The input voltages, E1 and E2 should be handled in the same manner as the resistors unless the designer has enough information about them to assume they have Gaussian distribution curves.

With a value for the 68% confidence limits of each parameter, these values are substituted into (A-2) to get:

$$\sigma\delta I_L = \left[\left(\frac{0.433}{R}\,\delta E1\right)^2 + \left(\frac{0.433}{R}\,\delta E2\right)^2 + \left(\frac{E1}{R^2}\,0.433\,\delta R1\right)^2 + \left(\left(\frac{E1}{R^2} + \frac{I_n}{R}\right) 0.433\,\delta R2\right)^2 + \left(\left(\frac{E1}{R^2} - \frac{\partial \Delta V}{R^2} - \frac{I_n}{R}\right) 0.433\,\delta R3\right)^2 + \left(\frac{E2}{R2}\,0.433\,\delta R4\right)^2 + \left(\frac{\partial}{R}\,0.333\,\delta\Delta V\right)^2 + (0.333\,\delta I_n)^2 + (0.333\,\delta I_p)^2\right]^{1/2} \quad \text{(A-3)}$$

Where the δ terms are the worst case values.

The actual significance of $\sigma\delta I_L$ is: given values for the parameters and their worst case errors, a minimum of 68% of all the values for I_L can be expected to fall within the interval $I_{LO} \pm \sigma\delta I_L$. It is possible that a much larger percentage will be within these limits, but we are guaranteed at least 68%.

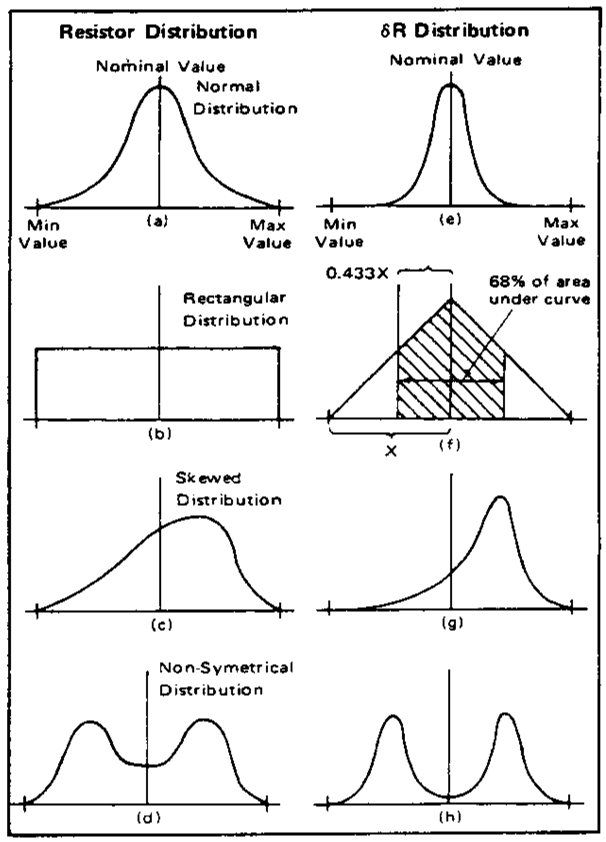

FIGURE A-1 — Parameter Distributions and Error Distributions

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