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070-0526-00

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# INTRODUCTION TO OPERATIONAL AMPLIFIERS

Functionally speaking, an operational amplifier is a device which, by means of negative feedback, is capable of processing a signal with a high degree of accuracy limited primarily only by the tolerances in the values of the passive elements used in the input and feedback networks.

Electronically, an operational amplifier is simply a high-gain amplifier designed to remain stable with large amounts of negative feedback from output to input.

General-purpose operational amplifiers, useful for linear amplification with precise values of gain, and for accurate integration and differentiation operations, have low output impedance and are DC-coupled, with the output DC level at ground potential.

The primary functions of the operational amplifier are achieved by means of negative feedback from the output to the input. This requires that the output be inverted (180° out of phase) with respect to the input. The conventional symbol for the operational amplifier is the triangle shown in Figure 1-a. The output is the apex of the triangle; the input is the side opposite the output. Negative feedback, through a resistor, capacitor, inductor, network or non-linear impedance, designated " $Z_f$ " is applied from the output to the input as shown in Figure 1-b. The input to which negative feedback is applied is generally termed "- input"\* or "- grid" (in the case of vacuum-tube operational amplifiers).

### Operational Amplifier Seeks Current Null at — Input

An operational amplifier, using negative feedback, functions in the manner of a self-balancing bridge, providing through the feedback element whatever current is necessary to hold the — input at null (ground potential). See Figure 1-b. The output signal is a function of this current and the impedance of the feedback element.

The – input, held to ground potential by the feedback current, appears as a very low impedance to any signal source. Using resistive feedback, for instance, the input appears to be the resistance of the feedback element, divided by the open-circuit gain of the operational amplifier.



Figure 1. Conventional Operational Amplifier Symbols.

- (a) The input is to the base of the triangular symbol, the output is from the apex opposite. The – input and output are out-of-phase (arrows).
- (b) Feedback element Z<sub>f</sub> provides the negative feedback to permit high-accuracy operations. The amplifier seeks a null at the input by providing feedback current through Z<sub>f</sub> equal and opposite to the input current l<sub>in</sub>. Output voltage is whatever is necessary to provide required balancing current through Z<sub>f</sub>.
- (c) Input element Z<sub>i</sub> converts a voltage signal (E<sub>in</sub>) to current, which is balanced by current through Z<sub>f</sub>.

If current is applied to the — input, it would tend to develop voltage across the impedance of the feedback element, and move the — input away from ground potential. The output, however, swings in the opposite direction,

<sup>\*</sup> The operational amplifiers of the Tektronix Type O Plugin unit also provide access to a non-inverting input. Uses of this "+ input" or "+ grid" are discussed later.

providing current to balance the input current and hold the - input at ground. If the impedance of the feedback element is high, the output voltage must become quite high to provide enough current to balance even a small input current.

### Input Element Z<sub>i</sub> Converts **Input Signal to Current**

Since we more often have to deal with voltage rather than current signals, an additional element is used in most operational amplifier applications, designated " $Z_i$ " (input impedance). This is an impedance placed in series with the - input, converting into current that parameter of the input signal which we want to appear as voltage at the output (Figure 1-c).

If  $Z_i$  and  $Z_f$  are both resistors (Figure 2), the operational amplifier becomes a simple voltage amplifier, the gain of which is  $-Z_f/Z_i$ .



Figure 2.

(a) Operational amplifier using resistors for both Z<sub>i</sub> and Z<sub>f</sub> becomes fixed-gain linear amplifier. Gain is  $\frac{-\mathbf{Z}_f}{\mathbf{Z}_i}$ .

(b) "See-Saw" operation of operational amplifier. System appears to pivot about a fulcrum (the null point B) whose "location" is determined by  $Z_f/Z_i$ .

Let's examine the mechanism by which this works. Referring again to Figure 2, we apply a voltage to point A, causing current to flow through  $Z_i$ . Were it not for the operational amplifier, this current would also flow through  $Z_f$  and to ground through the low impedance at point C, making  $Z_i$  and  $Z_f$  a voltage divider, and raising the voltage at point B. However, the operational amplifier operates to hold the voltage at point B (the - input) at ground potential. To do this, it must supply at point C a voltage which will cause a current to flow through  $Z_f$  which will just balance the current in the opposite direction flowing

through  $Z_i$ . When point B is thus held at ground potential, the voltage across  $Z_i$  is obviously equal to the applied voltage at A.

### **Output Voltage is** Input Current $\times$ Impedance of $Z_f$

The current through  $Z_i$  is equal to the applied voltage at A divided by the impedance (in this case, resistance) of  $Z_{i}$ , or  $E_{in}/Z_i$ . This same value of current must flow in the opposite direction through  $Z_f$  in order to keep point B at ground. The voltage at point C, then, must be  $E_{in}/Z_i$ (which is the value of the current in  $Z_f$ ) multiplied by  $Z_f$ . The output is inverted (of opposite polarity) from the input, so we say that  $E_{out} = (-E_{in}) \left( \frac{Z_f}{Z_i} \right)$ , and the voltage gain of this amplifier configuration is seen to be  $-\frac{Z_f}{7}$ .

### **See-Saw Operation**

As indicated in Figure 2-b, the operational amplifier with resistive input and feedback elements acts in see-saw fashion, the amplifier moving the output end of the seesaw in response to any motion of the input end, causing the system to pivot about an imaginary fulcrum, which is the "sensing point" (- input). The distance from the near end to the sensing point or fulcrum corresponds to the  $Z_i$ or input resistor, and the distance from the fulcrum to the far end corresponds to  $Z_{f}$ . The motion of the far end depends on the motion of the near end and the ratio of the two distances. This analogy suggests that the operational amplifier may be used to solve dynamic problems in mechanical engineering, and so it can. One of the principal uses of operational amplifiers has been in the rapid solution of complex mechanical or hydraulic problems by means of electronic analogs of mechanical or hydraulic systems: operational amplifiers are the basic components of an analog computer.

As may be expected, simple linear voltage amplification by precise gain factors is, though useful, not by any means the limit of the operational amplifier's capabilities.

#### Capacitor as Z<sub>i</sub> Senses Rate-of-Change

Remembering that an operational amplifier with a resistor as a feedback element responds with an output voltage equal to the product of the input current and the feedback resistance, let's consider what happens if a capacitor is used instead of a resistor as  $Z_i$  (Figure 3).

The current through a capacitor is proportional to the rate-of-change of the voltage across the capacitor. A steady state DC voltage across a capacitor (assuming an "ideal" capacitor) passes no current througn the capacitor, so no balancing current need be furnished by the output to hold the — input of the operational amplifier at ground. The output voltage then, is zero.

If the voltage at the input is changed, however, the change causes a current to flow through capacitor Z<sub>i</sub>. The amount of current that flows is directly proportional to



Figure 3. Operational Amplifier as Differentiator. Output is proportional to rate-of-change of input voltage.  $E_{out} = \frac{-dE_{in}}{dt} \times RC$ .

the capacitance of  $Z_i$  times the **rate of change** of the input voltage.

Let's assume that the potential at point A is +100 v DC, and that we change it smoothly to +95 v DC in five seconds. This represents a rate of change of one volt per second, the change taking place over a period of five seconds. If the value of  $Z_i$  is 1  $\mu$ f, then, a current of -1microampere will flow through  $Z_i$  for those 5 seconds.

The operational amplifier will cause an equal and opposite current to flow in  $Z_f$ . If we select a value of 1 megohm for  $Z_f$ , the one microampere current necessary to balance the circuit will require +1 v to appear at the output of the operational amplifier, during the time that  $1\mu$ a current flows through the capacitor.

This operation is **differentiation:** sensing the **rate-ofchange** of an input voltage, and providing an output voltage proportional to that rate of change.

The actual relationship of output to input is this:  $E_{out} = -\left(\frac{dE_{in}}{dt}\right)$  (RC), where the expression  $\frac{dE_{in}}{dt}$  indicates the rate of change (in volts per second) of the input signal at any given instant, and R and C are  $Z_f$  and  $Z_i$  respectively.

In our example, we used a constant rate of change, and obtained a constant voltage level out. Had the rate been less even, the output signal would have demonstrated this dramatically with wide variations in amplitude. The differentiator senses both the rate and direction of change, and is very useful in detecting small variations of slope or discontinuities in waveforms.

### Differentiator Has Rising Sine Wave Response Characteristic

In responding to sine-waves, the differentiator has a rising characteristic directly proportional to frequency, within its own bandwith limitations (see chart page 6). The output voltage is equal to  $(E_{in})$  ( $2\pi f$  RC), and the output waveform is shifted in phase by  $-90^{\circ}$  from the input (the phase shift across the capacitor is actually  $+90^{\circ}$ , but the output is inverted, shifting it another 180°).

### Capacitor as $Z_f$ Senses Input Amplitude and Duration

If we interchange the resistor and capacitor used for differentiation, and use a resistor for  $Z_i$  and a capacitor for  $Z_f$  (Figure 4) we obtain, as might be expected, the exact opposite characteristics from those obtained above. While in differentiation we obtained an output voltage proportional to the rate of change of the input, by swapping the resistor and capacitor, the output signal becomes a rate of change which is proportional to the input voltage.

This characteristic allows us to use the operational amplifier for integration, since the instantaneous value of output voltage at any time is a measure of both the amplitude and duration (up to that time) of the input signal—to be exact, a sum of all the amplitudes, multiplied by their durations, of the input waveform since the start of the measurement.

Here's how integration works: Let's assume the conditions of Figure 4 ( $Z_i = 1 \text{ meg}, Z_f = 1 \mu f$ ), and an input signal level of zero volts. No current flows through  $Z_i$ , so the operational amplifier needs to supply no balancing current through  $Z_f$ . Suppose now we apply a DC voltage of -1v to  $Z_i$ . This will cause a current of  $-1 \mu a$  to flow in  $Z_i$ , and the operational amplifier will seek to provide a balancing current through  $Z_f$ . To obtain a steady current of  $+1 \mu a$  through 1  $\mu f$ , the operational amplifier will have to provide a continually rising voltage at the output, the rate of rise required being 1 volt per second. It will continue to provide this rate of rise until the input voltage is changed or the amplifier reaches its swing limit ("bottoms out"), or approaches its open-loop gain.

Now, this rate-of-rise, though helpful in understanding the mechanism by which the operational amplifier performs integration, is not the "answer" we seek from an integrator. The significant characteristic is the exact voltage level at a certain time, or after a certain interval.

### **Integrator Holds Final Level Until Reset**

Before the amplifier reaches its output limit, suppose we remove the input voltage to  $Z_i$ . The output does not return to ground, but remains at the level it reached just before the signal was removed. The rate of rise has stopped because the necessity for providing  $\pm 1 \mu a$ through  $Z_f$  to maintain the null at the – input has been



Figure 4. Operational Amplifier as Integrator. Output rate of change is proportional to input voltage.  $\frac{dE_{out}}{dt} = \frac{-E_{in}}{RC}$ , or  $E_{out} = \frac{-1}{RC} \int E_{in} dt$ . RC in the example here is 1 second. Output, then, is 1 volt per second per volt input, and most important — the output level at anytime is one volt per volt-second input.

removed. With an ideal capacitor and amplifier, the output voltage would remain at the last level reached indefinitely, until an input signal of the opposite polarity was applied to  $Z_i$ , and a negative-going rate of change at the output was required to maintain the null at the – input.

If the positive input signal is greater than our original -1 volt, it will take less time for the output voltage to reach zero than it originally took to rise. If the positive signal is smaller, it will take more time.

The absolute output level of the integrator at the end of some interval is the sum of the products of all the voltages applied to  $Z_i$  since the output was at zero, times the durations of these voltages, that sum divided by -RC.

### Interpreting Answers Obtained From Integrator

The mathematical expression for the output level reached in a given interval of time  $(T_2 - T_1)$  is as follows:

$$E_{out} = \left(\frac{-1}{RC}\right) \int_{T_1}^{T_2} E_{in} dt$$

The integral sign indicates that the value to be used is the **sum** of all of the products ( $E_{in} X dt$ ) shown, between the limits ( $T_1$ ,  $T_2$ ) noted. The expression "dt" indicates infinitely small increments of time.

It is not necessary, however, to understand and be able to manipulate expressions in integral calculus to understand and make use of an operational amplifier integrator.

The integrator provides a voltage output proportional to the net number of volt-seconds applied to the input. If the total volt-seconds of one polarity is equalled by those of the opposite polarity, the output level at the end of the selected interval will be zero. Let's look at some examples.

#### Simple Example of Data From Integrator

First, we'll assume the signal we want to integrate is a simple one-volt positive pulse of one second duration (Figure 5). The sum of all voltages times durations between  $T_1$  and  $T_2$  is one volt-second. Using 1 megohm and 1 microfarad for  $Z_i$  and  $Z_f$ , the operational amplifier output will fall at the rate of one volt per second  $\left(\frac{-E_{in}}{RC}\right)$  for one second, reaching -1 v when the pulse ends, and remaining at that level.

In reading this output level at  $T_2$  we know that the input signal has amounted to 1 volt-second during the interval  $T_1$  to  $T_2$ . Note also that a later observation, at  $T_3$ , gives the same answer, since  $E_{in}$  has been 0 between  $T_2$  and  $T_3$ .

#### More Complex Cases

Now, take the more complicated case of the waveform in Figure 6-a. Its four voltage levels, of different duration, cause the integrator output to fall at four different rates, reaching a final level representing the total number of volt-seconds contained in the waveform. It should be apparent now that the integrator can measure the total volt-seconds contained in even the very complex waveform of Figure 6-b—something that would be difficult to measure by direct observation of the waveform. This type of operation is often referred to as "taking the area under the curve," since the area underneath a waveform plotted against time (i.e., the area bounded by  $T_1$ ,  $T_2$ , the waveform and the line representing 0 volts) is the number of volt-seconds involved. Note, too, that we needn't wait for  $T_2$  to obtain a reading: the instantaneous value of Eout at anytime is proportional to the input volt-seconds up to that time.

### Using Different Values of R and C

In the cases we've used for illustration, RC was 1 ( $10^6 \times 10^{-6}$ ), and the numerical value of the output voltage at the end of the integrating interval was the number of volt-seconds in the input waveform. Using other values of R and C requires some additional calculation. To find the actual input volt-seconds, multiply the output voltage by (-RC). Example: R is 200 k, C is .01  $\mu$ f and the output voltage after the selected interval is -2.5 volts. Multiplying -2.5 by ( $-2 \times 10^5 \times 1 \times 10^{-8}$ ) gives us  $5 \times 10^{-3}$ , or 5 millivolt-seconds, positive polarity. Note that because of the polarity-reversal in the amplifier, we multiply by (-RC), to obtain the proper sign in the answer.



Figure 5. Simple case of integrating 1-volt-second pulse. Integrator does not improve measurement accuracy in so simple a case.

#### Measuring Ampere-Seconds (Coulombs)

To measure ampere-seconds,  $Z_i$  is omitted, and the current source is applied directly to the - input. The output level reached in a given time  $(T_2 - T_1)$  is  $\frac{-1}{C} \int_{T_1}^{T_2} I_{in} dt$ .



Figure 6. Integrating more complex waveforms to determine "area under the curve" between  $T_1$  and  $T_2$ . Note that in (c) the negative portion of the input waveform *reduces* the net integral.

### Integrator Response to + and - Signals

If a waveform to be integrated contains both positive and negative polarity portions during the integrating interval, the output will be proportional to the **difference** between the volt-seconds of each polarity, the integrator being an averaging device. If it's desired to add the two polarities instead of allowing them to be subtracted, it is necessary to precede the integrator with an "absolutevalue amplifier" (full wave rectifier) which inverts one of the polarities.

### Necessity to "Reset" Integrator After T<sub>2</sub>

The "integrating interval" ( $T_1$  to  $T_2$ ) has been mentioned several times. Because we frequently deal with repetitive signals, and continued integration of a waveform which is not perfectly symmetrical with respect to zero volts will eventually drive the operational amplifier to its output voltage limit, it's desirable to have some way of returning the output to zero at or after  $T_2$ , the end of the desired interval.

For slow work, a pushbutton which can be used to discharge  $Z_f$  manually is usually sufficient. Other circuits which may be used to perform this function automatically are shown in the applications section, page 2-4. Where the integrating interval is quite short, RC networks may be placed around  $Z_f$  to return the output level to 0 v through a time constant much longer (e.g.,  $100 \times$ ) than the integrating interval.

In the Type O Unit, the "Integrator LF Reject" switch positions perform this function whenever  $Z_f$  is set to a capacitive value.

Since the LF Reject circuit operates continually to return the integrator output to zero, it is necessary not only to keep the integrating interval short with respect to the LF Reject time-constant, but also to measure  $E_o$  before it has had a chance to decay, whenever these circuits are used. The value of resistors used in the circuit will also limit the maximum output obtainable for any given amplitude input  $\left(Max \frac{E_{out}}{E_{in}} \approx \frac{R_r}{Z_i}\right)$ , where  $R_r$  is the resistance of the LF Reject circuit.



Figure 7. Average Gain-Frequency characteristics for integration and differentiation.

### Reset or LF Reject Imperative When $Z_f = C$ is Small

Use of resetting or LF reject circuits is usually imperative when small values of C are used for  $Z_f$ , since the small amount of grid current which flows in the – input grid even in the absence of an input signal is sufficient to cause a relatively rapid rise in output voltage as the operational amplifier tries to hold the – input null with balancing current through  $Z_f$ .

#### **Response of Integrator to Sine Waves**

For sine-waves, the gain of the integrator varies inversely with frequency, the actual gain being  $\frac{-1}{2\pi f RC}$ , except as limited by the open-loop DC gain (at low frequencies) and the open-loop gain-bandwidth product at high frequencies (see Figure 7). At low frequencies, the gain becomes less than the formula would indicate, the effect becoming noticeable at approximately the point where the formula indicates a gain of approximately 1/3 the open loop gain. At high frequencies, the error becomes significant above approximately 1/10 of the open-loop gain-bandwidth product. Except as limited above, the integrator shifts the phase of the input sine wave by +90°.

#### Use of The + Input

Many operational amplifiers (including those in the Type O unit) provide access to a non-inverting input, referred to as the + grid or + input. A positive-going signal injected at this point produces a positive-going signal at the output. Conventional identification of + and - inputs is shown in Figure 8.



Figure 8. Identification (a) of + and - inputs of an operational amplifier. If only one input is shown (b), it is always assumed to be the - input.

If the output is connected directly to the - input, the operational amplifier becomes a non-inverting gain-ofone voltage amplifier for a signal applied to the + grid, with very high input impedance and very low output impedance.

### Non-Inverting Amplifier With Gain > 1

With less than 100% negative feedback (Figure 9), obtained by putting the – input on a voltage divider between the output and ground, gains of greater than one may be realized, the actual gain being  $\frac{R_i + R_f}{R_i}$ , or 2 where  $R_i = R_f$ .



Figure 9. Gain of Two Using + Input. Very high input resistance  $(> 10^9 \ \Omega)$  for signals on the order of 1 v amplitude is possible. Other values of gain may be obtained using different ratios of  $R_i$  and  $R_f$ .

Feedback applied to the + input from the output is positive feedback, which tends to raise the input impedance of the + input toward infinity as the amplitude of the feedback approaches the amplitude of the input signal. If the loop gain (feedback amplitude compared to signal amplitude) exceeds 1 for any frequency, the amplifier becomes unstable (negative input resistance) and will oscillate at that frequency. If the loop gain exceeds 1 at DC, the amplifier will swing to its output voltage limit and stay there. The + input is useful for applications combining positive and negative feedback, and for use of the operational amplifier as an oscillator, waveform generator or multivibrator. The + input may also be used to provide a balanced or differential input, in which the operational amplifier responds only to the instantaneous difference between the signals applied to the + and - inputs. Other uses are suggested in the applications section.

#### **Operational Amplifier Limitations**

In performing linear operations with an operational amplifier, it is necessary to recognize and allow for the limitations of the amplifier and technique used, to obtain accurate results. The chief limitations are:

- 1. Open-loop gain.
- 2. Gain-bandwidth product.
- 3. Grid current (chiefly of concern
- during integration). 4. Output current and voltage capability.
- 5. Signal source impedance.

#### 1. Open Loop Gain

The accuracy of all operations is ultimately limited by the open-loop gain of the amplifier, which determines how closely the amplifier is capable of holding the — input null. An amplifier with infinite gain would provide a null of exactly 0 volts, and the impedance at the — input (using feedback) would be exactly 0 ohms.

With finite gain, the - input does not quite null, and does not appear as 0 ohms. With an open-loop gain of  $A^*$ , the - input moves  $\frac{1}{A}$  times the output voltage swing, and appears as an impedance which is  $\frac{Z_f}{1-A}$ . If this voltage swing of  $\frac{E_{out}}{A}$  is a significant fraction of the input signal  $E_{in}$ , or if the impedance  $\frac{Z_f}{1-A}$  is a significant fraction of  $Z_i$ , there will be a definite output signal error in addition to the error introduced by the tolerances of  $Z_i$  and  $Z_f$ . The exact value of this error is  $1 - \frac{A}{A-1-\frac{Z_f}{Z_i}}$ .

So long as  $\frac{Z_f}{Z_i}$  is small and A is large, the error is not serious. For instance, using the O-Unit's operational amplifiers (at low frequencies where A = -2500) in the simple fixed-gain amplifier mode with resistors for  $Z_i$  and  $Z_f$ , we see that the error for a gain of 1 ( $Z_i = Z_f$ ) is only  $1 - \frac{2500}{2502}$ , or less than 0.1%. For a gain of 100, however, the error becomes  $1 - \frac{2500}{2601}$ , or almost 4%. (A gain-correcting resistor is automatically shunted across  $Z_i$  in the O-Unit when the internal  $Z_i$  resistor is set to 10 K and  $Z_f$  to 0.5 or 1.0 meg.

Using external components, similar precautions should be observed when high gain is required).

#### \*NOTE

Common usage in the analog computer field assigns a **negative number** to the open-loop gain between the — input and output (and a positive number to the gain from the + input). Therefore, in calculating values from formulas involving A and the — input, it is necessary to keep in mind that **A** is a negative number, and the expression "1 - A" for instance, when A is - 2500, equals + 2501, not - 2499.

One simplification has been made in this article.

Closed-loop gain, commonly expressed as

$$\frac{-Z_f}{Z_i} \left[ \frac{1}{1 - \frac{1}{A} \left( 1 + \frac{Z_f}{Z_i} \right)} \right]$$
  
has been reduced to  
$$\frac{-Z_f}{Z_i} \left[ \frac{A}{A - 1 - \frac{Z_f}{Z_i}} \right]$$

It may also be written

$$\frac{-Z_f}{Z_i} \left[ \frac{1}{1 - \frac{1 + Z_f/Z_i}{A}} \right]$$

if this seems to indicate the effect of A on accuracy more clearly.

#### Approximate Error Calculation Using C for Z<sub>i</sub> or Z<sub>f</sub>.

Since it is not easy to assign a single impedance value in the error formula for  $Z_f$  or  $Z_i$  when one of them is a capacitor, it is convenient to use the ratio  $E_{out}/E_{in}$ , representing the actually obtained voltage gain, to compute the approximate error. The error  $\epsilon$  is found by this formula:

$$\epsilon = 1 - \left[\frac{\frac{E_{out}}{E_{in}} - A}{1 - A}\right], \text{ or, more simply,}$$

$$\epsilon = \frac{1 - \frac{E_{out}}{E_{in}}}{1 - A}, \text{ where } A, \text{ as before, is the}$$

open-loop gain, and  $E_{out}/E_{in}$  is the actually obtained voltage gain (Don't forget—both A and  $E_{out}/E_{in}$  are **negative numbers**). Example: where A is -1000 and the observed  $E_{out}/E_{in}$  is -50, the error has been 51/1001 or 5.095%. The output "-50" represents, then, 94.905% of the correct value, and the correct value is -50/0.94905, or almost -52.7.

For convenience, you may want to rearrange the terms as shown below, to determine how large an output signal to allow, for a given input and an arbitrarily selected maximum error:

$$\frac{Max E_{out}}{E_{in}} = 1 - \epsilon(1 - A)$$

Using the Tektronix Type O Operational Amplifier for integration, for instance, to keep error due to amplifier gain below 1%, the output voltage during or at the end of the integrating interval should not exceed the average value of the signal being integrated by more than a factor of  $1 - (.01 \times 2501)$ , or -24, for low frequencies. The same limitation should be observed during differentiation.

The minimum open loop gain required by an operational amplifier to operate within a given error even at "zero"  $(\epsilon - 1)$ 

 $Z_f/Z_i$  is  $A = \frac{(\epsilon - 1)}{\epsilon}$ , where  $\epsilon$  is the error expressed as a decimal fraction (.01 = 1%, 0.1 = 10%, etc.).

Where  $Z_f/Z_i$  is a finite number, the minimum open-loop gain required for a given maximum error is

$$A = \frac{(\epsilon - 1)(1 + Z_f/Z_i)}{\epsilon}.$$

The application of these formulae will be most useful in observing gain-bandwidth limitations, discussed below.

#### 2A. Gain-Bandwidth Product:

The gain-factor A varies with frequency, and it's important to know what the effective value of A is for the frequencies or signal frequency components being used. In the Type O, the gain factor A is constant (-2500) only

to about 1 KC, dropping off to -1000 at about 15 KC, and reaching a value of -1 at approximately 15 Mc.

The error introduced by the gain factor, then, becomes greater with frequency, and for accurate measurements the allowable ratio of  $E_{out}$  to  $E_{in}$  must be reduced as higher-frequency information is processed.

Although the drop in gain at high frequencies in the open-loop bandwidth characteristic follows the same pattern as that of an integrator, it must be remembered that this response is obtained **without** input and feedback elements. The effect of this rolloff will **add** to the effect of the integrating components, altering their effect.

At a frequency approximately 1/10 of the open-loop gain-bandwidth product, the open-loop gain will be insufficient (on the order of 10 or so) to provide accuracy better than 9% even at "zero" closed loop gain, or 16.7% when  $Z_f/Z_i$  is 1, (i.e.,  $E_{out} = E_{in}$ ). Above 1/10 of the open-loop gain-bandwidth product, answers will be only approximate, although the data may be useful for frequencies as high as 1/3 of the open-loop gain-bandwidth product.



Figure 10.

(a) Variation in open-loop gain after application of signal, for 0-Unit.
 (b) Nomograph for determining A-FACTOR, ERROR and Z<sub>f</sub>/Z<sub>i</sub>. Given any two factors, the third may be found. (Lay straightedge across chart.)

### 2B. Gain-versus-Time Factor – Complex Waveshapes:

In working with pulse and complex waveforms, openloop gain in terms of frequency is not too useful. Instead, the open-loop risetime characteristic, Figure 10(a) may be used to determine the time after the start of a signal at which the A-factor has reached a sufficiently high level to permit the desired accuracy. Figure 10(b) shows the A-factor required to support a given accuracy at a given attempted or "virtual" gain  $(Z_f/Z_i)$ .

"Virtual gain" (roughly,  $E_{out}/E_{in}$ ) in the case of integration or differentiation is the ratio between the RC time constant chosen and the time interval involved in the operation.

In the case of integration, virtual gain  $G_v$  will be:  $G_v = \frac{-t}{RC}$ , where t is the integrating interval—i.e., that span of time during which the integral continues to increase. The larger the values of integrating components, the smaller the virtual gain.

In the case of differentiation, the virtual gain will be:  $G_v = \frac{-RC}{t}$ , where t is that span of time during which the input signal has its steepest slope. The larger the values of differentiating components, the higher the virtual gain.

As can be seen from Figure 10(b), holding virtual gain to a value of one or so is a good general rule of thumb for accurate measurements.

#### NOTE

It should be kept in mind that the values of the internal 10 pf and 100 pf  $Z_f$  and  $Z_i$  components of the O-Unit have been adjusted under **dynamic conditions**, to compensate partially for the time-dependent errors indicated in Figure 10(a). For greatest measurement accuracy, standard waveforms involving a similar time interval and virtual gain as the signal to be measured should be used to determine the probable measurement error, or to trim the values of external components to provide direct readings for the particular waveform to be measured (comparison method). However, correction of this sort can be optimized for only a limited range of waveforms, and cannot extend the operating range of the system indefinitely.

#### 3. Grid Current:

During integration, any grid-current flowing in the – input will be integrated along with the current through  $E_{in}$ , except when this current is bucked out through a DC path from output to input (in the Type O Unit "Integrator LF Reject" circuits are provided for this purpose).

The amount of grid-current flowing in the —input circuit may be determined by switching out any "LF Reject" circuit and measuring the length of time it takes the output signal to rise or fall 1 v with a capacitor as  $Z_f$  (no signal input). The grid current  $I_g$  is found by this formula  $I_g = \frac{C}{t}$ , where t is the time (in seconds) required for the output to move one volt.



Figure 11. Operational amplifier connected as gain-of-one, non-inverting amplifier to drive low-input-impedance differentiator from high impedance signal source. If output current capability is 5 ma (as in the 0-Unit), driver amplifier will reproduce faithfully an input rate-of-change as high as 0.5 v/μsec into .01 μf, several orders of magnitude in excess of the amount necessary to obtain usable output from the differentiator. Component R<sub>1</sub> combines with the input C of the first operational amplifier to compensate its response in this mode. R<sub>2</sub> limits the current to the second operational amplifier to prevent overdriving, and reduces noise components possibly introduced by first amplifier.

In the Type O unit, a grid current of 300 picoamperes at the - grid is normal.

It is not usually practical to try to adjust an operational amplifier for "zero" input current, since this condition is not as stable as a fixed value of grid-current appropriate to the input tube type and amplifier design. In lowfrequency operational amplifiers using electrometer tubes as input elements, extremely low values of input grid current can be obtained with good stability. In wide-band units, higher values must be tolerated.

Once the grid current has been set to a known value, its effect on a given integrating operation can be computed. So long as the value of  $l_y$  is very small compared to the average value of the current through  $Z_i$  during the integrating interval, the effect of  $l_g$  can be largely ignored.

#### 4. Output Current and Voltage Limits:

Any operational amplifier is limited in the amount of current and voltage it can deliver to its feedback network and any external load with good linearity. If these limits are exceeded during any part of an operation, the accuracy of that part of the operation, at least, will be impaired.

In the case of the O-Unit, maximum output is  $\pm 50$  v and  $\pm 5$  ma. At high speeds, the maximum rate-of-change at the output will be limited by the available current, and should not exceed 20 v per  $\mu$ sec, when loaded by the O-Unit's oscilloscope preamplifier (47 pf) and 10 pf of other loading (e.g.,  $Z_f$ ).

#### 5. Input Signal Source Impedance:

A part of  $Z_i$ , the input element of the operational amplifier circuit, is the source impedance of the signal being processed. Linear operations using precision input and feedback components will be accurate only if the source impedance of the signal is very small compared to the impedance of the input component, or the value of the input or feedback component is trimmed to allow for the impedance of the signal source.

Where trimming of components is not practical, or the signal source impedance is not resistive and linear, the usual practice is to process the signal first through a gain-of-one, high input-impedance, low-output-impedance amplifier, such as that shown in Figure 11, to obtain a low-impedance signal source for the desired operation. In the case shown, the output impedance of the first amplifier is **too** low, making it capable of overdriving the second. A current-limiting resistor helps keep down noise as well as prevent overdriving.

### Shunt Impedance Across - Input

Though we tend to think of the - input or - grid as a "virtual ground", and that impedances between this point and ground will have a negligible effect on the performance of the operational amplifier, this is only partially true. The true impedance of this point is  $Z_f/(1 - A)$  and that instead of holding a perfect voltage null (as would be the case if A were infinite), its voltage excursions actually amount to  $E_{out}$  /A.

So long as A is large and  $Z_f$  has a fairly low value, an impedance across the – input which is large compared to  $Z_f$  or  $Z_i$  will have little effect on performance. However, in high-frequency work, where the effective value of A is low, more and more care must be exercised to assure that shunt impedances – particularly capacitive reactances, which become lower with increasing frequency—do not interfere with the operation (Figure 12).

The general expression for the closed loop gain of an operational amplifier  $\frac{E_{out}}{E_{in}} = \frac{-Z_f}{Z_i} \begin{bmatrix} A \\ A - 1 - \frac{Z_f}{Z_i} \end{bmatrix}$  may be

modified as follows to show the effect of shunt impedance  $Z_s$  across the – grid:

$$\frac{E_{out}}{E_{in}} = \frac{-Z_f}{Z_i} \left[ \frac{A}{A - 1 - \frac{Z_f}{Z_i} - \frac{Z_f}{Z_s}} \right],$$

keeping in mind that A is a negative number. As you can see, unless  $Z_s$  is very high compared to  $Z_f$ , its effect on accuracy may become comparable to that of  $Z_f/Z_i$ .

The terms in the above equation can be rearranged to show the effect of  $Z_s$  as related to  $Z_i$ :

$$\frac{E_{out}}{E_{in}} = \frac{-Z_f}{Z_i} \left[ \frac{A}{A - 1 - \frac{Z_f}{Z_i} \left( \frac{Z_s + Z_i}{Z_s} \right)} \right]$$



#### Figure 12.

(a) Shunt Impedance across - input. Where Z<sub>f</sub> is large compared to Z<sub>i</sub> and Z<sub>f</sub>, and open-loop gain A is high, effect of Z<sub>f</sub> is negligible.
(b) Where Z<sub>i</sub> or Z<sub>f</sub> is a resistor, and particularly if a large (> 100k) value, more serious errors may be caused by capacitance from the resistor body (highest impedance point) to ground, and, in the case of R<sub>i</sub> during integration, end-to-end capacitance of R<sub>i</sub>. Time constants involved in shunt capacitance C<sub>is</sub> and C<sub>fs</sub> are approximately RC/4.

### Correcting For The Effects of Stray C

In high-speed work, the accuracy of operations will be affected by  $C_s$  during the start of an operation when the effective value of A is low, and also by the end-to-end and distributed capacitance to ground of the resistors used for  $Z_i$  and  $Z_f$  (Fig. 12b).

To correct for strays and the variation in A, the 100 pf and 10 pf values of  $Z_i$  and  $Z_f$  in the Type O operational amplifiers are factory adjusted under dynamic conditions, and no external compensation of these components is generally required. If it is intended to use values in this range externally, they should be padded or trimmed as necessary under conditions similar to those of the contemplated measurements. The resistors used as  $Z_i$  and  $Z_f$ , however, can be given only partial compensation internally, since the optimum value of compensation varies with the application. For this reason, it is usually necessary in dealing with shortduration or high-frequency signals to add external compensation to  $R_f$  or  $R_i$  when these components are used in amplification and differentiation.

Figure 13 illustrates the corrections necessary to improve operational accuracy for each of the three basic operations.

Note, however, that except in the case of straight amplification (Fig. 13A), the compensation itself introduces possible errors which must be recognized and allowed for in interpretation of results.

#### **Compensated Amplifier**

In the case of amplification, selecting small values of capacitance (on the order of 2-25 pf) for  $C_1$  and  $C_2$ , the closed-loop risetime can be made to approach the slope of the open-loop risetime (Fig. 9), providing a gainbandwidth product about equal to the open-loop gainbandwidth product. Without compensation, the amplifier may typically achieve perhaps only 1/20 of this figure.

#### **Compensated Differentiator**

Without compensation, the differentiator (Fig. 13B) may respond to a sudden change in  $dE_{in}/dt$  by overshoot, followed by sinusoidal ringing, due to the fact that excess output voltage must be developed to charge via  $R_f$  the input capacitance and the distributed stray capacitance of  $R_f$  itself, as well as provide the current needed to obtain a null at the – input. As soon as the strays are charged, however, the excess current through  $R_f$  upsets the null, and the output must swing in the opposite direction to reestablish the null and discharge the capacitance associated with  $R_f$ —hence the ringing. A small capacitance across  $R_f$  provides the current needed to establish the null at the start of the waveform without having to develop excess voltage across  $R_f$ .

### Differentiator Compensation Limits Initial Accuracy

The presence of this capacitance, however, limits the output voltage maximum to approximately  $\frac{-dE_{in} C_i}{C_2}$ . After an abrupt change in the input waveform, then, when  $dE_{in}$  is small, but  $\frac{dE_{in}}{dt} \times RC$  may be quite large, the output voltage limitation of  $\frac{-dE_{in} C_i}{C_2}$  may result in a significant error. The solution in this case is to select a larger value of  $C_i$  and smaller values for  $R_f$  and  $C_2$  (keeping the  $R_f C_i$  time constant the same) to minimize the error, and keep its duration as short as possible.

#### Integrator Compensation Rarely Needed

Failure of the integrator to start integrating at the proper rate at the beginning of a fast-rise pulse or after a sharp step in the input waveform is usually due largely to the distributed stray capacitance to ground in  $R_i$ . This is infrequent; more commonly the error is in the opposite direction because of excessive capacitive coupling of the input waveform around  $R_i$  into the – grid directly, pro-

ducing a step of approximately  $\frac{-dE_{in} C_{in}}{C_f}$ .

The first (- error) waveform in Fig. 13C was obtained by deliberately putting a ground plane near the center of a 9 Megohm  $R_i$  and carefully shielding the - grid. Removing the ground plane and shield produced the third (+ error) waveform, using the same input signal (a rectangular pulse) and components.

Normally, the "undercompensated" effect would only occur when  $R_i$  is composed of several resistors in series, or when a high-value potentiometer is used as, or in series with,  $R_i$ .

The solution usually is to select a smaller value for  $R_i$ and a larger one for  $C_f$ , to maintain the same time-constant. Normally, if a signal source is capable of driving a large value of  $R_i$  with capacitive compensation, it is also capable of driving a smaller value of  $R_i$  without compensation. Theoretically—as when a potentiometer is used in conjunction with  $R_i$ —it is possible to compensate the RC losses in  $R_i$  by shunting  $R_i$  with a series RC network of the proper time-constant, or by using a small value of R in series with  $C_f$ . In practice, these added components usually add nearly as many stray-C problems as they cure, and "compensation" of this sort is not recommended. Compensating with simple capacitance across  $R_i$  produces a "step" error at any abrupt transition, and usually an error of greater magnitude than the one to be corrected.

If  $R_i$  is a single component, an environmental "guard" driven by the input signal (e.g., a short piece of wire soldered to the input end of  $R_i$  and dressed near the body of the resistor) can make some correction, but its use requires more complete shielding of the – grid and the – grid end of  $R_i$ .

### Using "Standard" Waveforms For Comparison

The use of standard waveforms (pulses and ramps) with known parameters, is of considerable help in adjusting compensation and assuring best accuracy for critical measurements near the limits of the instrument's capabilities. For many purposes, such "standard" waveforms may be obtained by attenuation of the oscilloscope gate and sawtooth output waveforms. Selection of time and amplitude parameters close to those of waveforms to be measured will give best assurance against possible system errors.



Figure 13.





### **BASIC OPERATIONS-I**









Notes



















<sup>\*</sup>Selected and matched. See appendix.









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Notes






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EoB (as above)

-0 R<sub>1</sub>

 $\xi R_2$ 

B

R

-22































## Semiconductors:

Some of the semiconductors referred to in these notes are non-registered types selected for special characteristics and available only from the original manufacturers.

In a few cases (Types 6045, 6061, 6075), the diodes referred to are made to Tektronix specifications, and may be procured in small quantities from Tektronix if suitable equivalents are not locally available.

## Germanium, General Purpose Signal Diodes:

, , , , , , , , , , , , , , , , , , , ,	PIV	Fwd ma	Leakage	Procure
T12G (Transitron, Raytheon) HD2607 (Hughes) CGD-1075 (Clevite)	75v	20 ma	30 μa@10v	Locally
Туре 6075	25v	40 ma	10 µa@ 6v	Tektronix (152-0075-00)
ED-2007 (Erie Resistor Corp.)	15v	5 ma	10 µa@ 6v	Locally
DR-746 (General Instruments) HD2948 (Hughes)	10-25v	40 ma	10 µa@ 6v	Locally
T13G (Transitron)	25v	50 ma	2 μa@10v	Locally

## Silicon Signal Diodes (Low leakage or high voltage):

<b>J</b>		•		
HD5000 (Hughes)	10v	12 ma	0.2 μa@ 5v	Locally
6045	125v	40 ma	.002 μa@ 9v	Tektronix
				(152-004 <b>5-</b> 00)
6061				
RD2594 (Rheem)	200v	100 ma	5 μa@175v	Tektronix
ED2927 <b>(E</b> rie)	2007	roo mu	5 µd@175V	(152-0061-00)
CSD2800 (Clevite) J				•
G 130 (Texas Instruments) "Les Di	in doll "Starkintor"			or Locally
G-130 (Texas Instruments) "Log Di	iode, stabistor.			Locally

# Zener Diodes:

Voltage	Current	Use	Tolerance	Mfr.	Procure
±6v <sup>*</sup>	25 ma	RT6*	±1v	Hoffman	Locally
6v	60 ma	1N753	5.6-6.8v	Texas Instrs.	Locally
7v	30 ma	1N707	6.2-8.0v	Hoffman	Locally
10v	25 ma	¼M10Z10	±10%	Motorola	Locally
15v	15 ma	1N718A	±5%	Pacific	Locally
*					

\*Double Zener

## Transistors:

2N1304 2N1305 General Instrument, Raytheon, Sylvania, Texas Instrs.	Locally
	,
$ \begin{array}{c} 2N706B \\ 2N970 \end{array} \right\} (Motorola, Fairchild) may be substituted for \left\{ \begin{array}{c} 2N1304 \\ 2N1305 \end{array} \right\} in \div 2 Multivibrator circuit $	
For Higher speed operation.	Procure locally
2N1302 2N1303 May also be used in ÷ 2 Multivibrator.	,

## **Selected Diodes:**

The wideband logarithmic amplifier requires careful selection and matching of diodes to obtain calibrated symmetrical output.

If unidirectional operation only is needed, only half the diodes are needed—selected on the basis of the criteria below. If symmetrical operation is needed, the selected diodes must be further matched in pairs.

	Se	Matching Criteria: Match Forward	
Circuit Location, Type	@ I fwd	Select for Forward Drop	Drop Within:
Input ED-2007	10 µa	0.10-0.13v	.004v
(Erie Resistor Corp.)	10 ma	0.35-0.50v	v 10.
Feedback ED-2007	10 µa	0.10-0.13v	.004v
	lma	0.25-0.30∨	v 10.
Feedback HD-5000 (Hughes)	10 $\mu_{a}$	0.25-0.40∨	.10 v
	100 µa	0.35-0.50∨	.01 v

Procure

Typically, it takes about 10 ED-2007's to obtain two matched pairs and 5 HD-5000's to obtain one matched pair. \*

#### Precision Resistors:

Precision carbon-film and metal film resistors in 1% and closer tolerance are available from most electronics suppliers at a cost of 50 cents to \$1.50 each. Use of smallest sizes (1/8w, 1/4w, etc.) compatible with power requirements is recommended for high-speed work. Use of large "meter" type wirewound precision resistors is not recommended for fast work because of excessive capacitance and (in some cases) inductance.

## **Oscilloscope Modifications:**

A slight modification of your oscilloscope may facilitate use of the Type O-Unit for some applications. Extensive modification should not be undertaken without consulting your local field engineer, however.

#### +Gate Output Modification:

For gated-integrator operation, it may be desirable to have the +Gate output from your oscilloscope drop to a negative DC level of 2 or 3 volts between sweeps, to avoid the use of a battery or integrating network to obtain this negative voltage. It is possible to modify your instrument as shown in the following schematic.

To modify the output, add a resistor between the cathode of the +Gate output cathode follower and -150v, the value of the resistor about 60 times the value of the cathode resistor already in the circuit.

## Example:

Type 545A or 535A, +Gate, Sweep A:



\*Yield figures are for diodes manufactured in 1963 and earlier. Yield figures for more recent production approach zero for the ED-2007 because of modified forward characteristics in this type. Instead of HD5000 diodes, 1N3605 diodes (G.E.) may be used with some modification of circuit values. There is no entirely adequate commercially available substitute for the early ED-2007, but by selecting resistor values and accepting some reduction in operational accuracy, adequate performance for many applications may be obtained from selected ED-2007's of current manufacture or from other similar germanium diodes such as the T13G. In most Tektronix oscilloscopes manufactured since 1960, the square-wave calibrator pentode section is a Type 6AU6, and so is individually removable (not so in earlier instruments using the triode-pentode Type 6U8). Removal of this tube causes the calibrator output to fly up to its precalibrated value and hold this DC level. Reference

DC levels from + 0.2 mv to + 100 v may thus be obtained from the front-panel calibrator output connector for use in operational amplifier applications. Current capabilities of this circuit are limited, however. Although up to 5 ma may safely be drawn from the + 100 v position, the high impedance divider may be damaged if the 50 v position is heavily loaded. Accurate calibrated output will be available only into high (meg ohm) impedances.

Negative reference voltages up to -50 v may be obtained by processing the + DC voltage from the modified calibrator through a -1 amplifier (one operational amplifier set up with  $R_i = 1$  M,  $R_f = 1$  M). Care should be exercised to avoid driving the amplifier with more than 50 v or overloading the output (5 ma max).

Where currents in excess of 5 ma are required, external supplies should be used, to avoid overloading the regulated DC supplies in the oscilloscope or damaging internal components.

# Obtaining Time Scaling Factors From Sampling Oscilloscopes

In integrating or differentiating waveforms obtained from the vertical signal output of sampling oscilloscopes, it is essential to preserve a known and stable real-time: equivalent time ratio, or scaling factor. The operations performed by the operational amplifier are in real time; the information contained in the sampling oscilloscope output is in equivalent time.

Unless some means is employed to **force** a fixed real time: equivalent time ratio, the ratio will be dependent on the repetition rate of the signal being sampled, up to about 100 KC; above this point by the sampling-rate limiter in the sampling oscilloscope. An erratic pulse repetition or sampling rate may cause significant errors.

To force a fixed relationship between real and equivalent time, it is only necessary to drive the external horizontal input (external scan) of the sampling oscilloscope with the real-time sawtooth of the oscilloscope used with the operational amplifier. The real-time sawtooth should be properly attenuated so that it provides the **same display** on both the sampling and the real-time oscilloscopes when the sampling oscilloscope vertical signal output is displayed on the real-time oscilloscope. The real-time sweep should be made slow enough to provide about 100 samples/cm or more on the sampling oscilloscope, particularly during differentiation. The horizontal scaling factor under these circumstances becomes the ratio of time/cm switch settings of the sampling and real-time oscilloscopes.

It is not necessary to trigger the real-time oscilloscope sweep; it may be run at any convenient repetition rate. The retrace and hold-off of the real-time sweep will not be blanked on the sampling oscilloscope, so free-running the real-time sweep at a slow time/cm setting is generally most satisfactory.

Vertical scaling factors require no special techniques. However, since accumulated calibration errors and loading effects may be serious in some cases, it's usually advisable to obtain a 1:1 scaling factor as follows:

- 1. Apply several cm of calibrator signal to the sampling oscilloscope input.
- 2. Using the same value  $Z_i$  as will be used in the intended operation, set the operational amplifier for  $\times$  (-1) amplification ( $Z_f = Z_i$ ).
- 3. Adjust the real-time oscilloscope to display the same number of centimeters as the sampling oscilloscope Volts/Cm and Calibrator settings indicate **should** be displayed on the sampling oscilloscope. The vertical scaling factor is now 1:1.  $Z_f$  may now be reset to the correct value for the intended operation. Since DC levels are critical in integration, the sampling oscil-

loscope DC offset (and/or vertical position or DC balance controls, depending on the make and model) should be carefully adjusted so that the vertical signal output is at 0 v DC when the signal waveform is at the reference level. The accuracy of this setting may be confirmed during adjustment of the vertical scaling factor, when the operational amplifier is set for  $\times$  (-1) operation.

### **Bibliography**:

Much valuable information concerning theory, construction and applications of operational amplifiers will be found in the following:

- G. A. Korn and T. M. Korn, Electronic. Analog Computers, McGraw-Hill Book Company, 1956.
- C. L. Johnson, Analog Computer Techniques, McGraw-Hill Book Company, 1956.
- N. R. Scott, Analog and Digital Computer Technology, McGraw-Hill Book Company, 1960.
- A. E. Rogers, T. W. Connolly, Analog Computation in Engineering Design, McGraw-Hill Book Company, 1960.
- 5. A Palimpsest on the Electronic Analog Art George A. Philbrick Researches, Inc. Boston, Massachusetts.

