6.2 Magnetic Units and Definitions

Magnetic Flux Φ

The magnetic flux Φ results as a product of electric voltage and time. The S.I. unit of magnetic flux is the Weber (Wb) or the Volt-second (Vs). If the magnetic flux Φ changes uniformly by 1 unit over a time of 1s, then a voltage of 1 V is induced in a conductive loop encircling it. Hence the following is valid:

$$1 \text{ Wb} = 1 \text{ Vs}$$

Magnetic Induction B

The magnetic induction B or the magnetic flux density is formed as a quotient from magnetic flux over area. The S.I. unit is Tesla (T). It is defined as the magnetic induction of a homogenous magnetic flux which perpendicularly cuts an area of 1 m^2 with a strength of 1 Wb. The following is also valid:

$$1 \text{ T} = 1 \text{ Wb/m}^2 = 1 \text{ V} \times \text{s/m}^2$$

Magnetic Field Strength H

The magnetic field strength H is the quotient of electric current strength and length. The S.I. unit of Magnetic field strength is the Ampere per meter (A/m). It is defined as the magnetic field strength which is generated in free space on the 1 m circumference of a circle concentric to a circular cross-section conductor of infinite length carrying a current of 1 A.

Magnetic Potential F

In a homogenous magnetic field, the magnetic potential F is the product of the magnetic field strength H and the path length under consideration along a field line. The unit of magnetic potential is the Ampere.

Magnetic potential is also known as **M**agneto **M**otive **F**orce (MMF).

Table 9

		SI Unit		Previous Unit		Relation
Induction	B	Tesla	T	Gauss	G	1 G = 10^{-4} T
Flux	Φ	Weber	Wb	Maxwell	M	1 M = 10^{-8} Wb
Magnetic field strength	H	$\frac{\text{Ampere}}{\text{metre}}$	A/m	Oersted	Oe	1 Oe = $10^3/4\pi$ A/m
Magneto Motive Force	F	Ampere	A	Gilbert	Gb	1 Gb = 1 Oe $\times$ cm

Electromagnetic Circuit

The magnetic field of an electrically stimulated magnetic circuit is arranged such that the magnetic circumferential potential, the current flow in Ampere turns and the combined magnetic flux is equal at each point.

For calculating such a magnetic circuit, the following equation is used:

$$\oint H \times \mathrm{dw} = I \times \mathrm{n}$$

For an annular magnetic circuit with an airgap δ and a constant cross section as shown in **figure 24a** we have, neglecting leakage flux:

$$H \times l = I \times \mathrm{n}$$

where: H = magnetic field strength
l = magnetic path length
I = excitation current
n = no. of turns

The path of the magnetic flux lies partly in iron and partly in air. It can be separated as follows

$$(H_{\mathrm{Air}} \times \delta) + H_{\mathrm{Fe}} \times l_{\mathrm{Fe}} = I \times \mathrm{n}$$

and as the magnetic flux is equal through the whole circuit, the flux density in air is the same as in iron

$$B_{\mathrm{Fe}} = B_{\mathrm{Air}}$$

and from the relation $B = \mu_r \times \mu_0 \times H$ we have

$$\frac{(B_{\mathrm{Air}} \times \delta)}{\mu_0} + \frac{(B_{\mathrm{Air}} \times l_{\mathrm{Fe}})}{\mu_r \times \mu_0} = I \times \mathrm{n} \qquad (\mu_{\mathrm{Air}} = 1)$$

and from this

$$B_{\mathrm{Air}} = \frac{\mu_0 \times I \times \mathrm{n}}{\delta + \frac{l_{\mathrm{Fe}}}{\mu_r}}$$

where $\mu_0 = 1.256 \times 10^{-6}$ Vs/Am and is the magnetic field constant. μ_r = relative permeability of the iron ring.

For magnetically soft magnetic material μ_r is very large (μ_r > 1000), so that airgaps of a few tenths of a millimeter can be neglected when dealing with iron path length of some centimeters. The airgap induction B_{Air} is then proportional to the current flow through the core, by the relation:

$$B_{\mathrm{Air}} = \frac{\mu_0 \times I \times \mathrm{n}}{\delta}$$

The specific magnetic properties of iron, described by the magnetic field dependence of the permeability, therefore do not arise. With increasing current flow the iron core reaches saturation, the permeability drops significantly and the induction in the airgap increases only very weakly with increasing current flow.

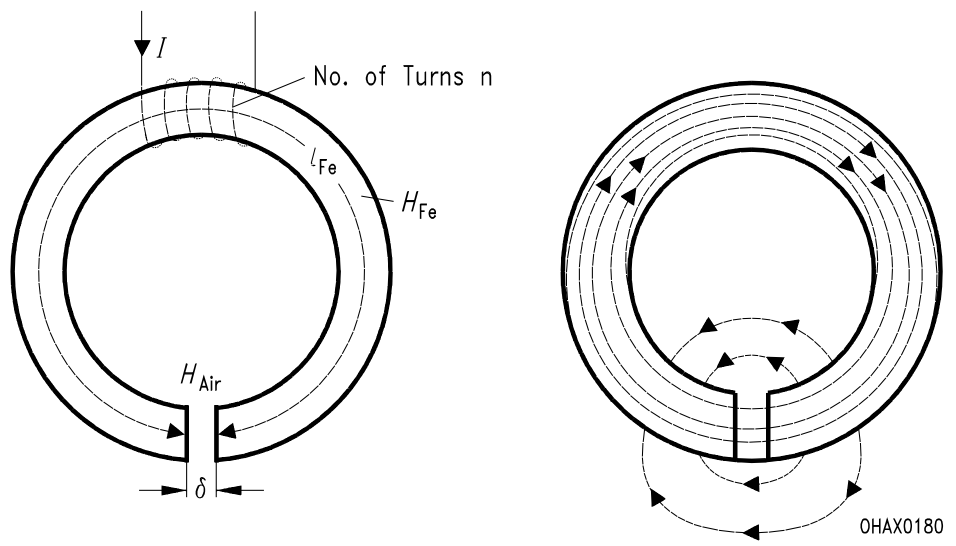

Figure 24a
Electrically excited Annular Magnetic Circuit with an Airgap

Figure 24b
Path of the Lines of Induction in a Magnetic Circuit with an Airgap

The equation is only valid under the condition of constant induction B_{Fe} throughout the iron path l_{Fe}. With the presence of an airgap, this condition is only approximately fulfilled. As **figure 24b** shows, the field lines start to spread out before reaching the airgap and fringe around the airgap. This effect is known as airgap fringing. It has the effect that the magnetic induction along the length of the iron path is not constant, and the maximum value of the magnetic induction is reached at a point in the iron core diametrically opposite to the airgap. Magnetic circuits used with Hall generators and magneto resistors are usually configured such that the airgap can be neglected. The equation given above therefore can be used to calculate the airgap induction to sufficient accuracy.

Permanent Magnetic Circuit

Figure 25 shows a magnetic circuit excited by a permanent magnet. It is assumed that the magnet of length l_m has a constant magnetization of M_m. The magnetization of a permanent magnet is dependent on the demagnetization curve (B/H curve) of the magnet material and on the working point on the curve of the magnet. The working point is determined by the magnet dimensions and on the geometry of the magnetic circuit. If H_m defines the magnetic field strength and B_m the magnetic induction within the permanent magnet then the following equation applies:

$$B_m = \mu_0 \times H + M_m$$

and as the current flow through the magnetic circuit is zero we have

$$H_{Air} \times \delta + H_{Fe} \times l_{Fe} + H_m \times l_m = 0$$

The airgap field strength H_{Air} and the field strength within the iron H_{Fe} have the same directional sense. As the magnetic ring voltage disappears, the field strength H_m within the permanent magnet must be in the direction opposite to that of the flux.

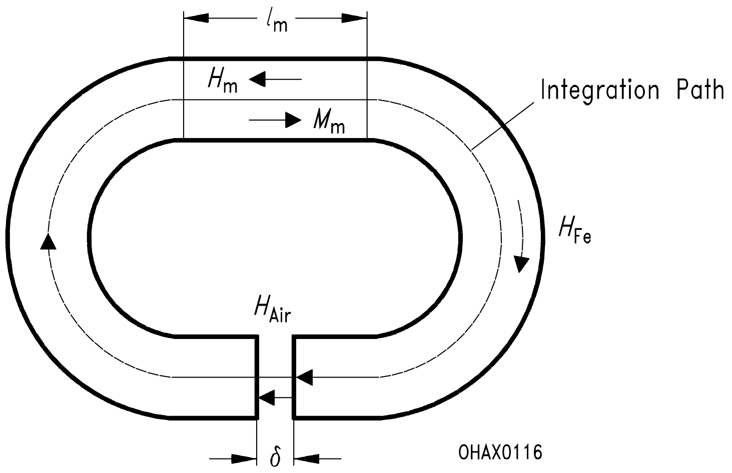

Figure 25
Magnetic Circuit excited by a Permanent Magnet

With the conditions: $B_{Air} = B_{Fe}$

and $B_m = B_{Fe}$

which arise from the source independent of the magnetic induction, the induction in the airgap is calculated by:

$$B_{Air} = \frac{M_m}{1 + \frac{\delta}{l_m} + \frac{l_{Fe}}{\mu \times l_m}}$$

In general, $\delta << l_m$ and the permeability μ of the iron circuit is very large ($\mu > 1000$). As opposed to an electrically excited circuit, a permanent magnet circuit under the influence of constant magnetization of a permanent magnet has an airgap induction which to a first approximation is independent of the size of the airgap δ. As already mentioned, however, the magnetization of M_m of the permanent magnet also depends on the position of the working point on the demagnetization curve. The working point, however, changes with the size of the airgap.

The assumption of the magnetization being constant is in practice inaccurate. With steep demagnetization curves this can therefore cause a large change in the airgap induction B_{Air} in dependance with the airgap size. To achieve higher airgap induction B_{AIR} the cross section S_m of the permanent magnet is, in technical applications, chosen to be greater than the airgap cross section S_{AIR}. The flux is then equated by

$$B_m \times S_m = B_{Air} \times S_{Air}$$

This gives the airgap induction:

$$B_{Air} = \frac{M_m}{\frac{S_{Air}}{S_m} + \frac{\delta}{l_m} + \frac{l_{Fe}}{\mu \times l_m}}$$

and if $\delta << l_m$ and μ is large, then as a first order approx.

$$B_{Air} = \frac{S_m}{S_{Air}} \times M_m$$

The airgap induction is therefore increased by a factor S_m/S_{Air} over the circuit of constant cross section.

To simplify estimation of magnetic fields under constant conditions in the airgap, the following equations can be used. They are not exact relations as such, as they contain simplifying parameters. The subscripts “m” and “Air” indicate whether the values relate to the magnet or the airgap. The values for the field strength and the induction in the magnet are given for the working point with ($H_a \times B_a$).

Calculation of the magnetic length (l = length)

$$l_m = \frac{B_{Air} \times \delta}{\mu_a \times H_a}$$

Calculation of the magnetic cross section (S = cross section)

$$S_{\mathrm{m}} = \frac{B_{\mathrm{Air}} \times S_{\mathrm{Air}}}{B_{\mathrm{a}}} \times \sigma$$

Calculation of the magnetic volume (V = volume)

$$V_{\mathrm{m}} = \frac{B_{\mathrm{Air}}^{2} \times V_{\mathrm{Air}}}{\mu_0 \times H_{\mathrm{a}} \times B_{\mathrm{a}}} \times \sigma \times \tau$$

for the given geometry of a magnetic circuit the airgap induction follows form the above equations.

$$B_{\mathrm{Air}} = \sqrt{\frac{\mu_0 \times H_{\mathrm{a}} \times B_{\mathrm{a}} \times V_{\mathrm{m}}}{V_{\mathrm{Air}}} \times \frac{1}{\sigma \times \tau}}$$

It is therefore dependent on the energy product at the working point. The maximum airgap induction is then achieved when the $(B \times H)_{\mathrm{max.}}$ point is chosen as the working point. It is useful to choose the working point somewhat higher than the $(B \times H)_{\mathrm{max.}}$ point, the value can be taken from the demagnetization curve of the relevant magnetic material. As B_{Air} increases with the square root of the volume and of the energy product, a doubling of either of these values only leads to an increase of about 40% in the airgap induction.

Using this formula, it is possible to get the dimensions of a permanent magnet with specific regard to leakage flux factor σ and the factor τ.

The leakage factor σ gives the relation of the total flux in the airgap. In general it lies somewhere between 1.2 and 5. Its reciprocal $1/\sigma$ corresponds approximately to the system efficiency and usually lies somewhere between 0.2 and 0.8.

The field decay along a flux conduction portion and in the center of the airgap of the magnetic circuit is accounted for by the factor τ. It lies in the region of 1 to 1.5 and a good average value would be 1.2.

Another method for dimensioning a permanent magnet is based on the choice of value of the demagnetization factor. This determines for example for a bar magnet an optimal length to the breadth ratio, from the requirement that the demagnetization line of gradient $-\mu_0/N$ cuts the demagnetization curve at the desired working point of (H_{a}, B_{a}). From this requirement the optimal demagnetization factor N for the relevant Magnetic material can be established.

Permanent Magnets

A wide selection of permanent magnet materials are available, each with useful properties depending on the application. The properties are often best described by depiciting the 2nd quadrant of the B-H hysteresis loop, known as the demagnetization curve. This curve shows the usable remnance field generated by a permanent magnet under various load conditions.

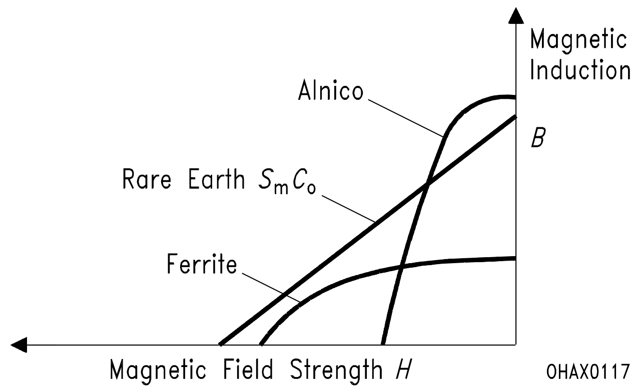

Figure 26

The loading of a magnet determines the point of the BH curve where the magnet will operate. An unloaded magnet is not really a practical proposition as it would need to be a toroid of magnetically saturated material with no airgaps. In this case the magnetic center line, i.e. any plane intersecting the toroid perpendicular to the magnetic axis would indicate a remnance of B_r, and this is a material dependent constant.

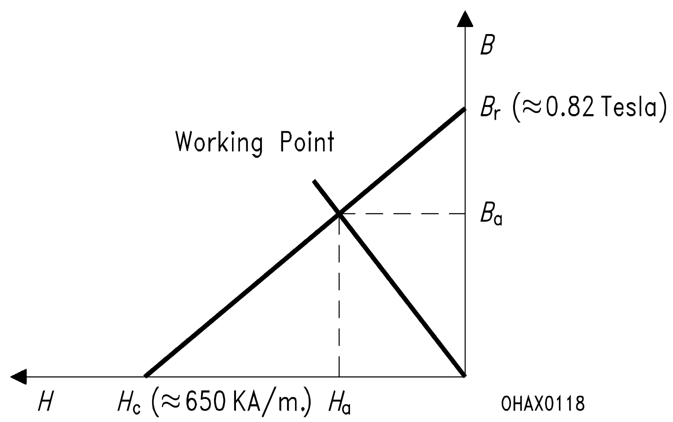

Figure 27
Demagnetization Curve for SmCo

If an airgap is introduced into the toroid then the magnet experiences a degree of loading, i.e. the airgap permeability is seen as a resistance to the flux, and the working point of the magnet, measured at the point on the toroid directly opposite the airgap is reduced to the value B_a. From this and from the preceeding equations it can be seen that the working point is dependent on magnet geometry and on the airgap size so that in practice a fully loaded magnet is effectively a short bar magnet (i.e. a short magnetic length with a very large airgap), e.g. see **figure 27**.

The addition of ferromagnetic flux linking parts to the magnet, or even the proximity of such parts, which will influence the permeability of the surrounding space, serve to raise the working point on the BH curve and hence the usable magnetic field produced by the magnet.

As the demagnetization curve is part of the hysteresis loop, it is evident that the working point of a magnet moves down the BH curve as the magnet is loaded, but does not retrace its original path when this loading is removed. Instead the working point traces a recoil loop from its lowest working point (the point to which it was unloaded). This recoil loop has a gradient approximately equal to that of the BH curve at the point B_r. From the various profiles of the BH curves for various materials it can be seen that this effect is of varying significance, e.g. it is very pronounced in Alnico type magnets, but for SmCo where the gradient of the BH curve is effectively constant it is of little importance. This is a factor which influences magnet geometry.